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EDITED BY

RALPH COPELAND, Ph. D., F.R.S.E., F.R.A.S., ETC.,

ASTRONOMER ROYAL FOR SCOTLAND AND PROFESSOR OF ASTRONOMY IN THE UNIVERSITY OF EDINBURGH.

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CONTENTS.

PREFACE.

THIS volume, the first of the Annals of this Observatory, contains four papers contributed by members of the staff of the former Observatory of Lord Crawford at Dun Echt and of the Edinburgh Royal Observatory.

The first two papers, by Dr. L. Becker, now Professor of Astronomy in the University of Glasgow, contain the results of an endeavour to turn to account the relatively large aperture of the Dun Echt Transit Circle by determining the positions of such faint objects as nebulæ or certain small stars in the immediate neighbourhood of the Pole, which for the greater part are beyond the reach of smaller meridian instruments. The observations of nebulæ embodied in the first paper, apart from their value as a contribution to our knowledge of the positions of a selection of the more symmetrical of these bodies, afford the materials for an interesting comparison between the personal equations of different observers. The chief value of the second paper will probably eventually centre in its accurate determination of the position of the celestial pole at the epoch of the observations.

The paper by Mr. J. G. Lohse contains an account of a new double image micrometer invented by him, and of the observations made with it. The instrument consists of a small heliometer, with a Barlow lens in front, attached to the eye-end of a refracting telescope. The chief advantage of this instrument over Airy's double image micrometer will be found in the fact that the object-glass of the heliometer here intercepts a broad cylinder of parallel rays, whereas in Airy's micrometer the separation of the images takes place near the apex of a cone. This change is advantageous in two most important respects: the loss of light is much less than in the older instrument, and the value of the scale is

almost absolutely uniform in all parts of the field. The actual instrument employed had only one outstanding deficiency—viz., a want of chromatic compensation. To improve the micrometer in this respect, the Barlow lens would require to be specially designed to suit the individual character of the object-glass of the telescope. The instrument permits of combining the optical grasp of a large refractor with the measuring accuracy and other advantages of the heliometer. Moreover, there seems to be no reason why the apparatus should not be adapted to a reflector, as the slight displacements of the speculum would cause no serious inconvenience.

The original observations for these three papers were made at Dun Echt, but the final reductions were completed after the whole outfit of the Dun Echt Observatory was presented to the Edinburgh Royal Observatory through the generosity of Lord Crawford.

The paper by Dr. J. Halm—"Contributions to the Theory of the Sun"—contains the main results of his theoretical researches into the causes of the periodicity of solar phenomena. Brief accounts of the chief principle upon which this new solar theory is founded have already appeared in the *Astronomische Nachrichten* and in *Nature*. The present paper deals with the application of this extremely simple principle to the explanation of some of the more prominent dynamical phenomena exhibited on the surface of the sun, and shows that several important but hitherto unsolved questions within the domain of solar physics may be satisfactorily answered from the point of view advocated in these researches. The Crawford Library of this Observatory, with its extensive collection of works and papers on the subject, afforded invaluable assistance in Dr. Halm's investigations.

RALPH COPELAND.

ROYAL OBSERVATORY,
EDINBURGH, *February* 1902.

OBSERVATIONS OF 217 NEBULÆ MADE WITH THE TRANSIT CIRCLE AT DUN ECHT OBSERVATORY.

By L. BECKER, Ph.D.,

Late Assistant at Lord Crawford's Observatory, Dunecht.

The bulk of these observations, which I began at the suggestion of Dr. Copeland, were made between September, 1886, and May, 1889, during eight months of each year. After the closing of the Observatory in the autumn of 1889, some supplementary observations were added during one lunation in 1890, and during three others at the beginning of 1891. Altogether there are 840 observations of 217 nebulæ; 12 objects were observed once and 46 twice, thus leaving an average of 4·6 observations to each of the remaining 159 nebulæ.

During the three periods of 8 months from 1886 to 1889 there were 96 suitable nights; of these 35 nights yielded each less than 5 observations of nebulæ, 31 nights from 5 to 10, and but 3 nights more than 20. The last months of 1887 count as exceptionally bad, with only 6 observing nights up to January.

It ought to be stated that from the outset the removal of the instruments to a more favourable locality was imminent, and that for this reason some desirable alterations of the instrument were not attempted. This refers especially to the illumination of the wires, which was unsatisfactory, and to the object-glass (aperture 215 mm.), which was covered with yellowish stains.

In Table I. the single positions are collected. Those obtained in the first season, September, 1886, to April, 1887, were observed by the eye-and-ear method. There were 27 transit wires and three declination wires, two of the latter being close together at a distance of 1′ south of the third, which was the one employed. This set of transit wires was replaced in August, 1887, by a frame with only 7 wires, and 2 declination wires were removed, because the nebulæ could be seen only with the greatest difficulty within the wire system. After November, 1887, I chronographed the transits over these 7 wires. Between the two kinds of observations I find the following systematic difference, if only the positions observed on at least 2 occasions by each method are taken into account.

$$\text{II.}-\text{I.} = -0^{s}{\cdot}262 \sec\delta \text{ (76 observations)} - 1''{\cdot}9 \text{ (74 observations)}$$

or excluding the very large and very faint objects :—

$$\text{II.—I.} = -\ 0^{s}{\cdot}233 \sec \delta\ -\ 1''{\cdot}8 \text{ (68 observations).}$$

The observations made from September, 1886, to April, 1887, have been corrected by the latter values before being combined with the later results. The mean values are given in the final catalogue (Table III.) which therefore refers to the system of the chronographed observations.

The accuracy of the results is represented by the following probable errors of one chronographed observation.

$$\Delta\,\alpha \cos\delta = \pm\ 0^{s}{\cdot}15 \qquad \Delta\,\delta = \pm\ 1''{\cdot}9 \text{ for 154 nebulæ.}$$
$$\pm\ 0{\cdot}36 \qquad \pm\ 5{\cdot}4 \text{ for 35 nebulæ; very large or very faint objects.}$$

A comparison of the positions resulting from my chronographed observations with those given in the more recent catalogues of nebulæ will be found in Table II. From this Table I deduce the following systematic differences and probable differences :—

	No Observations Excluded.					Very Large and Difficult Objects Excluded.				
	Systematic Difference.			Probable Difference.		Systematic Difference.			Probable Difference.	
	Δ α cos δ.	Δ δ.	No.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	No.	Δ α cos δ.	Δ δ.
	s	″		s	″	s	″		s	″
Schmidt . . —B	+0·40	+0·9	40	±0·25	±2·8	+0·35	+0·5	36	±0.24	±2.4
d'Arrest (Leipzig) . —B	−0·02	+1·9	59	0·42	7·6	−0·01	+2·0	52	0·37	7·3
d'Arrest (Copenhagen)—B	+0·35	+3·0	172	0·69	9·4	+0·35	+2·6	138	0·73	9·4
Rümker . . —B	+0·45	+2·0	34	0·39	4·0	+0·36	+0·6	25	0·32	3·7
Schönfeld I. . . —B	−0·03	+1·6	82	0·23	2·8	+0·00	+1·0	69	0·22	1·8
Schönfeld II.. . —B	+0·26	+2·4	86	0·21	3·2	+0·28	+1·9	67	0·18	2.5
Schultz . . . —B	+0·27	+1·6	110, 109	0·29	2·6	+0·27	+0·9	88, 87	0·17	1·7
Auwers . . . —B	+0·41	+2·5	15	0·18	1·9	+0·35	+2·3	13	0·16	2·0
Vogel I., II. . . —B	+0·23	+1·2	63	0·21	2·5	+0·26	+0·7	54	0·19	2·2
Engelmann . . —B	+0·42	+3·1	47	0·20	2·9	+0·41	+3·1	46	0·20	2·5
Engelhardt . . —B	+0·43	+1·1	46	0·25	3·4	+0·38	+0·8	39	0·22	2·5
Porter . . . —B	+0·28	−1·5	16	0·29	3·5	+0·29	+0·4	13	0·32	2·5

The systematic difference of the bright nebulæ with a central condensation is essentially the same as that given in cols. 6, 7. It appears that, compared with other observers, I have chronographed the transits of nebulæ $0^s{\cdot}30$ sec δ earlier than those of the fundamental stars in a bright field. This is probably due to the bad definition of the illuminated wires. It will be remarked that the eye-and-ear observations of the first year are devoid of this error. My declinations are $1''{\cdot}2$ too large on an average. The difference between the two catalogues by Schönfeld is confirmed by the comparison with my observations, since 26 nebulæ only of the 82 and 86 occur in both the catalogues. The figures given as the probable difference of the positions ought not to be considered as a measure of the relative accuracy of the several catalogues, because they are disfigured more or less by some very large deviations which are probably produced by differences in the points observed.

The last two columns are certainly the more trustworthy of the two determinations. The probable differences between some of the catalogues are given also by Schönfeld and Vogel. They find

	s.	″	
Schultz—Schönfeld II.	= ± 0·219	± 2·20.	
d'Arrest—Schönfeld I.	= ± 0·348	± 7·02.	
Schönfeld I.—Vogel II.	= ± 0·180	± 1·58,	16 of 57 nebulæ being excluded.
	= ± 0·145	± 1·11,	for the same comparison star.
Schultz—Vogel II.	= ± 0·189	± 1·62.	
Auwers—Vogel II.	= ± 0·140	± 1·38.	

Considering the catalogues of Schönfeld, Schultz, Vogel, and Auwers equally correct, I find the probable error of one catalogue position on an average $= \pm\ 0^s{\cdot}130$ sec $\delta \pm 1''{\cdot}20$. This combined with the figures given in the last two columns of the Table gives on an average $\pm\ 0^s{\cdot}130$ sec $\delta \pm 1''{\cdot}64$ as the probable error of one of my positions [mean of about three chronographed observations, the very large and difficult objects excluded]. As the eye-and-ear observations have not been included, the final positions of my catalogue will be slightly more accurate.

In conclusion, I can only endorse Dr. Engelmann's opinion that the positions of nebulæ observed with the Transit Circle cannot compete with somewhat carefully made micrometer observations, unless the Transit Circle possesses a very perfect illumination of the wires. There is also no considerable gain in time as compared with micrometric observations, as each observation of a nebula occupied about 23 minutes, if the time spent on fundamental stars, determination of the errors of the instrument, and unsuccessful observations be included.

Tables.

Table I. gives for each nebula in the first column the number of the General Catalogue by Herschel and of the new General Catalogue by Dreyer.

In the third and fifth columns the eye-and-ear and chronographed observations are separately combined to a mean, the first not being corrected for the systematic difference (*see* p. 2). The number of wires over which the transits have been observed and the number of bisections in declination are stated in cols. 4 and 6.

Table II. contains the comparison of the mean of my *chronographed* observations with the positions given by other observers. The second catalogues of Vogel and Engelhardt are indicated by italic numerals. For 10 nebulæ which were observed only by the eye-and-ear method, the differences are enclosed in brackets.

The final positions are found in Table III. They are the mean values of all the observations, the eye-and-ear positions having been reduced to the chronographed observations by the correction given on p. 2. The precession has been computed from Struve's constant.

[Table I.

TABLE I.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
8	1886	Sept. 27	0 3 24·03	9	+27 6 55·3	1	F
16	1886	Nov. 29	24·16	11	58·3	1	pB
	1886	Nov. 30	24·21	14	52·8	1	
	....	....	0 3 24·13	..	+27 6 55·5	..	
	1887	Dec. 7	23·75	3	64·6	2	F
	1888	Sept. 12	23·70	6	57·5	2	
	1888	Oct. 12	....	..	66·8	1	pF, clouds
	....	....	0 3 23·72	..	+27 6 63·0	..	
53	1888	Sept. 4	0 20 12·95	6	+28 36 12·0	2	pB, pS, diffic.
108	1888	Sept. 12	12·39	6	4·3	2	pF, pL, diffic.
	1888	Dec. 8	12·77	2	....	..	
	....	....	0 20 12·70	..	+28 36 8·2	..	
62	1887	Dec. 7	0 23 36·28	6	+2 15 21·7	2	pB, pS, = * 11-12
128	1888	Dec. 8	36·00	6	21·4	2	pB, moonlight
	....	....	0 23 36·14	..	+2 15 21·5	..	
90	1887	Dec. 7	0 32 52·62	7	+47 43 54·2	2	B, vL
185	1888	Sept. 4	52·18	6	50·7	2	pB, vL, bM
	1888	Sept. 12	51·39	7	41·5	2	pB, vL, cond. ?
	1888	Oct. 14	52·88	7	45·7	2	pB, vL
	....	....	0 32 52·27	..	+47 43 48·0	..	
105	1888	Dec. 8	0 34 22·28	4	+41 4 44·6	2	B, vL, vmbM, B * nf
205	1888	Dec. 9	22·94	2	43·9	2	B, vL, vmbM
	....	....	0 34 22·61	..	+41 4 44·2	..	
117	1886	Sept. 27	0 36 41·88	11	+40 15 43·7	2	vB, L, N
221	1886	Nov. 17	42·14	5	41·6	1	
	1886	Nov. 24	41·81	9	44·9	2	
	1886	Dec. 20	42·35	11	39·1	1	
	1886	Dec. 29	41·91	14	44·7	2	
	....	....	0 36 42·02	..	+40 15 42·8	..	
	1888	Sept. 13	41·92	7	43·4	3	vB, S, N
	1888	Nov. 14	42·01	7	40·3	3	
	1888	Dec. 29	41·67	5	40·6	3	vB, S, R
	....	....	0 36 41·87	..	+40 15 41·4	..	
116	1886	Nov. 30	0 36 43·76	12	+40 39 54·9	3	Andr. Nebula
224	....	....	....	..	....	..	
149	1886	Dec. 26	0 43 50·82	6	+31 40	..	pB
266	....	....	....	..	....	..	
	1887	Dec. 7	50·61	5	43·2	1	cB, pS
	1888	Sept. 4	51·03	7	34·9	1	pB, pS, B * sf
	1888	Sept. 12	50·07	7	41·7	2	pB
	1888	Oct. 14	50·70	6	42·8	2	pF, pS
	....	....	0 43 50·60	..	+31 40 40·6	.	
158	1886	Sept. 27	0 45 52·28	7	+46 56 66·5	2	
278	1886	Nov. 24	52·09	8	60·4	1	L
	1886	Nov. 25	52·05	16	55·1	1	cB, L
	....	....	0 45 52·14	..	+46 56 60·7	..	
	1888	Dec. 8	51·93	7	60·3	4	B, pS
	1888	Dec. 9	51·51	5	60·7	1	
	1888	Dec. 29	51·34	7	63·1	3	cB, S, R
	....	....	0 45 51·59	..	+46 56 61·4	..	

TABLE I.—Continued.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	Remarks.
			h m s		° ′ ″		
176	1887	Dec. 7	0 51	..	+29 45 26·3	2	* 9 nf
315	1888	Sept. 12	49·94	4	29·3	2	pB, pS
	1888	Oct. 14	50·13	2	19·8	1	pB, S
	1888	Dec. 26	50·52	5	22·4	1	cB
	1889	Jan. 1	....	..	26·2	2	
	....	....	0 51 50·20	..	+29 45 24·8	..	
218	1886	Sept. 27	1 3 19·28	7	+35 7 50·2	2	β Andromedæ near
404	1886	Nov. 24	18·97	9	42·3	1	L, Decl. ±
	1886	Nov. 25	19·11	11	43·3	1	L, Decl. ±
	1886	Nov. 29	19·18	8	50·1	1	B, L, Decl. ±
	1886	Nov. 30	18·94	7	38·3	1	Decl. ±
	1886	Dec. 26	19·45	6	40·7	1	diffic., Decl. ±
	....	....	1 3 19·15	..	+35 7 44·1	..	
	1888	Oct. 14	18·52	6	44·9	2	cB, RA±
	1889	Jan. 1	....	..	49·9	3	cB, pS, R, bM
	1889	Jan. 2	19·09	7	47·1	3	cB, pS, R, bM
	....	....	1 3 18·80	..	+35 7 47·3	..	
264	1888	Dec. 26	1 14 4·00	6	+2 49 59·1	2	pB, pL, diffic.
470	1888	Dec 29	4·25	4	....	..	v, diffic.
	....	....	1 14 4·12	..	+2 49 59·1	..	
269	1887	Dec. 7	1 14 25·10	4	+2 50 13·7	1	pF, pS
474	1888	Dec. 9	24·93	4	17·6	1	F
	....	....	1 14 25·02	..	+2 50 15·6	..	
307	1886	Sept. 27	1 19 1·65	8	+8 57 55·0	1	B
524	1886	Nov. 25	1·91	12	56·4	1	
	1886	Nov. 29	1·32	12	40·1	1	Decl. ±
	1886	Nov. 30	1·33	8	51·8	1	
	....	....	1 19 1·55	..	+8 57 50·8	..	
	1887	Dec. 17	....	..	55·4	1	
	1888	Dec. 29	1·16	5	54·4	2	pB, diffic.
	1889	Jan. 2	0·81	6	50·7	2	pB, through haze
	....	....	1 19 0·98	..	+8 57 53·5	..	
342	1886	Nov. 24	1 25 49·49	12	−7 26 6·1	1	
584	1886	Nov. 30	49·55	10	12·3	1	
	1887	Jan. 12	49·78	5	2·2	1	B, = * 10-11
	....	....	1 25 49·60	..	−7 26 6·9	..	
	1887	Dec. 7	49·40	7	3·7	1	B, pS
	1888	Dec. 8	49·34	7	8·7	3	
	1888	Dec. 29	49·19	6	10·3	3	cB
	....	....	1 25 49·31	..	−7 26 7·6	..	
385	1886	Sept. 27	1 35 23·30	6	+51 0 51·3	1	vB, L, bM
650	1886	Nov. 25	22·83	11	46·5	1	vB, L
	1887	Jan, 12	22·08	6	49·6	1	vB, L
	....	....	1 35 22·74	..	+51 0 49·1	..	
	1887	Dec. 7	22·93	7	46·4	1	
	1888	Dec. 26	22·82	7	50·1	3	vB, L, mbM
	....	....	1 35 22·87	..	+51 0 48·2	..	

TABLE I.—Continued.

G.C. N.G.C.	Date.	Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	Remarks.
		h m s		° ′ ″		
386	1886 Nov. 29	1 35 26·76	12	+51 1 23·1	1	
651	1886 Dec. 26	26·98	15	22·9	1	
	....	1 35 26·87	..	+51 1 23·0	..	
	1887 Dec. 7	26·28	7	21·4	1	
	1887 Dec. 17	26.41	6	31·4	2	not well separated from 385
	1888 Dec. 29	26·34	7	19·8	3	
	1889 Jan. 1	26·27	6	23·1	2	vB, vmbM
	....	1 35 26·33	..	+51 1 23·9	..	
430	1887 Dec. 7	1 47 30·54	7	+3 39 14·9	2	pB
718	1887 Dec. 10	30·09	5	....	..	
	1887 Dec. 17	29·97	6	13·0	2	=*11-12, *11 sp
	1888 Dec. 26	30·20	5	7·7	2	pB, S
	1889 Jan. 1	29·71	7	0·3	2	pB, S, diffic.
	1889 Jan. 2	30·06	5	0·8	2	pF, pS, diffic.
	1889 Jan. 5	29·70	5	4·4	1	pF, diffic.
	....	1 47 30·04	..	+3 39 6·8	..	
463	1886 Nov. 25	1 53 17·96	6	+18 28 20·5	1	B, Decl. ±
772	....	....	..	....	..	
526	1886 Nov. 25	2 15 28·78	8	+32 45 47·4	1	
890	1886 Nov. 29	29·11	8	....	..	pB
	....	2 15 28·95	..	+32 45 47·4	..	
	1888 Oct. 28	28·37	7	43·2	2	cB, S, bM
	1888 Oct. 29	28·64	6	41·4	2	pF, hazy
	1888 Dec. 8	28·26	7	43·5	3	
	1888 Dec. 26	28·27	7	42·3	3	B, pS, bM
	1889 Jan. 1	28·62	7	41·0	2	B
	1889 Jan. 2	28.00	2	38·3	1	pF, diffic, RA±
	....	2 15 28·39	..	+32 45 41·6	..	
544	1886 Dec. 24	2 22 1·49	14	−1 38 60·0	1	
936	....	....	..	....	..	
	1887 Dec. 7	1·35	7	56·8	2	B, L
	1887 Dec. 10	1·18	7	58·7	3	B, pL, smbM
	1888 Jan. 10	0·92	5	64·1	2	
	1889 Jan. 1	1·26	7	64·0	3	vB, L, vmbM
	1889 Jan. 5	1·25	7	64·3	2	
	....	2 22 1·19	..	−1 39 1·6	..	
549	1887 Dec. 17	2 24 4·47	7	+36 38 50·0	3	pB, pL
949	1888 Oct. 28	3·46	2	41·2	2	pB, vL, RA±
	1888 Dec. 8	3·89	6	46·1	2	pB
	1888 Dec. 26	3·22	3	46·8	1	pB, pL
	....	2 24 3·80	..	+36 38 46·0	..	
574	1889 Jan. 1	2 33 5·46	6	−7 9 5·7	1	pB, pL, diffic.
1022	1889 Jan. 5	5·24	6	11·7	1	
	....	2 33 5·35	..	−7 9 8·7	..	
575	1886 Nov. 25	2 33 31·14	11	+38 35 19·2	2	vB, N
1023	1886 Dec. 26	31·47	17	22·1	1	
	....	2 33 31·30	..	+38 35 20·6	..	
	1887 Dec. 10	30·61	7	21·1	3	B
	1888 Oct. 28	31·19	3	21·5	2	vB, N
	1888 Oct. 29	30·49	6	18·2	2	B, N
	....	2 33 30·76	..	+38 35 20·3	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
600	1889	Jan. 5	2 37 2·84	6	−0 28 57·3	3	vB, pS
1068	1889	Jan. 22	2·76	7	59·8	2	
	....	....	2 37 2·80	..	−0 28 58·5	..	
604	1887	Dec. 17	2 40 35·22	7	−8 2 32·4	3	B, pL, bM
1084	1888	Oct. 29	34·72	7	35·1	2	cB, L
	1889	Jan. 1	34·76	7	40·5	2	
	....	....	2 40 34·90	..	−8 2 36·0	..	
628=	1888	Oct. 29	2 53 54·34	6	+44 27 30·7	3	pB, S, 2 ** prec.
634	1888	Dec. 26	53·70	5	30·7	2	pB, diffic.
1161	1889	Jan. 5	53·96	7	37·9	2	cB, S, R
	1889	Jan. 22	54·22	6	36·7	2	pB, S
	....	....	2 53 54·06	..	+44 27 34·0	..	
648	1887	Jan. 21	3 0 54·41	6	−16 2 14·2	1	diffic.
1209	....	....	....	..	....	..	
	1888	Oct. 28	3 0 54·57	4	−16 2 14·7	2	cB, S, N ?
675	1886	Dec. 24	3 12 31·92	4	+41 6 39·4	1	cB, Decl. ±
1275	1886	Dec. 26	32·60	8	....	..	cB
	1887	Jan. 21	31·80	12	45·0	1	pB, pL
	....	....	3 12 32·11	..	+41 6 42·2	..	
	1888	Oct. 28	31·76	6	30·7	2	pB
	1888	Oct. 29	32·32	7	25·3	1	cB, pS, 2B ** pr.
	1888	Dec. 26	31·84	4	30·4	2	pB, S, bM
	1889	Jan. 22	32·88	2	26·2	1	RA±
	....	....	3 12 32·11	..	+41 6 28·2	..	
709	1889	Jan. 22	3 21 23·67	5	−21 43 16·8	2	= * 11-12
1332	1889	Jan. 29	23·79	5	18·9	2	pB, R, S
	....	....	3 21 23·73	..	−21 43 17·8	..	
752	1887	Jan. 23	3 35 14·56	8	−18 56 15·5	1	= * 10
1407	....	....	....	..	....	..	
	1888	Oct. 29	14·19	7	17·7	2	cB, pL
	1888	Dec. 26	13·49	6	6·1	2	pB
	1889	Jan. 22	14·15	6	13·8	2	
	1889	Jan. 29	13·63	7	7·6	2	= * 11-12
	1889	Feb. 1	14·53	6	13·4	2	
	....	....	3 35 14·00	..	−18 56 11·7	..	
778	1889	Jan. 22	3 40 57·58	5	−4 18 47·0	1	pB
1453	1889	Jan. 29	57·09	4	44·5	2	pF, B * nf, v diffic.
	....	....	3 40 57·33	..	−4 18 45·7	..	
801	1888	Jan. 11	3 57 30·68	4	+60 37	..	pB, R
1501	1889	Jan. 5	30·14	5	18·1	2	pB, R, 1′ diam., RA±
	1889	Jan. 29	32·40	6	21·0	3	= * 10, 20″ diam.
	....	....	3 57 31·07	..	+60 37 19·5	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
826	1886	Dec. 26	4 9 7·51	18	−13 1 1·8	1	
1535	1887	Jan. 21	7·94	12	0·5	2	
	1887	Jan. 23	7·58	11	....	..	
	....	....	4 9 7·68	..	−13 1 1·2	..	
	1888	Nov. 26	7·59	7	10·8	3	
	1889	Jan. 1	7·25	7	8·2	2	= * 10, R
	1889	Jan. 5	7·17	7	10·5	3	
	1889	Jan. 22	7·64	5	6·4	2	= * 10
	1889	Feb. 1	7·49	2	5·9	1	
	....	....	4 9 7·43	..	−13 1 8·4	..	
847	1887	Jan. 23	4 20 24·79	21	+64 36 19·6	1	B, * 9·5 nf
1569	....	....	....	..	....	..	
	1888	Oct. 29	25·51	7	12·6	3	pB
	1888	Nov. 26	24·70	6	14·0	2	pF, S
	1888	Dec. 5	24·65	7	11·8	3	
	1889	Jan. 5	.....	..	7·1	1	* 9·5 nf, Decl. ±
	1889	Jan. 29	25·16	6	11·5	3	diffic.
	1889	Feb. 1	23·93	6	11·9	3	
	....	....	4 20 24·79	..	+64 36 11·5	..	
866	1889	Jan. 1	4 26 14·36	6	−5 19 22·1	1	vF, diffic.
1600	1889	Jan. 22	14·00	7	20·1	2	pB, pS
	....	....	4 26 14·18	..	−5 19 21·1	..	
888	1887	Dec. 11	4 35 56·85	5	−3 4 9·1	1	pF, L, v diffic.
1637	1888	Feb. 9	57·48	6	28·5	1	pF, L
	1888	Oct. 29	58·13	3	26·8	1	F, pL, RA±
	1889	Feb. 1	56·64	6	21·4	2	pB, pL, * 11-12 nf
	....	....	4 35 57·27	..	−3 4 21·5	..	
932	1887	Dec. 11	4 51 30·21	7	−5 2 17·0	3	cB, pS, mbM
1700	1888	Oct. 29	29·91	3	19·4	2	pB, S
	1888	Nov. 26	30·20	7	19·8	2	pB, pS, * 10 sp, B * n
	1888	Dec. 5	30·36	7	19·3	2	cB, S
	1889	Jan. 5	30·25	7	20·1	3	cB, vmbM
	1889	Jan. 22	30·17	7	25·8	3	pB, pS
	1889	Feb. 1	30·50	7	22·1	3	cB, pS, bM
	....	....	4 51 30·23	..	−5 2 20·5	..	
1005	1887	Jan. 23	5 1 26·03	6	−3 30 13·7	1	
1788	....	....	....	..	....	..	
	1887	Dec. 11	25·98	5	3·5	3	pB, pL, vsbM
	1888	Feb. 15	25·86	7	1·7	2	pB, pL
	1888	Oct. 29	26·10	7	9·6	3	
	1888	Nov. 26	25·83	7	7·5	2	B * np
	1888	Dec. 5	25·95	6	8·2	2	cB, cL, vmbM
	1889	Feb. 1	26·14	7	8·7	3	cB, S, vmbM, N
	....	....	5 1 25·98	..	−3 30 6·5	..	
1137	1886	Dec. 29	5 24 9·77	17	+34 9 44·3	1	
1931	1887	Feb. 16	9·90	12	48·6	2	
	1887	Feb. 26	10·03	16	41·7	1	
	....	....	5 24 9·90	..	+34 9 44·9	..	
	1888	Feb. 15	9·63	7	43·5	3	vB, S
	1888	Dec. 5	9·55	6	41·2	3	= * 9·5 in pB neb.
	1888	Dec. 28	9·58	7	40·2	3	vB, pS, N
	1889	Jan. 22	9·69	7	38·5	3	
	1889	Feb. 1	9·74	7	36·5	3	
	....	....	5 24 9·64	..	+34 9 40·0	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
1157	1887	Jan. 23	5 27 52·90	12	+21 56	..	vL
1952	....	....	....	..	....	..	
	1887	Dec. 11	53·25	6	29·2	3	vB, vL, E, no cond.
	1888	Feb. 15	53·78	4	25·1	3	B, vL, no cond.
	1888	Nov. 26	53·27	6	21·5	1	B, vvL, no cond.
	....	....	5 27 53·43	..	+21 56 25·3	..	
1185	1889	Jan. 22	5 30 6·88	7	−5 20 23·6	3	= * 8·9, Bright field
1982	....	....	....	..	....	..	
1202	1888	Dec. 5	5 31 4·65	4	−6 47 9·5	2	= * 9·5 in neb.
1999	1888	Dec. 28	4·27	4	9·0	2	= * 9·5 in cB neb.
	1889	Feb. 1	4·50	5	8·7	2	
	....	....	5 31 4·47	..	−6 47 9·1	..	
1225	1886	Dec. 29	5 36 4·94	16	+9 1 53·0	1	= * 10
2022	1887	Jan. 23	4·93	10	45·8	1	
	....	....	5 36 4·93	..	+9 1 49·4	..	
	1888	Feb. 15	4·66	7	46·4	3	pB, pS, R, = * 10-11
	1888	Nov. 26	4·70	7	47·5	3	cB, S, R
	....	....	5 36 4·68	..	+9 1 47·0	..	
1267	1886	Dec. 19	5 41 5·22	14	+0 1 56·7	1	* 9·5 in neb., 2f ** np
2068	1886	Dec. 30	5·35	14	61·8	1	* 9·5 in neb.
	1887	Feb. 26	5·20	11	57·8	1	* 9·5 in neb.
	....	....	5 41 5·26	..	+0 1 58·8	..	
1270	1887	Feb. 16	5 41 29·42	8	+0 15 30·9	1	= * 9·5
2071	1887	Feb. 25	29·17	10	28·2	1	= * 11-12.
	....	....	5 41 29·30	..	+0 15 29·5	..	
1337	1886	Dec. 29	5 56 39·94	11	−10 35 58·7	2	3 Monocerotis
2142	1886	Dec. 30	39·88	13	59·7	3	
	....	....	5 56 39·91	..	−10 35 59·2	..	
1362	1887	Feb. 16	6 2 10·55	7	−6 23	..	
2170	1887	Feb. 19	10·30	10	....	..	* 10 pr
	1887	Feb. 26	10·42	12	....	..	
	....	....	6 2 10·42	..	....	..	
	1888	Dec. 28	10·02	7	11·1	3	= * 9-10, * 10 sp
	1889	Jan. 9	10·14	7	12·4	3	
	1889	Jan. 22	10·30	7	10·4	3	
	1889	Feb. 23	10·32	7	13·7	3	= * 10, * 10-11 sp
	....	....	6 2 10·20	..	−6 23 11·9	..	
1375	1888	Feb. 15	6 5 43·82	5	−6 11 15·5	1	= * 11, vS
2185	1888	Dec. 5	43·97	5	23·9	2	= * 11, 2F ** sp
	1888	Dec. 28	44·31	2	15·2	2	= * 11, 2F ** sp
	1889	Jan. 22	44·12	5	20·9	2	
	1889	Feb. 1	44·08	7	18·2	3	= * 11-12
	....	....	6 5 44·06	..	−6 11 18·7	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
1425	1886	Dec. 29	6 26 37·40	14	+10 14 21·4	1	
2245	1886	Dec. 30	37·47	13	18·4	3	
	1887	Feb. 16	37·59	4	16·3	1	
	1887	Feb. 19	37·70	11	19·6	2	= * 10 in B neb.
	1887	Feb. 26	37·35	12	19·3	1	
	....	....	6 26 37·50	..	+10 14 19·0	..	
	1888	Jan. 11	37·22	7	18·1	3	
	1888	Feb. 15	37·31	7	19·6	3	B, N, = * 10
	1888	Nov. 26	37·45	7	18·5	3	
	1888	Dec. 5	37·44	7	17·1	3	
	1888	Dec. 29	37·41	7	19·7	3	
	....	....	6 26 37·37	..	+10 14 18·6	..	
1437	1886	Dec. 29	6 33 9·45	3	+8 49 63·0	1	B, com, = * 11
2261	1887	Feb. 19	9·43	12	59·2	1	B, com
	1887	Feb. 26	9·32	15	60·4	1	B, com
	....	....	6 33 9·40	..	+8 49 60·9	..	
	1888	Jan. 5	9·25	6	60·4	2	B, pS, com, Ns
	1888	Jan. 11	8·88	6	62·5	2	B, pS, com, Ns
	1888	Feb. 15	9·26	6	58·5	2	B, pS, com, Ns
	1888	Nov. 26	9·38	7	59·7	3	B, pS, com, Ns
	1888	Dec. 5	9·34	7	54·4	3	B, pS, com, Ns
	1888	Dec. 29	9·17	7	55·2	3	B, pS, com, Ns
	....	....	6 33 9·21	..	+8 49 58·5	..	
1500	1887	Jan. 19	7 3 46·28	11	−0 37 53·2	1	= * 10
2346	1887	Jan. 23	46·02	12	48·4	1	= * 10
	1887	Feb. 19	46·32	5	47·0	1	= * 10
	1887	Feb. 26	46·37	15	49·0	2	= * 10
	....	....	7 3 46·25	..	−0 37 49·4	..	
	1888	Jan. 11	46·05	7	51·0	3	= * 10
	1888	Jan. 18	46·28	7	50·6	2	= * 10
	1888	Feb. 15	45·98	5	49·3	2	= * 10
	1888	Dec. 28	46·06	7	48·9	3	= * 10
	1888	Dec. 29	46·14	2	52·4	2	= * 10
	....	....	7 3 46·10	..	−0 37 50·4	..	
1519	1887	Jan. 23	7 18 37·08	13	+29 42 9·0	1	
2371	1887	Feb. 26	37·77	12	14·3	1	pL
	....	....	7 18 37·42	..	+29 42 11·7	..	
	1888	Jan. 11	37·14	7	9·5	2	cB, pS, pB neb. nf
	1888	Jan. 18	37·01	7	12·1	2	cB, pB neb. nf
	1888	Dec. 28	37·23	7	8·1	3	cB, cS, N
	1888	Dec. 29	37·58	5	6·7	2	pB, S
	1889	Jan. 6	37·15	7	3·7	2	B, N, pB neb. nf
	....	....	7 18 37·22	..	+29 42 8·0	..	
1532	1891	Mar. 10	7 22 40·48	6	+21 8 7·2	2	B, * 9M
2392	1891	Mar. 13	40·30	7	6·6	1	same
	....	....	7 22 40·39	..	+21 8 6·9	..	
1546	1887	Jan. 23	7 29 42·37	12	+35 28 54·8	1	pB, B * nf 7s
2415	1887	Feb. 26	42·79	8	....	..	
	1887	Feb. 28	42·84	8	66·8	1	
	....	....	7 29 42·67	..	+35 28 60·8	..	
	1888	Jan. 5	41·87	6	59·6	1	pB, S
	1888	Jan. 11	42·28	6	69·3	3	
	1888	Jan. 18	42·56	7	64·5	2	pB
	1888	Dec. 28	41·99	7	64·8	1	pB, S
	....	....	7 29 42·18	..	+35 28 64·5	..	

TABLE I.—Continued.

G.C. N.G.C.	Date.	Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	Remarks.
		h m s		° ′ ″		
1548	1886 Dec. 30	7 30 41·70	5	+39 7	..	pB, vL, * 7 pr
2419	1887 Jan. 19	.....	..	45·9	1	pB, vL, * 7 interferes
	1887 Feb. 19	40·41	6	....	..	
	....	7 30 41·05	..	+39 7 45·9	2	
	1888 Jan. 9	41·04	5	45·6	2	pB, pL
	1888 Dec. 29	40·91	6	17·4	3	pB, * 7 interferes.
	1889 Jan. 6	39·90	5	28·5	2	pB
	1891 Mar. 10	.....	..	37·3	2	pB, pL
	1891 Mar. 13	41·01	7	28·6	2	pB
	....	7 30 40·71	..	+39 7 31·5	..	
1567	1886 Dec. 30	7 37 0·85	17	−17 56 60·9	1	= * 8·9
2440	1887 Jan. 23	0·81	8	58·2	1	Decl. ±
	....	7 37 0·83	..	−17 56 59·5	..	
	1888 Jan. 18	0·37	5	59·2	1	R, d, =10″
	1888 Dec. 28	0·38	2	62·1	2	B, S, R, = * 9 out of focus, * 8·5 f 10s
	1888 Dec. 29	0·63	5	65·9	2	
	1889 Jan. 6	0·43	7	63·9	3	not visible in red illumination of field
	....	7 37 0·45	..	−17 56 62·8	..	
1596	1886 Dec. 24	7 50 38·40	6	+24 3 40·3	1	pB, Decl. ±
2481	1887 Feb. 28	.....	..	39·8	1	pF
	....	7 50 38·40	..	+24 3 40·1	..	
	1888 Jan. 9	37·89	6	40·2	2	pB, vS
	1888 Jan. 11	38·13	2	28·8	2	pF, diffic.
	1888 Jan. 18	38·24	7	35·5	2	pB, mbM
	1888 Dec. 28	38·31	4	26·4	2	pB, pS, diffic., S * sf
	....	7 50 38·14	..	+24 3 32·7	..	
1626	1887 Jan. 19	8 3 10·79	5	+34 16	..	pF, between 2 **
2532	....	....	..	....	..	
	1888 Jan. 11	11·97	4	46·2	1	pB, diffic.
	1888 Dec. 28	11·50	6	43·7	1	pF, diffic.
	....	8 3 11·73	..	+34 16 45·0	..	
1629	1887 Jan. 23	8 5 25·74	5	+46 19 12·5	1	
2537	....	....	..	....	..	
	1888 Jan. 9	25·59	6	9·1	3	pB, B * pr, S * sf
	1889 Feb. 1	25·78	6	3·0	2	pB, * 10 sf
	1891 Mar. 12	27·08	6	14·1	2	pB, pL
	....	8 5 26·15	..	+46 19 8·7	..	
1632	1887 Mar. 24	8 6 6·71	14	−12 36 6·3	3	= * 6, B field illumination
2542	....	....	..	....	..	
	1889 Jan 6	6·77	7	3·5	2	= * 6, B field illumination
	1891 Mar. 14	6·64	4	3·3	2	
	....	8 6 6·70	..	−12 36 3·4	..	
1660	1887 Feb. 28	8 20 31·60	12	+26 19 46·6	1	cB,= * 11-12
2592	1887 Mar. 19	31·46	15	49·8	1	cB, pL
	....	8 20 31·53	..	+26 19 48·2	..	
	1888 Jan. 9	30·89	6	53·8	1	pF, N, diffic.
	1888 Dec. 28	31·27	6	42·9	2	pB
	1889 Jan. 6	30·96	6	41·2	2	
	....	8 20 31·04	..	+26 19 46·0	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
1673	1887	Jan. 19	8 28 38·63	5	−12 47	..	
2612	1887	Feb. 19	.....	..	45·0	1	
	1887	Feb. 28	39·59	4	35·9	1	
	....	....	8 28 39·11	..	−12 47 40·5	..	
	1889	Jan. 6	38·25	5	....	..	
	1891	Mar. 10	38·68	5	37·5	1	diffic.
	1891	Mar. 12	38·13	2	....	..	diffic.
	....	....	8 28 38·35	..	−12 47 37·5	..	
1684	1887	Jan. 23	8 35 43·52	10	+50 35 50·9	1	
2639	1887	Feb. 19	43·10	11	61·4	1	
	1887	Feb. 26	43·08	12	73·8	1	
	1887	Mar. 19	43·48	13	55·4	1	B, S, E, psmbM
	....	....	8 35 43·30	..	+50 35 60·4	..	
	1888	Jan. 9	42·29	6	55·5	2	pB, diffic.
	1888	Jan. 11	42·63	6	56·6	3	pB, vmbM
	1888	Mar. 19	43·03	6	72·4	1	cB, pL
	1888	Dec. 28	43·16	5	45·5	1	
	1889	Jan. 6	43·21	4	42·4	2	
	1891	Mar. 12	42·50	7	53·5	3	
	1891	Mar. 13	....	..	51·8	2	pF, v diffic.
	....	....	8 35 42·80	..	+50 35 54·0	..	
1691	1887	Mar. 23	8 41 7·58	20	+78 38 9·6	4	vB, N
2655	....	....	....	..	....	..	
	1888	Mar. 19	6·81	4	6·9	2	B, pL, moonlight
	1889	Feb. 23	7·25	7	4·4	3	B, S, vmbM
	....	....	8 41 7·03	..	+78 38 5·6	..	
1704	1887	Jan. 19	8 43 5·16	2	+19 28 53·6	1	pB, Decl.±
2672	....	....	....	..	....	..	
	1889	Jan. 6	5·13	6	41·2	2	pB
	1891	Mar. 12	4·98	4	47·1	2	
	1891	Mar. 13	5·86	4	34·8	1	pF
	....	....	8 43 5·32	..	+19 28 41·0	..	
1711	1887	Mar. 24	8 45 37·80	16	+51 43	..	
2681	....	....	....	..	....	..	
	1888	Jan. 9	37·51	4	40·7	1	cB
	1888	Jan. 11	37·06	6	37·3	2	
	1888	Feb. 12	37·51	6	37·9	2	pB, S
	....	....	8 45 37·36	..	+51 43 38·6	..	
1713	1887	Feb. 26	8 45 51·64	7	+33 49 53·4	1	B, vL, vmE, diffic.
2683	....	....	....	..	....	..	
	1888	Dec. 28	50·67	6	42·0	3	B, L, mE, bM
	1891	Mar. 10	51·10	6	55·9	3	B, vL, mE, diffic.
	1891	Mar. 14	51·55	6	63·2	2	B, vL
	....	....	8 45 51·10	..	+33 49 53·7	..	
1720	1887	Mar. 19	8 49 6·15	12	+51 46 4·3	1	B, S, R
2693	1887	Mar. 24	6·24	8	4·2	1	
	....	....	8 49 6·20	..	+51 46 4·2	..	
	1888	Jan. 9	....	..	1·5	1	pF
	1888	Jan. 11	....	..	2·9	1	pF
	....	....	....	..	+51 46 2·2	..	

TABLE I.—Continued.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	Remarks.
			h m s		° ′ ″		
1728	1887	Jan. 23	8 50 58·64	8	+54 11 48·0	1	
2701	....	....	....	..	....	..	
	1889	Jan. 6	58·18	3	48·8	2	pB, pL, R
	1891	Mar. 12	57·92	4	54·7	2	
	1891	Mar. 13	58·49	4	46·3	2	pB
	....	....	8 50 58·20	..	+54 11 49·9	..	
1765	1887	Jan. 23	9 3 3·53	15	+60 29 18·3	1	
2768	1887	Mar. 19	3·50	13	11·0	2	B, cL, R
	1887	Mar. 24	3·59	17	7·3	2	B, cL, N
	....	....	9 3 3·54	..	+60 29 12·2	..	
	1888	Feb. 12	2·64	4	26·7	1	pB, cL, diffic.
	1891	Mar. 12	2·74	7	0·5	1	
	1891	Mar. 13	3·13	6	15·2	3	B
	1891	Mar. 14	3·61	6	3·9	2	
	....	....	9 3 3·03	..	+60 29 11·6	..	
1771	1887	Feb. 26	9 4 29·64	6	+7 28	..	
2775	1887	Mar. 23	29·35	11	71·4	2	
	....	....	9 4 29·50	..	+7 28 71·4	..	
	1888	Feb. 15	29·18	5	69·7	2	pB, pL
	1889	Jan. 6	28·87	7	56·2	2	cB, pL, R, lbM
	1889	Feb. 23	29·14	5	62·2	2	
	....	....	9 4 29·06	..	+7 28 62·7	..	
1781	1888	Mar. 19	9 9 20·02	7	+69 39 55·5	1	
2787	1891	Mar. 12	21·81	2	51·0	2	
	1891	Mar. 14	20·54	2	....	..	
	....	....	9 9 20·60	..	+69 39 53·2	..	
1811	1887	Feb. 26	9 13 3·96	5	+34 12	..	F, = * 12
2830	1887	Mar. 24	3·66	5	50·7	1	cB, 2 ** 10 sf, B * f
	....	....	9 13 3·81	..	+34 12 50·7	..	
	1888	Feb. 15	3·38	4	46·8	1	
	1891	Mar. 13	3·19	6	39·8	1	pF
	....	....	9 13 3·28	..	+34 12 43·3	..	
1823	1887	Mar. 19	9 14 26·16	8	+51 26 35·9	1	vvB, pL, N
2841	1887	Mar. 23	26·22	9	29·4	1	vB, L, mbM
	....	....	9 14 26·19	..	+51 26 32·6	..	
1848	1887	Jan. 23	9 20 58·38	10	+62 58 16·0	1	* 9·5 nf
2880	1887	Feb. 18	58·43	5	18·4	1	
	1887	Feb. 26	58·66	10	16·0	1	= * 11
	....	....	9 20 58·49	..	+62 58 16·8	..	
	1891	Mar. 12	57·24	3	8·6	1	
	1891	Mar. 13	58·33	6	9·3	3	B
	....	....	9 20 57·78	..	+62 58 9·0	..	
1861	1887	Feb. 19	9 25 56·18	16	+21 58 67·2	1	vB, N
2903	1887	Mar. 19	56·33	15	69·7	2	vB, L, N
	1887	Mar. 23	56·29	11	74·1	1	
	1887	Mar. 24	56·30	12	68·0	2	
	....	....	9 25 56·28	..	+21 58 69·8	..	
	1888	Feb. 15	55·78	6	62·7	3	cB, pL
	1891	Mar. 31	55·99	3	58·2	2	
	....	....	9 25 55·89	..	+21 58 60·5	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
1896	1887	Feb. 19	9 36 22·83	9	+32 20 62·6	1	vL, R
2964	1887	Feb. 26	23·25	14	54·9	1	cB, vL
	1887	Mar. 24	23·02	12	58·4	2	
	....	....	9 36 23·03	..	+32 20 58·6	..	
	1888	Feb. 15	22·62	4	56·5	2	
	1889	Mar. 6	22·68	4	46·9	2	cB, pL
	1891	Mar. 11	23·37	3	45·7	2	
	1891	Mar. 31	22·57	7	39·7	2	
	....	....	9 36 22·81	..	+32 20 47·2	..	
1904	1889	Mar. 6	9 37 0·46	6	−3 11 45·0	2	cB, * 9 sp
2974	....	....	....	..	....	..	
1944	1887	Feb. 28	9 44 25·32	9	+34 3 66·9	1	* 10 sf, 5ˢ
3021	....	....	....	..	....	..	
	1889	Mar. 6	24·77	7	66·8	3	pB, S
	1891	Mar. 12	24·89	6	57·4	3	pB
	1891	Mar. 13	25·28	5	59·2	2	
	....	....	9 44 24·98	..	+34 3 61·1	..	
1949	1888	April 6	9 46 27·31	7	+69 34 64·4	3	= * 10, pL, R
3031	1889	Mar. 22	26·92	7	58·7	3	vB, L, smbM
	....	....	9 46 27·12	..	+69 34 61·5	..	
1973	1887	Feb. 26	9 51 54·53	8	+32 53 44·6	1	pB, B * f
3067	1887	Mar. 24	.....	..	48·7	2	
	....	....	9 51 54·53	..	+32 53 46·6	..	
	1888	Feb. 15	.....	..	39·0	1	pB
	1891	Mar. 12	54·48	5	38·4	1	
	1891	Mar. 13	54·08	5	35·7	2	
	....	....	9 51 54·28	..	+32 53 37·7	..	
2008	1887	Feb. 26	9 59 45·66	7	−7 11	..	vB, N, = * 10
3115	1887	Feb. 28	45·41	14	3·4	1	vB
	1887	Mar. 23	45·34	12	4·9	2	vB, L, N
	1887	Mar. 24	45·48	12	9·0	1	
	....	....	9 59 45·47	..	−7 11 5·8	..	
	1889	Mar. 22	45·18	4	3·0	2	vB, N
	1891	Mar. 12	45·42	6	3·0	3	
	1891	Mar. 13	45·50	6	6·9	3	vB, N
	1891	Mar. 31	45·18	6	11·0	3	vB
	....	....	9 59 45·32	..	−7 11 6·0	..	
2024	1888	April 6	10 7 26·83	6	+73 56 54·9	3	cB, S, R, = * 11
3147	1891	Mar. 13	26·86	5	51·2	3	B
	....	....	10 7 26·85	..	+73 56 53·0	..	
2041	1887	Feb. 28	10 8 32·59	9	+4 0 48·8	1	B, S, = * 11, * 10-11 nf
3169	1887	Mar. 24	32·37	12	38·2	2	B, S, = * 11-12
	....	....	10 8 32·48	..	+4 0 43·5	..	
	1888	Feb. 15	32·54	5	43·2	2	B, * 10 nf
	1889	Mar. 22	32·39	7	41·9	3	B, S
	....	....	10 8 32·46	..	+4 0 42·5	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
2058	1887	Feb. 26	10 12 1·56	14	+22 22 64·4	2	
3190	1887	Mar. 23	1·67	14	58·1	1	B
	....	....	10 12 1·62	..	+22 22 61·2	..	
	1891	Mar. 31	1·18	7	53·5	3	
	1891	April 9	1·33	5	60·7	2	cB, S
	....	....	10 12 1·25	..	+22 22 57·1	..	
2102	1887	Mar. 24	10 19 28·04	12	−18 5 1·7	1	vvB, d=1ˢ·5
3242	....	....	....	..	....	..	
	1891	April 9	28·38	7	7·1	4	
	1891	April 13	28·21	7	5·0	4	vvB, d=2ˢ
	....	....	10 19 28·30	..	−18 5 6·0	..	
2104	1887	Feb. 26	10 21 8·00	8	+29 3 66·6	1	B
3245	1887	Mar. 23	7·77	14	65·3	2	vB
	....	....	10 21 7·88	..	+29 3 66·0	..	
	1888	Feb. 15	6·99	7	66·6	1	cB pL
	1891	Mar. 31	7·56	7	57·7	3	vB, pL
	1891	April 7	7·37	4	61·5	2	
	....	....	10 21 7·31	..	+29 3 61·9	..	
2112	1887	Feb. 28	10 23 8·37	12	+30 3 8·5	1	cB, L
3254	....	....	....	..	....	..	
	1888	April 6	7·88	6	13·7	1	cB, 2B ** f
	1889	Mar. 22	8·17	6	14·6	3	pB, pL, lbM, diffic.
	1889	April 2	....	..	13·0	2	pB, diffic.
	....	....	10 23 8·02	..	+30 3 13·8	..	
2134	1889	Mar. 6	10 26 45·84	7	+29 4 35·5	3	cB, S, mbM
3277	1891	Mar. 31	46·08	4	38·9	2	B
	1891	April 7	46·36	7	39·4	2	
	1891	April 9	46·00	6	37·3	2	
	....	....	10 26 46·07	..	+29 4 37·8	..	
2145	1887	Feb. 18	10 29 54·78	6	+37 53 48·4	1	cB, vL
3294	1887	Mar. 23	55·28	5	48·8	1	vL
	....	....	10 29 55·03	..	+37 53 48·6	..	
	1888	April 6	10 29 55·92	7	+37 53 26·1	1	cB, L, no cond.
2150	1887	Feb. 19	10 30 56·29	11	+22 26 59·0	1	
3301	1887	Feb. 26	56·30	8	73·0	1	Decl. ±
	....	....	10 30 56·30	..	+22 26 66·0	..	
	1888	Feb. 15	55·83	4	69·0	2	cB, pS
	1891	April 7	56·29	2	63·4	2	
	....	....	10 30 56·06	..	+22 26 66·2	..	
2158	1887	Feb. 28	10 31 55·13	11	+54 4 27·9	1	vB, pL, R, N = * 10
3310	1887	Mar. 24	55·08	14	26·3	2	
	....	....	10 31 55·10	..	+54 4 27·1	..	
	1889	Mar. 22	54·53	7	25·6	3	cB, pL, vmbM
	1889	April 2	.....	..	21·7	3	B, S, = * 10-11
	1891	Mar. 31	54·95	3	24·1	2	vB
	1891	April 9	54·83	2	24·7	2	
	....	....	10 31 54·77	..	+54 4 24·0	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
2184	1887	Feb. 19	10 38 9·29	11	+12 16 47·7	1	B, L
3351	1887	Mar. 23	9·16	14	49·6	2	
	....	....	10 38 9·22	..	+12 16 48·6	..	
	1891	Mar. 13	9·11	5	51·3	2	
	1891	Mar. 31	9·15	2	42·5	1	
	....	....	10 38 9·13	..	+12 16 46·9	..	
2182	1887	Feb. 24	10 38 52·87	16	+73 25 14·2	1	B, S
3348	....	....	....	..	....	..	
	1888	April 6	10 38 53·15	5	+73 25 11·7	3	B, S, * 10 pr
2201	1887	April 10	10 41 52·08	12	+14 33 50·8	1	
3377	....	....	....	..	....	..	
	1888	Feb. 15	51·64	6	55·1	1	
	1889	Mar. 22	52·12	7	57·0	3	B, vmbM
	....	....	10 41 51·88	..	+14 33 56·0	..	
2203	1888	Mar. 19	10 42 0·72	7	+13 9 42·7	1	vB, cL, R, psbM
3379	1891	April 7	1·47	6	43·6	2	RA±
	1891	April 9	0·34	7	36·9	3	
	....	....	10 42 0·84	..	+13 9 40·8	..	
2231	1889	Mar. 6	10 45 34·40	4	+6 24 35·5	1	pB, vL, bM
3423	....	....	....	..	....	..	
2274	1887	Feb. 28	10 54 23·30	9	+29 33 55·6	1	B, L, iF
3486	1887	April 10	22·00	14	59·9	2	B, L, R, bM
	....	....	10 54 22·65	..	+29 33 57·7	..	
	1888	Feb. 15	23·52	7	48·7	1	B
	1889	Mar. 6	23·08	5	49·8	2	cB
	1891	Mar. 31	23·78	5	61·9	2	pB
	....	....	10 54 23·46	..	+29 33 53·5	..	
2276	1887	Feb. 19	10 54 30·86	13	+14 29 25·1	1	vB, BN
3489	1887	Mar. 23	30·78	12	30·9	2	
	1887	Mar. 24	30·37	10	26·8	2	
	....	....	10 54 30·67	..	+14 29 27·6	..	
	1888	Mar. 19	30·28	7	28·7	2	
	1888	Mar. 20	30·92	7	29·5	2	
	1889	Mar. 22	30·52	5	27·2	3	vB, S, N
	1889	Mar. 27	30·07	7	25·8	3	cB, S, N, through haze
	....	....	10 54 30·45	..	+14 29 27·8	..	
2287	1888	April 6	10 57 12·08	7	+28 33 53·9	2	
3504	1890	Mar. 23	12·68	7	48·3	3	
	1891	Mar. 31	12·42	7	51·1	2	cB
	1891	April 13	13·68	2	51·8	1	through clouds, RA±
	....	....	10 57 12·58	..	+28 33 51·3	..	
2301	1888	Mar. 19	11 0 9·94	7	+0 33 29·8	2	B, pL, N
3521	1888	Mar. 20	10·33	3	31·9	1	pB
	1890	Mar. 11	10·53	6	30·9	3	
	....	....	11 0 10·27	..	+0 33 30·9	..	
2343	1888	Mar. 9	11 8 22·08	5	+55 36 54·6	2	B, vvL, R, no cond.
3587	1891	April 13	24·64	4	80·9	2	cB, vL, no cond.
	....	....	11 8 23·36	..	+55 36 67·7	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
2347	1887	April 10	11 8 51·49	14	+13 24 59·3	2	B, cL, R
3593	....	....	....	..	....	..	
	1890	Mar. 23	.....	..	67·9	1	Decl.±
	1891	Mar. 31	51·35	4	59·5	2	pB, Decl.±
	1891	April 11	51·83	6	72·5	2	Decl.±
	....	....	11 8 51·59	..	+13 24 66·6	..	
2358	1887	Feb. 28	11 11 7·26	12	+18 39 6·9	1	
3607	1887	Mar. 22	7·16	11	4·3	1	vB, L, vmbM
	....	....	11 11 7·21	..	+18 39 5·6	..	
	1889	Mar. 22	6·78	7	4·7	3	
	1889	Mar. 27	6·35	6	2·5	3	B, pL, vmbM
	....	....	11 11 6·56	..	+18 39 3·6	..	
2373	1888	Mar. 19	11 13 10·88	7	+13 41 38·2	2	B, pL, N
3623	1890	Mar. 23	10·80	7	39·7	2	
	....	....	11 13 10·84	..	+13 41 39·0	..	
2376	1887	Mar. 19	11 14 16·27	11	+18 57	..	cB, N
3626	....	....	....	..	....	..	
	1891	Mar. 31	16·45	7	27·4	3	B, S, sbM
	1891	April 11	16·22	7	24·1	2	
	....	....	11 14 16·33	..	+18 57 25·7	..	
2396	1887	April 10	11 17 8·77	6	+17 11 38·1	1	
3655	....	....	....	..	....	..	
	1891	April 13	11 17 9·13	7	+17 11 42·7	2	pB, pL
2404	1890	Mar. 23	11 18 44·78	6	+39 21 58·2	3	cB
3665	1891	April 11	44·84	3	56·9	1	cB, pS
	....	....	11 18 44·81	..	+39 21 57·5	..	
2413	1887	Feb. 24	11 20 7·66	18	+44 11 22·7	1	vB, cL
3675	1887	Mar. 22	7·71	11	17·1	1	through clouds
	....	....	11 20 7·68	..	+44 11 19·9	..	
	1888	Mar. 9	6·55	6	25·3	1	
	1889	Mar. 22	6·82	7	21·8	3	B, mbM
	1889	Mar. 27	6·68	5	22·9	2	cB, L
	....	....	11 20 6·68	..	+44 11 23·3	..	
2421	1890	Mar. 11	11 21 17·36	4	+57 28 55·1	2	pB
3683	1891	Mar. 31	17·12	5	49·8	1	pB, Decl.±
	....	....	11 21 17·24	..	+57 28 52·4	..	
2426	1888	Mar. 19	11 22 21·61	5	+26 15 58·2	1	pB, pL
3689	1891	April 11	.....	..	37·9	1	pF, diffic.
	....	....	11 22 21·61	..	+26 15 48·0	..	
2445	1889	April 2	11 27 20·20	5	+47 37 73·4	2	pB, vL, * 11 n
3726	1891	Mar. 31	21·24	4	....	..	pB, diffic.
	1891	April 11	20·33	5	74·0	1	pB, vL, * np
	1891	April 13	22·84	2	58·4	1	RA±, v diffic.
	....	....	11 27 20·91	..	+47 37 68·6	..	
2454	1888	Mar. 31	11 29 44·82	4	+55 7 53·7	1	pB, pL, 2F ** nf
3738	1890	Mar. 11	43·68	4	60·1	2	pB
	1890	Mar. 17	....	..	57·1	2	
	1890	Mar. 23	43·57	4	52·0	2	cB
	....	....	11 29 44·02	..	+55 7 55·7	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
2499	1887	Feb. 19	11 35 18·59	1	+12 4 60·4	1	cB, Decl.±
3810	1887	Feb. 28	18·85	7	48·2	1	cB, vL, E
	....	....	11 35 18·75	..	+12 4 54·3	..	
	1888	Mar. 19	17·98	2	....	..	
	1891	Mar. 31	17·78	6	52·1	1	B, L
	1891	April 11	17·63	7	55·6	2	
	....	....	11 35 17·80	..	+12 4 53·8	..	
2501	1888	Mar. 20	11 35 29·15	6	+37 9 19·5	1	pF, cL, moonlight
3813	1889	April 2	28·66	6	23·1	3	cB
	....	....	11 35 28·91	..	+37 9 21·3	..	
2520	1887	Mar. 19	11 38 16·68	17	+58 33 31·9	1	pB, S, R, mbM
3838	1887	April 10	16·77	13	29·7	2	pB, pS, R, well seen
	....	....	11 38 16·72	..	+58 33 30·8	..	
	1889	Mar. 22	17·57	5	34·2	1	pF, v diffic.
	1890	Mar. 23	14·04	6	42·8	2	v diffic.
	1891	April 13	14·79	6	33·7	1	pF, F * s
	....	....	11 38 15·47	..	+58 33 36·9	..	
2545	1888	Mar. 31	11 40 14·98	6	+48 5 63·2	1	cB, cL, lbM, diffic.
3877	1889	Mar. 6	16·61	4	76·8	3	bad definition
	1889	Mar. 30	.....	..	59·3	2	pB
	1891	April 7	17·29	5	67·9	2	B, L, no cond.
	1891	April 11	16·87	5	76·4	3	
	....	....	11 40 16·44	..	+48 5 68·7	..	
2564	1888	Mar. 19	11 43 22·93	7	+56 41 42·4	1	B, mbM
3898	1889	April 2	22·58	6	42·2	3	vB, N
	....	....	11 43 22·75	..	+56 41 42·3	..	
2566	1887	Feb. 19	11 43 26·79	4	+27 37 56·6	1	cB, RA±
3900	....	....	....	..	....	..	
	1888	Mar. 20	26·41	7	61·3	1	cB, N
	1891	Mar. 31	26·99	6	55·7	1	
	....	....	11 43 26·70	..	+27 37 58·5	..	
2597	1888	Mar. 9	11 47 4·25	7	+44 43 65·5	1	B, pL, N
3938	1890	Mar. 23	4·05	5	52·9	2	pB, L
	1891	April 7	3·35	3	66·0	2	pB, diffic.
	1891	April 13	.....	..	43·7	2	v diffic.
	....	....	11 47 3·88	..	+44 43 57·0	..	
2600	1887	Feb. 28	11 47 12·78	15	+37 35 53·7	1	
3941	1887	Mar. 19	12·64	13	58·6	1	vB, N
	1887	Mar. 20	13·24	13	54·0	2	
	....	....	11 47 12·89	..	+37 35 55·4	..	
	1889	Mar. 22	12·14	6	50·1	3	vB, pL, N
	1889	Mar. 27	.....	..	52·6	3	vB, N
	1891	April 11	12·21	7	54·1	3	vB, N
	....	....	11 47 12·17	..	+37 35 52·3	..	
2635	1887	April 10	11 51 52·96	13	+53 59 13·1	2	pB, L, N, bright moon-[light
3992	....	....	....	..	....	..	
	1889	April 2	.....	..	8·9	2	vL, vmbM
	1891	Mar. 31	53·42	5	8·5	2	cB, vL, N
	1891	April 11	52·05	4	13·1	2	
	....	....	11 51 52·73	..	+53 59 10·2	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
2660	1887	Feb. 24	11 53 44·34	22	+51 34 27·8	1	vB, N = * 10
4026	1887	Mar. 22	43·99	15	24·8	3	vB, cL, stellar N
	....	....	11 53 44·16	..	+51 34 26·3	..	
	1888	Mar. 20	43·56	7	23·3	1	vB, pL, N, moonlight
	1890	Mar. 23	43·76	6	24·9	3	
	....	....	11 53 43·66	..	+51 34 24·1	..	
2663	1887	Feb. 19	11 54 45·91	11	−0 29 8·1	1	cB, L, mbM
4030	1887	Mar. 23	45·79	16	13·5	1	
	....	....	11 54 45·85	..	−0 29 10·8	..	
	1891	April 7	45·61	6	18·3	3	cB, L, bM
	1891	April 13	45·75	4	22·0	2	moonlight
	....	....	11 54 45·68	..	−0 29 20·2	..	
2680	1887	Feb. 28	11 57	..	+45 8 40·6	1	B
4051	1887	Mar. 19	31·39	12	50·7	1	cB, psbM
	....	....	11 57 31·39	..	+45 8 45·6	..	
	1888	Mar. 9	31·04	7	40·5	1	B, pL, N, B * pr
	1889	Mar. 22	31·19	3	38·0	2	B, diffic.
	....	....	11 57 31·11	..	+45 8 39·2	..	
2717	1890	Mar. 23	12 0 47·13	6	+53 19 23·4	2	B, pS, R
4102	1891	Mar. 31	47·97	4	19·2	2	2F ** pr. 3ˢ
	1891	April 7	47·42	5	29·5	3	
	....	....	12 0 47·51	..	+53 19 24·0	..	
2723	1887	April 1	12 1 27·79	17	+43 40 44·4	2	vB, = * 10
4111	....	....	....	..	....	..	
	1888	Mar. 19	27·05	7	41·5	1	vB, pL, B * nf
	1890	Mar. 17	27·36	3	44·3	2	
	....	....	12 1 27·20	..	+43 40 42·9	..	
2752	1887	Mar. 20	12 4 29·77	13	+19 9	..	
4147	....	....	....	..	....	..	
	1889	April 2	29·53	7	10·6	4	B, L, mbM
	1891	April 7	30·61	1	14·2	1	
	1891	April 13	30·62	7	12·2	4	
	1891	April 14	28·58	5	11·2	2	diffic., moonlight
	....	....	12 4 29·92	..	+19 9 12·0	..	
2765	1888	Mar. 20	12 6 17·39	5	+24 44 0·4	1	pF, pL, B * sp, RA ±
4162	1891	April 11	16·30	3	19·3	1	F, through clouds, * 9·5 sp
	....	....	12 6 16·85	..	+24 44 9·8	..	
2786	1887	Feb. 19	12 8 12·06	3	+15 30 42·0	1	
4192	1887	Mar. 23	12·09	11	47·0	1	vL, vmbM
	....	....	12 8 12·07	..	+15 30 44·5	..	
	1888	Mar. 31	11·73	6	34·4	1	cB, vL, N
	1889	Mar. 27	11·83	7	38·7	2	cB
	....	....	12 8 11·78	..	+15 30 36·5	..	
2796	1887	Feb. 24	12 9 32·25	17	+33 48 36·2	1	vB, S, = * 10-11
4203	....	....	....	..	....	..	
	1889	Mar. 30	.....	..	30·2	3	
	1890	Mar. 17	32·52	2	....	..	
	1890	Mar. 23	32·16	6	31·1	3	
	....	....	12 9 32·34	..	+33 48 30·6	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
2806	1887	April 10	12 10 18·44	14	+13 45	..	Decl. wire disappears in [moonlight
4216	....	....	....	..	....	..	
	1888	Mar. 19	18·05	7	41·7	1	vB, vL, N
	1891	Mar. 31	18·46	4	39.1	1	
	1891	April 13	18·75	6	40.9	3	
	....	....	12 10 18·42	..	+13 45 40.6	..	
2841	1887	Mar. 20	12 13 30·90	17	+47 54 50·7	2	
4258	....	....	....	..	....	..	
	1889	April 25	.....	..	51·6	3	vB, vL, N
	1890	Mar. 17	30·18	3	56·0	2	cB
	1890	Mar. 23	30·64	4	54·7	2	
	1891	April 7	30·94	7	49·6	3	
	1891	April 11	(31·10)	1	51·7	1	vL
	1891	April 14	30·75	7	50·6	3	
	....	....	12 13 30·63	..	+47 54 52·4	..	
2904	1889	Mar. 30	12 17 57·85	7	+6 41 30·7	2	cB, S
4339	1891	Mar. 31	58 54	6	29·3	1	
	....	....	12 17 58·20	..	+6 41 30·0	..	
2921	1887	Feb. 24	12 18 52·08	15	+7 55 39·6	2	vB, N
4365	1887	Mar. 23	52·06	17	38·2	2	B, cL
	....	....	12 18 52·07	..	+7 55 38·9	..	
	1889	Mar. 27	.....	..	34·7	3	cB, S
	1891	April 13	52·64	4	37·4	1	
	....	....	12 18 52·64	..	+7 55 36·0	..	
2924	1888	Mar. 19	12 19 9·04	3	+39 59 40·1	1	pB, pS
4369	1891	April 11	9·26	6	38·9	3	cB, S
	....	....	12 19 9·15	..	+39 59 39·5	..	
2961	1888	Mar. 31	12 20 37·03	6	+13 33 13·5	1	cB, L, N
4406	1890	Mar. 23	37·29	7	20·6	3	
	1891	April 7	37·35	6	17·8	3	
	1891	April 14	37·62	5	20·3	3	pB, moonlight
	....	....	12 20 37·32	..	+13 33 18·0	..	
3025	1888	Mar. 20	12 24 28·38	6	+14 14 39·0	2	cB, pS, N
4477	1889	Mar. 27	.....	..	43·5	2	cB, pL
	1891	April 11	28·62	4	23·5	1	Decl. ±
	....	....	12 24 28·50	..	+14 14 37·7	..	
3042	1890	Mar. 23	12 25 13·18	4	+12 14 66·1	2	B, vL, no cond.
4490	1891	April 7	13·86	4	46·6	1	vL, no cond., through [clouds
	....	....	12 25 13·52	..	+12 14 56·3	..	
3035	1887	Mar. 20	12 25 15·34	11	+12 59 55·0	2	vB
4486	....	....	....	..	....	..	
	1889	Mar. 30	15·09	6	53·6	2	cB, S
	1891	April 13	15·82	1	58·6	1	vB, L
	....	....	12 25 15·46	..	+12 59 56·1	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
3075	1887	Feb. 19	12 28 27·98	13	+8 18	..	
4526	1887	Feb. 24	27·87	14	22·3	2	vB, vL, N
	1887	Mar. 23	27·65	14	20·0	2	vB, pL, N
	....	....	12 28 27·83	..	+8 18 21·2	..	
	1889	April 25	27·46	6	18·7	3	B, pL
	1891	April 11	27·41	4	16·6	3	vB, mbM
	....	....	12 28 27·43	..	+8 18 17·6	..	
3106	1890	Mar. 23	12 30 53·14	6	+26 35 28·2	2	B, pL
4565	1891	April 13	53·30	7	31·6	3	
	1891	April 14	53·22	3	39·2	2	
	....	....	12 30 53·22	..	+26 35 33·0	..	
3121	1888	Mar. 20	12 32 10·22	6	+12 25 27·5	1	B, pL, N
4579	1888	April 4	9·34	6	22·0	1	F in clouds, RA±
	1889	Mar. 27	9·86	6	23·8	3	B, pL, N
	....	....	12 32 9·81	..	+12 25 24·4	..	
3132	1887	April 14	12 34 16·60	12	−11 1 8·1	1	vB, vL, N
4594	....	....	....	..	....	..	
	1891	April 11	12 34 16·50	6	−11 1 7·1	3	vB, vL, N
3165	1888	Mar. 31	12 36 43·31	5	+33 9 4·1	1	vB, vL, N = * 11-12
4631	1889	April 25	.....	..	11·8	2	pB, through clouds
	1890	Mar. 17	43·75	2	....	..	
	1891	Mar. 31	43·94	4	9·2	2	vL, N
	1891	April 13	44·16	3	7·8	2	
	....	....	12 36 43·79	..	+33 9 8·2	..	
3169-3170	1887	Feb. 24	12 37 13·35	12	+3 17 23·8	1	B, L, mbM
	1887	Mar. 19	13·19	13	30·3	2	
4636	....	....	12 37 13·27	..	+3 17 27·0	..	
3182	1888	Mar. 19	12 38 7·25	7	+12 9 20·4	1	vB, pL, R
4649	1891	April 14	7·76	7	18·6	3	
	....	....	12 38 7·50	..	+12 9 19·5	..	
3193	1887	Mar. 23	12 38 59·40	15	+11 47 35·7	2	B, S, N
4660	....	....	....	..	....	..	
	1889	Mar. 27	58·73	3	32·3	1	B
	1890	Mar. 23	59·62	6	34·8	3	
	1891	April 11	59·20	3	34·2	1	
	....	....	12 38 59·18	..	+11 47 33·8	..	
3198	1888	April 4	12 39 29·38	5	+0 8 11·8	1	through clouds
4666	....	....	....	..	....	..	
3227	1888	May 4	12 42 54·75	5	−5 11 59·8	2	twilight
4697	1890	Mar. 17	54·74	5	61·6	3	
	1890	Mar. 23	55·04	4	62·4	2	
	....	....	12 42 54·84	..	−5 11 61.3	..	
3249	1888	Mar. 20	12 45 2·89	7	+26 6 0·8	1	vB, vL, N
4725	1891	April 13	3·57	6	4·1	3	
	....	....	12 45 3·23	..	+26 6 2·5	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
3258	1887	April 14	12 45 41·98	12	+41 43	..	vB, L
4736	....	....	....	..	....	..	
	1888	Mar. 31	41·61	7	8·4	2	vB, S, N
	1889	April 25	.....	..	5·2	4	vB, N = * 10
	1891	April 11	41·94	6	8·4	4	
	1891	April 14	42·42	1	6·7	2	vB, RA±
	....	....	12 45 41·90	..	+41 43 7·2	..	
3274	1888	April 4	12 46 45·41	7	+11 54 36·5	2	vB, pS, N, 2 ** pr
4754	1889	Mar. 27	45·45	7	41·9	3	B, pS
	....	....	12 46 45·43	..	+11 54 39·2	..	
3300	1887	Mar. 23	12 49 21·58	8	+29 31 79·3	1	pB, pL, * 8 n 1′
4793	....	....	....	..	....	..	
	1890	Mar. 23	20·28	4	49·9	1	RA±
	1891	Mar. 31	20·89	3	51·4	2	
	....	....	12 49 20·58	..	+29 31 50·6	..	
3315	1888	Mar. 19	12 50 36·42	6	+58 56 22·6	1	pB, pL
4814	....	....	....	..	....	..	
3321	1887	Mar. 19	12 51 19·43	15	+22 16 44·4	2	vB, vL, N
4826	1887	April 14	19·47	13	41·6	2	vB, vL, N
	....	....	12 51 19·45	..	+22 16 43·0	..	
	1890	Mar. 17	19·45	7	38·1	3	
	1891	April 11	19·06	6	38·7	3	
	1891	April 13	19·55	2	38·7	2	
	....	....	12 51 19·35	..	+22 16 38·5	..	
3342	1888	Mar. 20	12 53 58·32	7	+14 45 46·6	1	cB, pL, 2 condensations;
4866	1889	April 25	.....	..	50·0	2	pF, diffic. [observed f
	1890	Mar. 23	58·43	3	45·7	1	2 condensat'ns, observed f
	....	....	12 53 58·37	..	+14 45 47·4	..	
3356	1888	April 4	12 55 4·28	7	+3 5 6·6	1	observed * 10 att
4900	1891	April 13	5·08	4	5·2	2	same
	....	....	12 55 4·68	..	+3 5 5·9	..	
3395	1888	Mar. 31	12 59 50·49	6	+35 46 4·4	1	pB, pS, bM, * n f
4956	1889	April 21	.....	..	1·8	1	pB, pS, smbM
	1891	April 11	51·10	6	3·6	3	cB
	1891	April 13	51·73	2	7·8	1	pF, RA±
	....	....	12 59 50·98	..	+35 46 4·4	..	
3397	1887	Mar. 19	13 0 5·43	11	−7 25	..	
4958	1887	Mar. 23	5·47	12	52·6	1	vB, S, N
	....	....	13 0 5·45	..	−7 25 52·6	..	
	1888	Mar. 19	4·97	5	49·6	1	
	1889	Mar. 27	5·10	7	54·2	2	vB, S, N
	....	....	13 0 5·03	..	−7 25 51·9	..	
3437	1887	Mar. 19	13 5 51·38	18	+37 38 44·5	2	vB, N
5005	1887	Mar. 22	51·01	9	42·8	2	vL, through clouds
	....	....	13 5 51·20	..	+37 38 43·6	..	
	1888	April 4	50·48	5	36·8	3	
	1889	April 25	50·73	6	33·4	3	B, pL
	1890	Mar. 17	50·76	7	36·3	3	
	1890	Mar. 23	50·89	7	35·1	3	
	....	....	13 5 50·71	..	+37 38 35·4	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.			Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
				h m s		° ′ ″		
3453	1888	Mar.	20	13 7 31·83	7	+18 45 3·0	3	vvB, vL, vmbM
5024	1891	April	11	31·67	4	2·4	4	vvB
	....	....		13 7 31·75	..	+18 45 2·7	..	
3459	1887	Mar.	23	13 8 23·08	13	+37 10 32·2	2	
5033	....	....		....	..	....	..	
	1888	Mar.	19	23·19	7	38·3	1	vB, pL, N
	1889	Mar.	27	....	..	35·8	2	B, N
	1890	Mar.	23	22·82	6	36·3	3	
	....	....		13 8 23·00	..	+37 10 36·8	..	
3474	1888	Mar.	31	13 10 51·49	7	+42 36 37·1	2	B, vL, N
5055	1890	Mar.	17	52·59	6	42·3	3	
	1891	Mar.	31	53·17	5	37·2	2	vB, vL, N
	1891	April	13	53·49	7	35·7	3	
	....	....		13 10 52·69	..	+42 36 38·1	..	
3505	1888	April	4	13 15 37·45	5	+43 39 32·9	1	pB, pS, * 7 interferes
5103	1889	April	21	.....	..	36·0	3	pB, S, B * n 1′
	1889	April	25	37·31	6	31·1	3	pB
	....	.. ..		13 15 37·38	..	+43 39 33·3	..	
3524	1887	Mar.	23	13 18 38·30	7	+32 8 33·2	1	pB, 2 ** 11-12 n p
5127	....	....		....	..	....	..	
	1889	Mar.	27	.....	..	25·1	1	pB, diffic.
	1891	Mar.	31	39·06	1	29·1	1	diffic., RA±
	1891	April	11	37·72	4	27·9	1	pB, diffic.
	....	....		13 18 38·17	..	+32 8 27·4	..	
3572	1888	Mar.	31	13 25 14·11	7	+47 45 43·5	2	vB, N (s p condensation)
5194	1888	May	4	13·55	5	44·9	1	B, twilight
	1889	Mar.	27	.....	..	45·5	2	
	1889	April	21	.....	..	44·1	3	vB
	1891	Mar.	31	14·23	6	47·7	3	
	1891	April	13	14·29	7	....	..	
	....	....		13 25 14·05	..	+47 45 45·1	..	
3592	1888	April	4	13 28 23·29	6	+63 20 10·3	1	pF, diffic.
5218	1890	Mar.	17	24·23	4	11·2	2	
	1891	April	11	23·97	5	5·5	2	v diffic.
	....	....		13 28 23·83	..	+63 20 9·0	..	
3615	1887	Feb.	24	13 32 3·65	14	+9 26 50·2	1	B, N
5248	1887	Mar.	23	3·65	9	54·6	2	B, N
	....	....		13 32 3·65	..	+9 26 52·4	..	
	1889	April	21	....	..	45·9	2	B, pL, N
	1891	Mar.	31	3·57	4	....	..	cB, L
	1891	April	11	2·93	1	44·2	1	RA±
	....	....		13 32 3·36	..	+9 26 45·1	..	
3636	1887	Feb.	24	13 37 7·39	7	+28 55	..	vvB, vL, 2 N?
5272	1887	Mar.	19	7·14	12	....	..	vvB, vL, N
	....	....		13 37 7·26	..	....	..	
	1888	Mar.	20	6·51	7	42·3	2	Observed cond. in s.p. part
	1888	Mar.	31	6·71	7	51·0	2	vB, vL, N
	1888	May	2	6·41	6	58·9	1	vB, vvL, N
	1891	April	11	6·54	6	57·8	4	
	....	....		13 37 6·54	..	+28 55 52·5	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.	Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
		h m s		° ′ ″		
3637	1887 April 14	13 37 15·39	11	+36 12 31·8	1	cB, pL
5273	....	....	..	....	..	
	1888 April 16	14·32	5	31·9	1	cB, pL
	1888 May 4	14·83	5	29·4	2	cB, mbM
	1891 April 13	16·04	3	40·1	1	RA±
	....	13 37 14·87	..	+36 12 33·8	.	
3645	1887 Mar. 23	13 40	..	+42 15 54·9	1	pB, E
5290	....	....	..	....	..	
	1888 April 4	38·15	6	47·0	1	pL
	1889 April 25	....	..	48·0	1	pB
	....	13 40 38·15	..	+42 15 47·5	..	
3671	1888 Mar. 19	13 45 33·54	5	+60 44 16·8	1	B, L, RA±
5322	1888 May 4	35·26	5	15·8	2	B, pL
	1889 Mar. 27	....	..	13·4	2	vB, vS, N
	1890 Mar. 17	34·81	5	17·6	2	
	....	13 45 34·74	..	+60 44 15·9	..	
3702	1887 Mar. 23	13 50 35·76	13	+5 47 39·5	2	vB, * 8·5 n f 17ˢ
5363	1887 April 14	36·17	5	32·8	1	cB, pL
	....	13 50 35·96	..	+5 47 36·2	..	
	1890 Mar. 23	35·55	6	40·7	1	
	1891 Mar. 31	36·04	7	32·5	3	
	....	13 50 35·80	..	+5 47 36·6	..	
3716	1888 April 4	13 51 53·82	7	+47 46 17·2	1	cB, L, N
5377	1891 April 11	54·73	3	35·5	2	±
	1891 May 9	55·33	2	29·4	1	} diffic., twilight
	1891 May 10	54·06	2	....	..	
	....	13 51 54·49	..	+47 46 27·4	..	
3770	1889 April 25	13 59 16·29	5	+54 52 20·4	2	pB, vL, N=12, * 11-12 [n f
5457	....	....	..	....	..	
*11-12 nf 3770 5457	1887 Mar. 23	13 59 20·82	10	+54 53 57·5	2	
	... .	....	..	....	..	
	1888 May 4	20·44	4	54·5	2	N of 3770 seen in dark [field
	1890 Mar. 17	20·28	4	60·2	2	
	1890 Mar. 23	19·71	6	55·8	2	
	1891 Mar. 31	19·81	3	60·2	2	
	1891 April 11	20·10	6	60·1	3	
	....	13 59 20·07	..	+54 53 58·2	..	
3794	1888 May 2	14 3 20·90	4	+55 31 31·1	1	pB, S, clouds
5485	1891 April 13	22·50	7	28·3	4	
	1891 May 9	22·68	2	26·7	1	
	1891 May 11	22·14	2	16·3	2	vF in twilight
	....	14 3 22·06	..	+55 31 25·6	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.	Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
		h m s		° ′ ″		
3846	1887 April 14	14 14 48·07	10	+4 26	..	
5566	....	....	..	....	..	
	1888 May 4	47·74	7	22·9	1	B, brighter than * f
	1889 Mar. 27	47·64	7	23·1	2	cB, * 12 s f
	1889 April 25	47·34	5	19·6	2	B, S, Decl. ±
	1890 Mar. 17	47·75	7	18·6	2	
	1890 Mar. 23	47·75	7	22·5	2	
	....	14 14 47·64	..	+4 26 21·3	..	
3854	1889 April 21	14 16	..	+40 11 51·2	2	pB, N, B * s f, F * s p
5582	1891 April 13	15·17	4	48·7	3	
	1891 May 9	14·29	5	55·3	2	cB
	1891 May 10	14·52	5	48·6	2	
	1891 May 11	14·62	3	35·6	1	Decl. ±
	....	14 16 14·65	..	+40 11 49·3	..	
3897	1887 April 14	14 23	..	+57 4 36·5	2	
5631	....	....	..	....	..	
	1888 Mar. 20	....	..	31·0	1	
	1888 May 4	7·37	6	37·3	2	B, S, R
	1889 Mar. 27	6·60	6	32·4	2	B
	1890 Mar. 17	7·10	6	....	..	
	....	14 23 7·02	..	+57 4 33·6	..	
3942	1888 April 4	14 31 35·48	7	+49 13 17·4	1	cB, pL, mbM
5689	1889 Mar. 27	35·57	5	19·7	2	
	1890 Mar. 17	35·81	5	13·6	2	
	1890 Mar. 23	35·58	6	20·0	3	
	....	14 31 35·61	..	+49 13 17·7	..	
3956	1888 May 4	14 33 41·37	4	+5 50 22·2	1	pB, diffic.
5701	1891 May 9	42·36	6	11·1	1	pB
	1891 May 10	41·89	1	17·5	1	pB, RA±
	1891 May 11	42·14	5	13·3	1	pB
	....	14 33 41·95	..	+5 50 16·0	..	
3987	1887 Mar. 23	14 39 21·53	13	+2 25 19·9	2	cB, L, bM
5746	....	....	..	....	..	
	1889 Mar. 27	21·01	7	17·6	1	pB, Decl. too large
	1890 Mar. 17	20·95	7	8·7	2	
	1890 Mar. 23	21·10	5	9·3	1	
	....	14 39 21·02	..	+2 25 11·9	..	
4021	1888 May 4	14 54 25·36	5	+2 19 41·6	1	pF, pL, Decl. ± diffic.
5806	1891 April 13	25·36	4	23·5	1	
	1891 May 10	25·39	7	28·6	2	pB, RA good
	1891 May 11	26·06	5	37·2	1	
	....	14 54 25·54	..	+2 19 32·7	..	
4045	1887 Mar. 23	15 0 55·02	14	+2 1 54·1	2	
5846	....	....	..	....	..	
	1890 Mar. 17	54·21	6	51·3	2	
	1891 April 11	53·87	7	48·2	2	vL
	1891 May 9	54·65	5	48·2	2	vB, L
	1891 May 11	54·70	5	54·8	2	
	....	15 0 54·36	..	+2 1 50·6	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
4058	1888	May 11	15 3 27·60	7	+56 11 13·7	2	
5866	1891	April 13	27·78	7	8·4	4	
	1891	May 10	27·61	6	14·2	4	
	1891	May 11	28·63	5	10·8	3	
	....	....	15 3 27·90	..	+56 11 11·8	..	
4064	1887	April 15	15 6 55·48	14	+57 24 65·3	2	
5879	....	....	....	..	....	..	
	1888	April 4	54·09	3	55·2	1	
	1891	April 11	53·65	6	53·7	3	
	1891	May 9	55·31	5	57·2	3	
	....	....	15 6 54·35	..	+57 24 55·4	..	
4077	1887	Mar. 23	15 11 4·17	6	+42 27 25·4	1	pB, pL
5899	....	....	....	..	....	..	
	1891	April 11	....	..	30·1	1	Decl. ±
	1891	April 13	4·97	5	18·6	2	pB, v diffic.
	1891	May 10	....	..	2·5	2	Decl. ±
	1891	May 11	4·38	4	21·9	1	diffic. good obs.
	....	....	15 11 4·67	..	+42 27 18·3	..	
4128	1887	April 15	15 36 26·11	12	+59 42 50·1	3	B, pS, cond. = * 10-11
5982	....	....	....	..	....	..	
	1888	May 4	25·39	6	53·3	2	pB, S, R, Decl. ±
	1891	April 11	25·21	7	45·6	4	
	1891	April 13	25·28	4	50·0	3	
	1891	May 9	24·71	7	45·9	2	
	1891	May 10	24·60	6	49·8	3	
	1891	May 11	25·00	5	47·5	1	
	....	....	15 36 25·03	..	+59 42 48·7	..	
4234	1888	May 4	16 39 52·95	6	+24 0 14·8	2	vB, S, R, = * 9·5. Red [field illumination
6210	1891	April 11	53·08	6	13·5	4	Through clouds. Bright wire illumination
	1891	April 13	53·18	7	13·4	3	
	1891	May 9	53·06	7	16·1	3	
	1891	May 11	52·88	6	13·0	3	= * 9·5. Red field illumination
	....	....	16 39 53·03	..	+24 0 14·2	..	
4256	1888	May 4	16 51 22·16	6	−3 55 19·7	1	N, another cond. nf, diffic. in twilight
6254	1891	April 13	21·39	4	24·4	1	
	1891	May 9	22·07	5	15·6	1	
	1891	May 11	22·33	7	....	..	
	....	....	16 51 21·99	..	−3 55 19·9	..	
4625	1886	Sept. 27	20 56 21·20	9	+15 44 57·4	1	
7006	....	....	....	..	....	..	
	1888	Oct. 12	20 56 20·60	7	+15 45 19·2	3	B, pS, R, moonlight
4670	1886	Sept. 27	21 24 39·37	15	+11 41 4·5	2	vB, vL, mbM
7078	....	....	....	..	....	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
4678	1886	Oct. 22	21 27 47·36	14	−1 18 37·6	2	vvB, vL, d = 4ˢ
7089	1886	Nov. 16	47·19	12	44·9	1	vB, vL, iR
	1886	Nov. 18	47·09	10	38·3	2	vvB, vL, iR, pmbM
	....	....	21 27 47·21	..	−1 18 40·3	..	
	1888	Oct. 12	46·75	6	37·1	3	vvB, L, lbM, moonlight
	1888	Oct. 14	46·76	6	43·1	2	vL, vmbM, strong moonlight
	1888	Oct. 29	46·97	7	41·5	3	vB, vL, iR, mbM
	....	....	21 27 46·83	..	−1 18 40·6	..	
4734	1886	Oct. 22	21 55 26·77	7	+17 12 33·8	1	F
7177	1886	Nov. 18	26·43	7	28·4	1	F
	1886	Nov. 27	26·02	5	....	..	F, diffic.
	1886	Nov. 29	26·53	6	....	..	
	....	....	21 55 26·44	..	+17 12 31·1	..	
	1888	Oct. 12	....	..	32·8	2	pB, pS
	1888	Oct. 14	26·43	4	27·2	1	pB, S, moonlight
	1888	Oct. 23	26·19	6	30·6	2	pF, pS
	1888	Oct. 29	26·23	7	35·7	2	cB, pS, mbM
	....	....	21 55 26·28	..	+17 12 31·6	..	
4760	1886	Aug. 26	22 2 56·76	7	+30 49 16·2	1	
7217	1886	Oct. 22	57·06	7	10·8	1	
	1886	Oct. 31	56·78	12	12·8	1	
	1886	Nov. 27	56·60	14	15·9	1	
	1886	Nov. 29	56·86	13	....	..	B, mbM
	....	....	22 2 56·81	..	+30 49 13·9	..	
	1888	Sept. 5	56·17	7	20·3	2	B, pL, mbM
	1888	Oct. 23	56·22	7	20·0	2	cB, pL, moonlight
	1888	Oct. 29	56·14	5	17·0	2	B, pL
	....	....	22 2 56·18	..	+30 49 19·1	..	
4815	1886	Sept. 27	22 32 2·16	13	+33 50 42·9	2	
7331	1886	Oct. 22	2·20	12	45·1	1	
	1886	Oct. 31	2·41	10	46·4	1	cB
	....	....	22 32 2·26	..	+33 50 44·8	..	
	1888	Sept. 4	1·93	6	45·8	3	B, S, mbM
	1888	Sept. 5	1·95	7	41·8	3	B, S, mbM
	1888	Sept. 14	2·12	6	46·0	3	cB, pL
	1888	Oct. 14	2·28	7	46·9	2	
	....	....	22 32 2·07	..	+33 50 45·1	..	
4821	1886	Nov. 27	22 32 9·74	8	+23 13 35·3	1	
7332	....	....	....	..	....	..	
	1888	Oct. 23	9·40	7	39·0	1	cB, S
	1888	Oct. 29	9·57	7	39·0	2	B, pS, mbM
	....	....	22 32 9·48	..	+23 13 39·0	..	
4827	1886	Oct. 22	22 36	..	+60 42 48·5	1	
7354	1886	Nov. 25	14·62	4	41·5	1	Decl. ±
	....	....	22 36 14·62	..	+60 42 45·0	..	
	1888	Aug. 30	14·53	5	45·7	3	B, S, R
	1888	Sept. 12	14·10	7	43·5	2	B, pL, lbM
	1888	Nov. 28	13·86	6	41·5	1	pF in haze, B*sf
	....	....	22 36 14·16	..	+60 42 43·6	..	
4845	1888	Sept. 5	22 44 25·72	6	+11 1 27·6	1	pF, pL, *np
7385	1888	Oct. 29	25·24	5	26·0	1	pF, S
	....	....	22 44 25·48	..	+11 1 26·8	..	

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
4879	1888	Sept. 5	22 54 35·90	7	+15 23 23·8	2	pB, pL, lbM, *f10ˢ
7448	1888	Oct. 23	36·04	7	17·2	1	cB, pL, mbM
	1888	Oct. 29	36·60	7	26·0	1	cB, pL, bM, diffic.
	....	....	22 54 36·18	..	+15 23 22·3	..	
4883	1886	Sept. 27	22 55 44·28	5	+29 33	..	
7457	1886	Nov. 25	....	..	2·4	1	pB, L
	1886	Nov. 27	44·12	5	....	..	pB
	1886	Nov. 29	44·10	6	10·8	1	pB, d = 3ˢ
	1886	Nov. 30	....	..	5·9	1	pB
	....	....	22 55 44·17	..	+29 33 6·4	..	
	1887	Nov. 15	43·66	6	....	..	pF, 2**f in par.
	1888	Aug. 30	43·64	6	17·8	2	cB, pS
	1888	Sept. 4	43·82	7	14·2	3	
	1888	Sept. 12	43·70	6	13·6	2	cB, pL, mbM
	....	....	22 55 43·70	..	+29 33 15·2	..	
4903	1888	Sept. 4	23 7 17·22	6	+12 4 55·2	2	pF, pL, diffic., * 9·5 nf
7515	1888	Sept. 12	17·13	4	....	..	pF, pL
	1888	Oct. 29	17·75	6	46·1	1	pF, L, v diffic.
	....	....	23 7 17·37	..	+12 4 50·6	..	
4921	1888	Aug. 30	23 10 22·88	4	+ 6 5 17·3	2	pB, pS
7562	1888	Oct. 23	22·62	2	....	..	pF, in moonlight
	....	....	23 10 22·75	..	+ 6 5 17·3	..	
4928	1886	Sept. 27	23 12 20·96	4	− 5 15 1·3	1	pB, pS
7585	....	....	....	..	....	..	
4936	1888	Sept. 5	23 14 39·85	7	+ 7 36 13·0	2	cB
7619	1888	Sept. 12	39·84	6	7·8	2	B, pL, mbM
	1888	Sept. 14	39·95	7	19·7	2	cB, S
	....	....	23 14 39·88	..	+ 7 36 13·5	..	
4939	1887	Nov. 15	23 15 0·14	4	+16 37 25·6	1	pB, B * f
7625	....	....	....	..	....	..	
4964	1886	Nov. 25	23 20 37·22	13	+41 55 47·0	3	vB, R, d = 10″ *)
7662	1886	Nov. 27	37·32	10	46·9	2	
	1886	Nov. 29	37·04	14	49·2	3	Red field illumination
	1886	Nov. 30	37·06	16	47·2	3	same
	....	....	23 20 37·16	..	+41 55 47·6	..	
	1888	Aug. 30	36·91	7	49·2	3	same
	1888	Sept. 4	37·04	7	48·6	3	same
	1888	Sept. 5	36·88	4	47·9	2	same
	1888	Oct. 12	....	..	48·8	3	same
	1888	Oct. 23	36·97	7	49·3	2	same
	....	....	23 20 36·95	..	+41 55 48·8	..	
4993	1888	Sept. 5	23 30 36·78	5	+ 1 32	..	
7714	1888	Sept. 12	36·38	7	48·2	2	pB, pS, bM, B * s f
	1888	Oct. 12	.....	..	44·3	1	pB
	1888	Oct. 29	36·47	7	43·1	1	pF, S
	....	....	23 30 36·54	..	+ 1 32 45·2	..	
5000	1886	Nov. 27	23 34 12·80	5	−12 54 2·7	1	F, Decl. ±
7727	1886	Nov. 29	12·34	10	12·8	1	Decl. ±
	1886	Nov. 30	12·60	1	15·6	1	cB, Decl. ±
	....	....	23 34 12·58	..	−12 54 10·4	..	

*) N.G.C. 7662 appears in a dark field 0·5 mg fainter than neighbouring star 7-8 mg. It is not visible when the field is illuminated by yellow light, although this shows stars of the 9 mg.

TABLE I.—CONTINUED.

G.C. N.G.C.	Date.		Right Ascension 1890·0.	Number of Wires.	Declination 1890·0.	Number of Bisections.	REMARKS.
			h m s		° ′ ″		
5005	1888	Aug. 30	23 38 39·87	7	+10 9 23·7	2	S, =* 11-12, * 12 s f
7742	1888	Sept. 4	39·68	5	24·9	3	cB, pS
	1888	Dec. 8	39·37	7	20·5	2	pB, diffic. in moonlight
	....	....	23 38 39·64	..	+10 9 23·0	..	
5006	1886	Nov. 27	23 38 44·98	..	+ 9 19 32·3	1	F, * sp, Decl. ±
7743	....	....	....	..	....	..	
	1888	Oct. 29	23 38 44·94	4	+ 9 19 26·7	1	
5015	1888	Sept. 12	23 43 38·45	6	+30 22 10·2	2	cB, S, bM
7760	....	....	....	..	....	..	
5029	1887	Dec. 7	23 49 41·23	6	+ 5 18 19·8	2	
7785	1888	Oct. 29	41·71	6	11·3	1	pF, pL, no cond., Decl. ±
	1888	Dec.	40·82	6	15·7	2	pF, pS, * 7 pr, 2 F ** sf,
	....	....	23 49 41·27	..	+ 5 18 15·6	..	[diffic.
5038	1887	Nov. 15	23 53 48·38	3	+20 8 21·9	1	F, S, * 10 sp 3ˢ
7798	....	....	....	..	....	..	
5046	1888	Aug. 30	23 57 36·92	4	+15 31 57·9	1	F in moonlight
7814	1888	Sept. 4	36·12	7	54·0	2	pB, cL
	1888	Sept. 14	36·15	7	56·9	1	pB, diffic.
	....	....	23 57 36·40	..	+15 31 56·3	..	
5050	1888	Oct. 29	23 58 51·76	5	+ 4 35 1·8	1	F, S, diffic.
7820	1888	Dec. 8	50·59	7	....	..	F, v diffic.
	....	....	23 58 51·17	..	+ 4 35 1·8	..	

TABLE II.

TABLE

No.	G.C.	N.G.C.	Number of chrono-graphed Observations.	Schmidt.		D'Arrest (Leipsic).		D'Arrest (Copenhagen).		Rümker.		Schönfeld I.	
				Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
				s	″	s	″	s	″	s	″	s	″
1	8	16	2 3	......	......	......	......	+ 2·6	− 11	......	......	......	......
2	53	108	3 2	......	......	......	......	+ 3·4	+ 15	......	......	......	......
3	62	128	2	......	......	......	......	− 0·1	− 6	......	......	......	......
4	90	185	4	......	......	......	......	− 0·9	+ 60	......	......	......	......
5	105	205	2	......	......	− 1·1	− 5	+ 0·4	+ 8	+0·10	+17·9	−0·09	+15·5
6	117	221	3	+0·14	+ 2·8	+ 0·5	− 7	+ 0·5	− 7	−0·05	+ 3·1	−0·04	+ 5
7	116	224	..	......	......	(− 0·2)	(+ 5)	(− 0·4)	(+ 18)	(+0·47)	(+ 0·6)	......	......
8	149	266	4	......	......	......	......	+ 1·0	− 7	......	......	......	......
9	158	278	3	......	......	......	......	+ 1·1	− 29	......	......	......	......
10	176	315	3 5	......	......	......	......	+ 1·1	+ 14	......	......	+0·50	+ 2·3
11	218	404	2 3	......	......	......	......	+ 0·4	+ 12	+0·87	+ 8·3	......	......
12	264	470	2 1	......	......	......	......	......	......	......	......	......	......
13	269	474	2	......	......	......	......	+ 0·7	+ 9	......	......	......	......
14	307	524	2 3	......	......	− 1·0	− 15	+ 0·2	+ 1	......	......	−0·29	− 0·6
15	342	584	3	......	......	− 0·4	+ 13	+ 0·9	− 11	......	......	+0·55	+ 2·4
16	385	650	2	......	......	......	......	......	......	+0·08	0·0	......	......
17	386	651	4	......	......	......	......	......	......			......	......
18	430	718	7 6	......	......	......	......	− 0·8	+ 35	......	......	......	......
19	463	772	..	......	......	(− 0·9)	(− 2)	(+ 1·4)	(+ 15)	......	......	(−0·86)	(+ 5·9)
20	526	890	6	......	......	......	......	+ 0·4	+ 7	......	......	......	......
21	544	936	5	......	......	− 0·3	+ 3	+ 0·7	+ 5	......	......	+0·11	+ 0·7
22	549	949	4	......	......	......	......	− 0·7	+ 6	+0·50	− 0·6	+0·09	+ 3·3
23	574	1022	2	......	......	......	......	− 0·8	− 23	......	......	......	......
24	575	1023	3	+0·69	− 0·5	+ 0·7	− 17	+ 0·2	− 1	+0·80	+ 3·5	+0·12	+ 2·1
25	600	1068	2	+0·61	+ 0·1	+ 0·2	− 2	+ 0·5	− 7	......	......	+0·22	+ 1·7
26	604	1084	3	......	......	− 0·7	− 2	− 1·7	− 11	......	......	......	......
27	628	1161	4	......	......	......	......	......	......	......	......	......	......
28	648	1209	1	......	......	......	......	+ 1·5	+ 24	......	......	......	......
29	675	1275	4	......	......	......	......	+ 0·8	− 1	......	......	......	......
30	709	1332	2	......	......	......	......	......	......	......	......	......	......
31	752	1407	5	......	......	+ 0·5	+ 2	+ 0·4	+ 20	......	......	......	......
32	778	1453	2	......	......	......	......	+ 1·8	+ 5	......	......	......	......
33	801	1501	3 2	......	......	......	......	......	......	+1·08	− 9·8	......	......
34	826	1535	5	−0·01	+ 3·5	− 0·1	+ 14	− 0·5	+ 5	......	......	+0·15	+ 3.8
35	847	1569	5 6	......	......	......	......	......	......	......	......	......	......
36	866	1600	2	......	......	− 1·0	− 15	− 0·4	− 1	......	......	−0·43	− 2·0
37	888	1637	4	......	......	......	......	− 1·0	− 10	......	......	−1·11	+ 9·1
38	932	1700	7	......	......	......	......	− 0·3	− 5	......	......	......	......
39	1005	1788	6	......	......	......	......	− 1·2	− 7	......	......	......	......
40	1137	1931	5	......	......	......	......	0·0	− 2	......	......	......	......
41	1157	1952	3	+0·94	+ 6·0	+ 1·1	+ 1	+ 0·7	0	......	......	+0·22	+ 0·3
42	1185	1982	1	......	......	......	......	......	......	......	......	......	......
43	1202	1999	3	+0·12	− 6·8	......	......	0·0	− 13	......	......	......	......
44	1225	2022	2	+0·36	+ 9·4	+ 0·1	+ 35	− 1·2	+ 12	......	......	+0·30	+ 3·8
45	1267	2068	..	......	......	(− 0·2)	(+ 8)	......	......	......	......	......	......

II.

No.	Schönfeld II.		Schultz.		Auwers.		Vogel I., II.		Engelmann.		Engelhardt I., II.		Porter.	
	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
	s	"	s	"	s	"	s	"	s	"	s	"	s	"
1	+0·51	− 0·5	+0·44	− 1·3	......	......	......	......	......	......	*+0·81*	*− 1·2*	......	......
2	+0·64	+ 3·0	+1·23	+ 1·9	......	......	......	......	......	......	......	......	......	......
3	+0·02	− 2·2	+0·61	......	......	......	......	......	......	......	......	......	......	......
4	+0·21	+10·8	......	......	......	......	......	......	......	......	......	......	......	......
5	+0·36	+13·5	+0·61	+ 6·8	......	......	+0·03	− 8·3	......	......	......	......	......	......
6	......	......	+0·07	+ 2·3	......	......	+0·22	+ 3·3	+0·07	+ 2·6	+0·11	+ 1·0	......	......
7	(+0·39)	(+ 2·9)	(+0·38)	(+ 0·7)	......	......	(+0·53)	(− 0·8)	(+0·29)	(+ 2·1)	......	......	......	......
8	+0·15	+ 7·3	+0·37	0·0	......	......	......	......	......	......	......	......	......	......
9	+0·57	+ 8·2	......	......	......	......	......	......	......	......	......	......	......	......
10	+0·45	+ 1·7	+0·50	+ 1·4	......	......	+0·53	+ 1·2	......	......	......	......	......	......
11	+0·10	+ 9·0	+0·12	+4·9	......	......	......	......	......	......	......	......	......	......
12	......	......	−0·04	−5·6	......	......	......	......	......	......	......	......	......	......
13	......	......	+1·06	+1·3	......	......	......	......	......	......	......	......	......	......
14	......	......	+0·14	+0·6	......	......	+0·18	+ 1·7	+0·69	− 2·2	......	......	......	......
15	+0·58	+ 2·3	......	......	......	......	+0·77	+ 2·5	+0·70	+ 1·9	......	......	+1·14	− 5·8
16	−0·20	+ 3·9	......	......	......	......	......	......	......	......	......	......	......	......
17	−0·32	+ 7·7	......	......	......	......	......	......	......	......	......	......	......	......
18	+0·43	+ 3·3	......	......	......	......	......	......	......	......	......	......	......	......
19	(−0·63)	(+ 1·6)	(−0·42)	(+ 2·0)	......	......	(−1·10)	(+10·7)	(+0·36)	(−25·2)	......	......	......	......
20	+0·21	+ 4·5	+0·91	+ 3·7	......	......	......	......	......	......	......	......	......	......
21	......	......	......	......	......	......	+0·46	+ 1·5	+0·62	+ 4·5	+0·44	+ 1·4	+0·16	− 1·1
22	......	......	+0·46	− 1·2	......	......	......	......	......	......	......	......	......	......
23	+0·67	− 7·9	......	......	......	......	......	......	......	......	......	......	+0·49	−11·3
24	......	......	+0·22	+ 0·7	......	......	+0·27	− 4·6	+0·28	+ 6·0	+0·34	+ 1·4	......	......
25	......	......	......	......	+0·59	+ 0·8	+0·56	+ 0·9	......	......	......	......	+0·48	− 0·7
26	+0·36	+ 1·3	......	......	......	......	......	......	+0·54	+10·7	......	......	−0·34	− 1·4
27	+0·25	− 0·4	......	......	......	......	......	......	......	......	......	......	......	......
28	......	......	......	......	......	......	......	......	......	......	*+0·17*	− 1·5	......	......
29	+0·09	+ 7·4	......	......	......	......	......	......	......	......	......	......	......	......
30	+0·64	+ 2·0	......	......	......	......	......	......	......	......	......	......	+0·86	+ 0·4
31	+0·53	+ 1·6	......	......	......	......	......	......	......	......	+1·22	− 6·5	+0·99	+ 2·2
32	......	......	......	......	......	......	......	......	......	......	......	......	......	......
33	+0·48	− 5·2	......	......	......	......	......	......	......	......	......	......	......	......
34	......	......	......	......	......	......	+0·27	+ 2·1	+0·24	+ 3·1	......	......	+0·24	+ 5·2
35	+0·04	+ 3·2	......	......	......	......	......	......	......	......	−0·03	+ 0·9	......	......
36	−0·06	+ 0·9	......	......	......	......	......	......	......	......	......	......	+0·19	− 6·1
37	−0·55	+ 8·6	......	......	......	......	......	......	......	......	......	......	......	......
38	+0·39	− 1·0	......	......	......	......	......	......	......	......	......	......	−0·19	+ 3·6
39	+0·15	− 1·1	......	......	......	......	......	......	......	......	......	......	......	......
40	......	......	+0·74	+ 1·7	......	......	......	......	+0·34	+ 4·6	*+0·71*	*+ 2·3*	......	......
41	......	......	......	......	+0·95	+ 1·3	+0·68	− 1·8	......	......	......	......	......	......
42	......	......	......	......	......	......	......	......	+0·08	− 0·7	......	......	......	......
43	+0·57	− 0·9	......	......	......	......	......	......	......	......	+0·50	− 2·0	......	......
44	......	......	−0·06	+ 0·4	......	......	+0·49	+ 3·0	+0·54	+ 2·8	*+0·51*	*+ 2·1*	......	......
45	......	......	(+0·20)	(+ 2·8)	......	......	......	......	(+0·18)	(+ 3·2)	......	......	......	......

TABLE II.—

No.	G.C.	N.G.C.	Number of chrono-graphed Observations.	Schmidt.		D'Arrest (Leipsic).		D'Arrest (Copenhagen).		Rümker.		Schönfeld I.	
				Δ α cos δ.	Δ δ	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
				s	″	s	″	s	″	s	″	s	″
46	1270	2071	..	......	......	......	......	(− 0·1)	(+ 22)	......	......	......	......
47	1362	2170	4	0·00	− 1·0	+ 0·1	+ 14	− 0·4	+ 5	......	......	+0·31	+ 0·2
48	1375	2185	5	......	......	− 0·7	+ 6	− 0·2	− 13	......	......	+0·30	+ 2·8
49	1425	2245	5	+0·32	− 0·2	......	......	− 0·1	+ 30	......	......	+0·27	+ 0·1
50	1437	2261	6	+0·38	+ 1·7	+ 0·7	+ 6	+ 0·2	+ 5	......	......	+0·43	+ 0·3
51	1500	2346	5	......	......	......	......	......	......	......	......	......	......
52	1519	2371	5	......	......	......	......	+ 0·4	− 33	......	......	......	......
53	1532	2392	2	0·00	+ 4·6	0·0	+ 9	− 0·2	+ 9	−0·06	+ 2 6	+0·03	+ 2·4
54	1546	2415	4	......	......	......	......	......	......	......	......	......	......
55	1548	2419	4 5	......	......	......	......	......	......	+1·79	+ 2·7	......	......
56	1567	2440	4	+0·48	− 2·1	+ 0·2	− 1	− 2·1	+ 7	......	......	......	......
57	1596	2481	4	......	......	......	......	− 2·4	− 2	......	......	......	......
58	1626	2532	2	......	......	......	......	......	......	......	......	......	......
59	1629	2537	3	......	......	......	......	......	......	+0·12	+ 0·4	......	......
60	1632	2542	2	......	......	......	......	......	......	......	......	......	......
61	1660	2592	3	......	......	......	......	+ 2·6	− 12	......	......	......	......
62	1673	2612	3 1	......	......	......	......	......	......	......	......	......	......
63	1684	2639	7	......	......	......	......	+ 1·5	0	......	......	......	......
64	1691	2655	2	......	......	......	......	+ 0·7	− 3	......	......	......	......
65	1704	2672	3	......	......	......	......	− 0·1	+ 15	......	......	−0·32	+16·4
66	1711	2681	3	......	......	......	......	+ 1·3	+ 12	+0·95	− 2·2	......	......
67	1713	2683	3	......	......	......	......	+ 0·5	+ 12	+0·30	+ 7·0	−0·40	+ 1·3
68	1720	2693	..	......	......	......	......	(+ 1·4)	(+ 8)	......	......	......	......
69	1728	2701	3	......	......	......	......	+ 1·5	+ 26	......	......	......	......
70	1765	2768	4	......	......	......	......	+ 0·4	+ 25	......	......	......	......
71	1771	2775	3	+0·51	+ 0·5	0·0	+ 9	− 0·3	+ 41	......	......	−0·12	+ 5·8
72	1781	2787	3 2	......	......	......	......	+ 0·6	− 4	−0·16	− 9 4	......	......
73	1811	2830	2	......	......	......	......	+ 2·3	− 14	+0·82	+ 6·6	......	......
74	1823	2841	..	......	......	......	......	− 0·3	− 4	(+0·08)	(+ 2·7)	......	......
75	1848	2880	2	......	......	......	......	+ 1·0	− 11	......	......	......	......
76	1861	2903	2	......	......	+ 2·3	+ 5	+ 2·0	+ 7	......	......	+0·16	+ 5·9
77	1896	2964	4	......	......	......	......	+ 0 8	+ 18	......	......	+0·27	+ 9·2
78	1904	2974	1	+0·36	− 5·0	− 0·3	+ 3	0·0	− 7	......	......	−0·37	− 1·0
79	1944	3021	3	......	......	......	......	− 1·3	− 5	......	......	......	......
80	1949	3031	2	......	......	......	......	+ 0·2	+ 3	+0·03	+ 1·5	−0·13	− 2·1
81	1973	3067	2 3	......	......	......	......	− 0·1	− 3·6	......	......	......	......
82	2008	3115	4	+0·50	− 2·6	+ 0·4	− 22	− 0·9	+ 1·6	......	......	+0·25	− 0·5
83	2024	3147	2	......	......	......	......	+ 0·8	− 1	+0·07	− 5·6	−0·07	− 5·0
84	2041	3169	2	......	......	− 0·6	0	− 2·1	+ 5	......	......	−0·18	+ 2·6
85	2058	3190	2	......	......	− 0·6	+ 3	+ 0·8	+ 24	......	......	+0·11	+ 0·7
86	2102	3242	2	+0·09	− 1·5	0·0	− 3	+ 2·0	− 29	......	......	......	......
87	2104	3245	3	......	......	+ 0·7	0	+ 0·5	+ 4	......	......	+0·10	0
88	2112	3254	2 : 3	......	......	......	......	+ 2·1	− 1	......	......	......	......
89	2134	3277	4	......	......	......	......	+ 1·2	− 3	......	......	......	......
90	2145	3294	1	......	......	......	......	− 0·5	+ 20	......	......	......	......

CONTINUED.

No.	Schönfeld II.		Schultz.		Auwers.		Vogel I., II.		Engelmann.		Engelhardt I., II.		Porter.	
	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
	s	"	s	"	s	"	s	"	s	"	s	"	s	"
46	......	......	......	......	......	......	......	......	......	......	(−0·38)	(− 5·1)	......	......
47	......	......	......	......	......	......	+0·22	− 1·1	......	......	+0·32	− 0·5	......	......
48	......	......	......	......	......	......	+0·51	+ 1·4	......	......	−0·07	+ 0·7	......	......
49	......	......	+0·12	+ 1·1	......	......	+0·33 +0·34	+ 0·4 + 0·7	......	......	+0·23	+ 0·1	......	......
50	......	......	+0·24	− 0·9	......	......	+0·36	− 1·7	......	......	+0·46	− 1·1	......	......
51	+0·52	− 0·1	......	......	......	......	......	......	......	......	+0·41	− 1·0	......	......
52	+0·25	+ 3·4	+0·48	+ 5·1	......	......	......	......	......	......	......	......	......	......
53	......	......	−0·22	+ 3·5	−0·05	+ 6·2	0·00	+ 3·2	−0·30	+ 5·0	−0·17	+ 2·6	......	......
54	+0·71	+ 3·0	+0·65	+ 3·0	......	......	......	......	......	......	......	......	......	......
55	+0·54	+ 8·2	+0·44	+ 4·8	......	......	......	......	......	......	......	......	......	......
56	+0·33	+ 0·1	......	......	......	......	......	......	......	......	+0·03	− 1·1	+0·19	− 4·9
57	......	......	......	......	......	......	......	......	......	......	......	......	......	......
58	......	......	+0·42	− 3·6	......	......	......	......	......	......	......	......	......	......
59	......	......	......	......	......	......	......	......	......	......	+0·75	+ 4·4	......	......
60	......	......	......	......	......	......	......	......	......	......	......	......	......	......
61	+0·43	− 0·2	......	......	......	......	......	......	......	......	+0·79	+ 1·5	......	......
62	......	......	......	......	......	......	......	......	......	......	+1·72	−18·5	......	......
63	+0·33	+ 0·5	......	......	......	......	......	......	......	......	......	......	......	......
64	......	......	......	......	......	......	......	......	+0·19	− 0·3	......	......	......	......
65	+0·09	+16·5	+0·23	+ 9·6	......	......	......	......	......	......	+0·44	+13·6	......	......
66	+0·47	− 3·3	+0·65	− 0·7	......	......	......	......	+0·80	+ 0·4	......	......	......	......
67	−0·29	+ 5·3	−0·08	+ 6·4	......	......	−0·27	− 4·5	......	......	......	......	......	......
68	......	......	......	......	......	......	......	......	......	......	......	......	......	......
69	......	......	......	......	......	......	......	......	......	......	+0·68	+ 1·2	......	......
70	......	......	......	......	......	......	......	......	......	......	......	......	......	......
71	......	......	+0·29	+ 3·1	......	......	−0·03	+ 4·0	......	......	......	......	......	......
72	......	......	......	......	......	......	......	......	......	......	+0·39	− 6·1	......	......
73	+0·83	− 0·5	......	......	......	......	......	......	......	......	......	......	......	......
74	(−0·35)	(+ 3·5)	(−0·18)	(+ 2·5)	......	......	......	......	(+0·14)	(− 0·9)	......	......	......	......
75	......	......	+0·15	− 3·8	......	......	......	......	......	......	......	......	......	......
76	......	......	+0·30	+ 4·2	+0·59	+ 7·4	+0·49	+ 4·5	+0·37	+ 7·3	+0·58	+ 6·7	......	......
77	+0·07	+ 6·9	+0·15	+ 8·0	......	......	......	......	......	......	......	......	......	......
78	......	......	−0·37	− 4·8	......	......	+0·05	− 2·2	......	......	......	......	......	......
79	+0·46	+ 4·9	+0·43	− 0·4	......	......	......	......	......	......	......	......	......	......
80	......	......	+0·21	− 2·7	......	......	......	......	+0·26	− 2·4	......	......	......	......
81	+0·04	− 1·2	......	......	......	......	......	......	......	......	......	......	......	......
82	......	......	+0·02	− 0·9	......	......	+0·33	− 0·9	......	......	......	......	......	......
83	+0·08	− 8·8	......	......	......	......	......	......	......	......	+0·39	− 6·3	......	......
84	......	......	+0·08	− 1·5	......	......	......	......	......	......	......	......	......	......
85	......	......	+0·20	− 2·9	......	......	......	......	......	......	......	......	......	......
86	......	......	......	......	+0·07	− 1·5	−0·08	− 1·0	−0·06	+ 5·1	+0·14	+ 1·8	......	......
87	......	......	+0·21	− 1·6	......	......	......	......	......	......	......	......	......	......
88	......	......	+0·23	− 0·7	......	......	......	......	......	......	......	......	......	......
89	+0·27	+ 5·6	+0·41	+ 0·3	......	......	......	......	......	......	......	......	......	......
90	......	......	−1·14	+12·5	......	......	......	......	......	......	......	......	......	......

TABLE II.—

No.	G.C.	N.G.C.	Number of chrono-graphed Observations.	Schmidt.		D'Arrest (Leipsic).		D'Arrest (Copenhagen).		Rümker.		Schönfeld I.	
				Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
				s	"	s	"	s	"	s	"	s	"
91	2150	3301	2	......	......	......	......	+ 0·1	− 5	......	......	−0·07	− 2·3
92	2158	3310	3 4	......	......	......	......	......	......	......	......	......	......
93	2184	3351	2	+0·40	− 2·1	......	......	+ 1·5	− 12	......	......	−0·11	+ 2·0
94	2182	3348	1	......	......	......	......	+ 0·2	− 11	......	......	......	......
95	2201	3377	2	+1·14	− 5·0	+ 0·4	− 11.	− 0·5	0	......	......	+0·37	− 0·9
96	2203	3379	3	+0·18	+ 1·8	− 0·3	+ 15	+ 0·5	+ 1	......	......	−0·19	+ 0.8
97	2274	3486	3	......	......	− 0·4	− 1	− 0·1	− 3	......	......	......	......
98	2276	3489	4	+0·21	− 3·4	......	......	− 0·1	− 16	......	......	+0.11	− 1·9
99	2287	3504	4	......	......	+ 0.5	− 4	+ 0·9	+ 32	......	......	......	......
100	2301	3521	3	+0·66	− 5·3	− 0·1	− 3	+ 0·5	+ 17	......	......	+0·71	− 2·5
101	2343	3587	2	......	......	......	......	......	......	......	......	......	......
102	2347	3593	2 3	......	......	......	......	0·0	− 5	......	......	......	......
103	2358	3607	2	......	......	......	......	− 0·4	− 21	......	......	−0·21	+ 3·2
104	2373	3623	2	......	......	+ 0·8	− 17	+ 1·1	− 10	......	......	−0·11	− 3·6
105	2376	3626	2	......	......	......	......	+ 3·3	+ 1	......	......	−0·10	+ 5·4
106	2396	3655	1	......	......	− 1·2	+ 25	− 1·6	+ 6	......	......	......	......
107	2404	3665	2	......	......	......	......	+ 1·2	− 20	+2·21	+ 3·9	......	......
108	2413	3675	3	......	......	......	......	+ 0·7	+ 8	......	......	......	......
109	2421	3683	2	......	......	......	......	− 0·1	− 7	......	......	......	......
110	2426	3689	1 2	......	......	......	......	+ 1.5	+ 3	......	......	......	......
111	2454	3738	4	......	......	......	......	+ 0·4	+ 22	......	......	......	......
112	2499	3810	3 2	+0·83	− 3·5	− 0·3	− 10	− 2·3	+ 4	......	......	−0·15	− 1·7
113	2501	3813	2	......	......	......	......	+ 1·9	− 11	......	......	......	......
114	2520	3838	3	......	......	......	......	+ 2·8	− 12	......	......	......	......
115	2545	3877	4 5	......	......	......	......	− 0·3	+ 21	......	......	......	......
116	2564	3898	2	......	......	......	......	+ 0·1	+ 6	......	......	......	......
117	2566	3900	2	......	......	......	......	+ 0·4	+ 25	......	......	......	......
118	2600	3941	2 3	......	......	......	......	− 0.6	+ 19	+0·14	− 7·3	+0·07	− 1·8
119	2660	4026	2	......	......	......	......	+ 0·5	+ 15	......	......	......	......
120	2663	4030	2	......	......	......	......	− 0·6	+ 39	......	......	......	......
121	2680	4051	2	......	......	......	......	+ 1·7	+ 9	......	......	......	......
122	2717	4102	3	......	......	......	......	+ 0·5	− 7	......	......	......	......
123	2723	4111	2	......	......	......	......	+ 0·1	− 4	+0·43	− 1·3	......	......
124	2752	4147	4	......	......	......	......	− 1·0	+ 5	......	......	−0·69	+ 3·7
125	2786	4192	2	+0·29	− 2·4	− 6·1	+ 24	+ 1·4	+ 11	......	......	+0·47	+ 7·4
126	2796	4203	2	+0·17	+ 3·4	+ 1·0	+ 13	+ 0·9	− 6	......	......	−0·09	+ 2·2
127	2806	4216	3	−0·11	− 1·6	− 0·1	+ 8	− 0·8	− 14	......	......	−0·41	− 1·4
128	2841	4258	5 6	......	......	......	......	0·0	+ 29	......	......	......	......
129	2904	4339	2	......	......	......	......	+ 0·8	− 5	......	......	+0·26	− 0·5
130	2921	4365	1 2	−0·44	+ 1·4	− 0·1	+ 12	+ 1·0	+ 18	......	......	−0·95	+ 2·1
131	2924	4369	2	......	......	......	......	+ 0·9	− 24	......	......	......	......
132	2961	4406	4	+0·05	+ 8·1	− 0·1	+ 3	0·0	− 5	......	......	−0·46	+ 2·3
133	3025	4477	2 3	......	......	......	......	+ 0·3	+ 3	......	......	−0·42	+ 2·8
134	3042	4490	2	......	......	......	......	+ 2·4	+ 16	......	......	......	......
135	3035	4486	2	......	......	+ 0·5	− 5	+ 0·3	− 12	......	......	−0·16	− 2·0

CONTINUED.

No.	Schönfeld II.		Schultz.		Auwers.		Vogel I., II.		Engelmann.		Engelhardt I., II.		Porter.	
	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
	s	″	s	″	s	″	s	″	s	″	s	″	s	″
91	+0·05	+ 2·2	+0·18	+ 0·7	......	......	......	......	......	......	......	......	......	......
92	+0·32	+ 3·0	......	......	......	......	......	......	+0·56	+ 1·7	......	......	......	......
93	......	......	+0·08	+ 1·3	......	......	*+0·33*	*+ 2·0*	......	......	......	......	......	......
94	......	......	......	......	......	......	......	......	......	......	+0·38	− 9·9	......	......
95	......	......	+0·71	− 0·6	+0·69	+ 2·6	*+0·20*	*+ 0·5*	......	......	......	......	......	......
96	......	......	−0·05	− 2·7	+0·07	+ 0·5	{ −0·11 *−0·13*	+0·9 *+0·7* }	+0·24	+ 2·1	......	......	......	......
97	+0·11	+ 3·3	+0·16	+ 1·2	......	......	......	......	......	......	......	......	......	......
98	......	......	+0·31	− 3·1	......	......	*+0·21*	*− 6·7*	+0·28	+ 0·9	......	......	......	......
99	......	......	+0·56	+ 0·4	......	......	......	......	......	......	......	......	......	......
100	......	......	+0·19	− 3·4	+0·21	− 3·0	......	......	......	......	......	......	+0·02	− 0·1
101	......	......	+1·26	−12·4	......	......	......	......	......	......	......	......	......	......
102	+0·62	− 7·4	+0·32	− 5·6	......	......	*+0·26*	*− 9·1*	......	......	......	......	......	......
103	......	......	+0·15	+ 2·1	......	......	......	......	......	......	......	......	......	......
104	......	......	+0·36	− 2·0	+0·27	+ 2·4	*+0·22*	*− 0·5*	......	......	......	......	......	......
105	......	......	+0·25	+ 3·9	......	......	......	......	......	......	......	......	......	......
106	+0·03	− 7·8	......	......	......	......	......	......	......	......	......	......	......	......
107	......	......	+1·46	+ 4·5	......	......	......	......	......	......	......	......	......	......
108	......	......	......	......	......	......	......	......	+0·71	+10·7	......	......	......	......
109	......	......	+0·93	+ 2·8	......	......	......	......	......	......	......	......	......	......
110	......	......	+1·44	+ 8·7	......	......	......	......	......	......	......	......	......	......
111	......	......	......	......	......	......	......	......	......	......	......	......	......	......
112	......	......	−0·11	− 4·2	......	......	*+0·27*	*− 2·2*	......	......	......	......	......	......
113	......	......	+0·35	+ 0·8	......	......	......	......	......	......	......	......	......	......
114	......	......	......	......	......	......	......	......	......	......	......	......	......	......
115	......	......	......	......	......	......	......	......	......	......	......	......	......	......
116	......	......	......	......	......	......	......	......	......	......	......	......	......	......
117	+0·05	+ 3·5	+0·33	− 0·5	......	......	......	......	......	......	......	......	......	......
118	+0·34	+ 0·2	−0·36	+ 5·6	......	......	......	......	......	......	......	......	......	......
119	......	......	+0·50	+ 2·7	......	......	......	......	+0·60	− 7·8	......	......	......	......
120	+0·16	+ 2·4	......	......	......	......	......	......	......	......	......	......	......	......
121	......	......	......	......	......	......	......	......	......	......	......	......	......	......
122	......	......	......	......	......	......	......	......	......	......	......	......	......	......
123	+0·45	+ 0·3	+0·51	− 3·5	......	......	......	......	......	......	......	......	......	......
124	......	......	−0·10	+ 4·8	......	......	......	......	+0·30	+ 6·3	+0·01	+ 5·4	......	......
125	......	......	+0·41	+ 6·0	......	......	*+0·32*	*+ 0·3*	......	......	......	......	......	......
126	......	......	−0·12	+ 0·5	......	......	......	......	......	......	......	......	......	......
127	......	......	−0·18	+ 0·1	......	......	*+0·02*	*− 2·8*	......	......	......	......	......	......
128	+0·19	+ 0·9	......	......	......	......	......	......	+0·46	+ 3·8	......	......	......	......
129	......	......	+0·19	+ 0·5	......	......	......	......	......	......	......	......	......	......
130	......	......	−0·32	+ 3·3	......	......	......	......	......	......	......	......	......	......
131	......	......	......	......	......	......	......	......	......	......	......	......	......	......
132	......	......	+0·04	+ 1·2	+0·36	+ 0·5	{ +0·09 *−0·01*	+3·2 *+2·3* }	+0·22	+ 4·5	......	......	......	......
133	......	......	−0·11	+ 0·7	......	......	*+0·33*	*+ 3·9*	......	......	......	......	......	......
134	......	......	......	......	......	......	......	......	......	......	......	......	......	......
135	......	......	+0·10	+ 1·7	......	......	*+0·52*	*− 0·1*	......	......	......	......	......	......

TABLE II.—

No.	G.C.	N.G.C.	Number of chrono-graphed Observations.	Schmidt.		D'Arrest (Leipsic).		D'Arrest (Copenhagen).		Rümker.		Schönfeld I.	
				Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
				s	"	s	"	s	"	s	"	s	"
136	3075	4526	2	+0·55	+ 5·1	+ 0·7	+ 2	+ 0·5	+ 3	......	......	+0·26	+ 6·1
137	3106	4565	3	......	......	......	......	− 0·2	+ 3	......	......	−0·33	+ 2·0
138	3121	4579	3	+0·97	− 0·9	+ 0·3	0	+ 0·8	+ 2	+0·66	− 0·3	+0·37	+ 0·8
139	3132	4594	1	......	......	......	......	− 1·3	+ 14	......	......	−0·29	+ 0·7
140	3165	4631	4	......	......	......	......	+ 5·3	+ 33	......	......	......	......
141	3169	4636	..	......	......	......	......	(− 0·1)	(+ 14)	......	......	(−0·67)	(+ 4·5)
142	3182	4649	2	−0·06	+ 0·3	− 0·6	+ 1	− 0·3	− 4	−0·26	+ 1·0	−0·57	− 0·7
143	3193	4660	3	......	......	......	......	− 1·2	+ 5	......	......	......	......
144	3198	4666	1	......	......	......	......	+ 1·1	+ 16	......	......	+0·29	+13·0
145	3227	4697	3	......	......	......	......	− 0·4	+ 8	......	......	−0·52	− 0·4
146	3249	4725	2	......	......	......	......	+ 2·1	− 2	......	......	......	......
147	3258	4736	3 4	......	......	......	......	+ 1·0	− 16	+0·66	− 0·3	......	......
148	3274	4754	2	+0·58	+ 2·9	......	......	+ 0·8	− 15	......	......	−0·05	+ 4·6
149	3300	4793	2	......	......	......	......	......	......	......	......	......	......
150	3321	4826	3	......	......	......	......	+ 0·2	+ 17	......	......	......	......
151	3342	4866	2 3	......	......	......	......	− 0·7	− 17	......	......	......	......
152	3356	4900	2	......	......	......	......	− 0·4	+ 24	......	......	......	......
153	3395	4956	3 4	......	......	......	......	+ 0·7	+ 7	......	......	......	......
154	3397	4958	2	+0·47	− 2·7	......	......	+ 0·8	+ 4	......	......	+0·37	+ 0·7
155	3437	5005	4	......	......	......	......	+ 1·0	+ 17	......	......	......	......
156	3453	5024	2	+0·62	+ 5·0	......	......	− 0·2	+ 6	−0·58	+ 5·8	−0·60	+ 4·0
157	3459	5033	2 3	......	......	......	......	+ 0·3	+ 15	......	......	......	......
158	3474	5055	4	......	......	......	......	+ 0·7	+ 4	+0·99	+ 6·8	......	......
159	3505	5103	2 3	......	......	......	......	......	......	......	......	......	......
160	3524	5127	2 3	......	......	......	......	− 0·8	+ 3	......	......	......	......
161	3572	5194	4 5	+0·70	+ 4·3	+ 1·0	− 14	+ 0·5	+ 5	+0·84	+ 3·6	......	......
162	3592	5218	3	......	......	......	......	......	......	......	......	......	......
163	3615	5248	2	......	......	......	......	− 0·7	+ 7	......	......	+0·18	+ 0·6
164	3636	5272	4	+0·87	+ 0·5	+ 1·0	+ 19	+ 1·2	+ 1	+0·33	+ 4·0	−0·34	+ 6·6
165	3637	5273	3	......	......	......	......	......	......	......	......	......	......
166	3645	5290	1 2	......	......	......	......	+ 0·6	+ 2	......	......	......	......
167	3671	5322	3 4	......	......	......	......	+ 0·3	− 3	+0·12	− 1·7	−0·62	− 2·6
168	3702	5363	2	......	......	......	......	− 0·2	− 1	......	......	+0·02	− 1.7
169	3794	5485	4	......	......	......	......	+ 0·2	+ 1	......	......	+0·29	+ 3·0
170	3846	5566	5	......	......	+ 0·3	+ 8	+ 1·0	+ 16	......	......	+0·28	− 2·3
171	3854	5582	4 5	......	......	......	......	− 1·4	+ 3	......	......	......	......
172	3897	5631	3	......	......	......	......	......	......	......	......	......	......
173	3942	5689	4	......	......	......	......	......	......	......	......	......	......
174	3956	5701	4	......	......	......	......	+ 0·4	+ 22	......	......	......	......
175	3987	5746	3	......	......	− 0·3	− 9	+ 0·6	+ 6	......	......	......	......
176	4021	5806	4	+0·68	+14·9	+ 0·9	− 7	+ 0·7	+ 9	......	......	+0·60	+16.3
177	4045	5846	4	+0·41	+ 1.5	+ 1·0	+ 8	+ 0·7	+ 17	......	......	−0·50	− 0·9
178	4058	5866	4	......	......	......	......	+ 2·6	− 39	+0·50	+ 5·0	......	......
179	4064	5879	3	......	......	......	......	+ 0.9	− 22	+0·49	+11·3	......	......
180	4077	5899	2 4	......	......	......	......	......	......	+0·19	+12·4	......	......

CONTINUED.

No.	Schönfeld II.		Schultz.		Auwers.		Vogel I., II.		Engelmann.		Engelhardt I., II.		Porter.	
	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
	s	"	s	"	s	"	s	"	s	"	s	"	s	"
136	......	......	+0·36	+ 5·4	+0·46	+ 5·0	+0·64	+ 5·1	+0·74	+ 9·3	......	......	......	......
137	−0·34	+ 1·7	−0·04	+ 1·7	......	......	+0·20	− 3·7	......	......	*+0·02*	*+ 3·0*	......	......
138	......	......	+1·41	+ 1·3	......	......	*+0·89*	*+ 0·7*	......	......	......	......	......	......
139	......	......	......	......	......	......	......	......	+0·37	+ 4·9	......	......	−0·18	+ 2·2
140	......	......	......	......	......	......	......	......	......	......	*+0·04*	*+ 3·1*	......	......
141	......	......	(−0·29)	(+ 1·8)	......	......	......	......	......	......	......	......	......	......
142	......	......	−0·24	− 0·5	......	......	*+0·10*	*+ 4·3*	+0·09	+ 4·5	......	......	......	......
143	......	......	......	......	......	......	*+0·12*	*+ 6·4*	......	......	......	......	......	......
144	......	......	......	......	......	......	......	......	......	......	......	......	......	......
145	......	......	+0·09	− 1·9	......	......	......	......	......	......	......	......	+0·45	+ 2·0
146	+0·01	− 5·4	−0·12	− 3·8	......	......	......	......	......	......	*+0·31*	*− 1·5*	......	......
147	+0·14	+ 2·6	......	......	......	......	......	......	+0·19	+ 2·7	*+0·39*	*+ 0·9*	......	......
148	......	......	+0·68	+ 3·6	......	......	*+0·64*	*+ 1·6*	......	......	......	......	......	......
149	......	......	+0·74	+15·3	......	......	......	......	......	......	......	......	......	......
150	......	......	+0·13	+ 2·9	......	......	......	......	+0·29	+ 3·7	......	......	......	......
151	......	......	−0·41	+ 7·9	......	......	......	......	......	......	......	......	......	......
152	......	......	......	......	......	......	......	......	......	......	......	......	......	......
153	......	......	......	......	......	......	......	......	......	......	......	......	......	......
154	......	......	......	......	......	......	......	......	......	......	......	......	−0·01	+ 3·3
155	......	......	−1·57	+ 9·0	......	......	......	......	......	......	......	......	......	......
156	......	......	......	......	+0·33	+ 3·5	+0·59	+ 4·1	+0·26	+ 5·0	......	......	......	......
157	......	......	......	......	......	......	......	......	......	......	......	......	......	......
158	+0·47	+ 4·8	......	......	......	......	......	......	+0·59	+10·5	......	......	......	......
159	......	......	......	......	......	......	......	......	......	......	*+1·00*	*+ 6·9*	......	......
160	......	......	......	......	......	......	......	......	......	......	......	......	......	......
161	+0·29	+ 6·0	+0·21	+ 3·0	+0·43	+ 2·6	+0·42	+ 3·1	+0·57	+ 4·2	*+0·36*	*+ 3·8*	......	......
162	......	......	......	......	......	......	......	......	......	......	......	......	......	......
163	+0·43	+ 2·8	+0·55	+ 0·7	......	......	*−0·04*	*− 8·1*	......	......	......	......	......	......
164	......	......	......	......	+0·67	+ 6·2	+0·69	+ 8·3	+0·65	+ 3·4	+0·58	+ 3·6	......	......
165	......	......	−0·53	− 1·0	......	......	......	......	......	......	......	......	......	......
166	......	......	......	......	......	......	......	......	......	......	......	......	......	......
167	−0·14	− 0·3	−0·16	− 3·7	......	......	......	......	−0·01	− 4·4	−0·08	− 1·3	......	......
168	+0·21	− 1·5	−0·23	+ 0·4	......	......	......	......	......	......	......	......	......	......
169	+0·63	+ 3·6	......	......	......	......	......	......	......	......	......	......	......	......
170	+0·64	− 0·3	+0·58	− 1·3	......	......	+0·28	− 1·3	......	......	......	......	......	......
171	+0·25	+ 3·5	......	......	......	......	......	......	......	......	......	......	......	......
172	+0·19	+ 2·3	......	......	......	......	......	......	......	......	+0·75	+ 4·2	......	......
173	+0·72	+ 3·2	......	......	......	......	......	......	......	......	......	......	......	......
174	......	......	+0·54	+ 2·9	......	......	......	......	......	......	......	......	......	......
175	......	......	+0·33	− 1·0	......	......	......	......	......	......	......	......	......	......
176	+0·80	+13·6	+0·57	+12·5	......	......	......	......	......	......	......	......	......	......
177	......	......	+0·31	+ 1·6	......	......	−0·65	− 1·8	......	......	+0·45	+ 3·6	......	......
178	+0·26	+ 4·0	......	......	......	......	......	......	......	......	......	......	......	......
179	+0·78	+10·4	......	......	......	......	......	......	......	......	......	......	......	......
180	......	......	......	......	......	......	......	......	......	......	−0·13	+ 8·4	......	......

TABLE II.—

No.	G.C.	N.G.C.	Number of chrono-graphed Observations.	Schmidt.		D'Arrest (Leipsic).		D'Arrest (Copenhagen).		Rümker.		Schönfeld I.	
				Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
				s	"	s	"	s	"	s	"	s	"
181	4128	5982	6	......	......	......	......	+ 0·3	− 2	−0·27	− 6·7	......	......
182	4234	6210	5	−0·16	+ 1·3	0·0	+ 6	− 0·1	+ 24	......	......	......	......
183	4256*)	6254	4 3	−0·45	+25·9	+ 0·1	+ 31	+ 0·6	− 16	......	......	......	......
184	4625	7006	1	+0·92	− 2·1	0·0	+ 18	− 1·0	− 10	......	......	+0·26	+ 1·4
185	4670	7078	..	(+0·56)	(+ 2·8)	(0·0)	(+ 8)	(+ 0·2)	(+ 12)	(+0·25)	(+ 4·3)	(+0·26)	(+ 3·3)
186	4678	7089	3	+0·98	− 0·5	− 0·1	− 11	+ 0·6	+ 8	+0·86	− 0·6	+0·08	+ 0·8
187	4734	7177	3 4	......	......	− 0·3	+ 11	+ 0·5	− 2	......	......	−0·04	+ 4·9
188	4760	7217	3	......	......	+ 0·4	− 1	− 0·7	0	......	......	+0·39	+ 0·2
189	4815	7331	4	......	......	+ 0·4	− 2	− 0·5	− 11	+0·68	+ 3·4	+0·24	+ 0·4
190	4821	7332	2	......	......	+ 0·1	+ 7	+ 1·1	+ 11	......	......	+0·06	+ 3·8
191	4827	7354	3	......	......	......	......	− 0·9	− 6	......	......	......	......
192	4845	7385	2	......	......	......	......	......	......	......	......	......	......
193	4879	7448	3	......	......	− .07	+ 12	− 0·9	− 1	......	......	−0·24	+ 3·7
194	4883	7457	4 3	......	......	− 0·4	− 22	− 0·1	− 2	......	......	......	......
195	4903	7515	3 2	......	......	......	......	− 0·5	+ 15	......	......	......	......
196	4921	7562	2 1	......	......	......	......	+ 0·5	+ 4	......	......	......	......
197	4928	7585	..	......	......	......	......	(− 0·7)	(− 4)	......	......	−0·14	− 4·8
198	4936	7619	3	......	......	......	......	+ 1·4	+ 1	......	......	+0·21	+ 3·5
199	4939	7625	1	......	......	......	......	− 1·0	− 2	......	......	(−0·11)	(+ 2·0)
200	4964	7662	4 5	−0·05	+ 0·9	− 0·2	− 4	0·0	− 3	+0·01	+ 2·0	−0·06	− 1·0
201	4993	7714	3	......	......	......	......	− 0·1	− 8	......	......	−0·25	− 1·5
202	5000	7727	..	......	......	......	......	(− 1·5)	(+ 36)	......	......	......	......
203	5005	7742	3	......	......	......	......	+ 0·6	+ 7	......	......	−0·05	− 1·6
204	5006	7743	1	......	......	......	......	+ 2·0	− 6	......	......	−0·32	+ 1·0
205	5015	7760	1	......	......	......	......	− 0·9	− 21	......	......	......	......
206	5029	7785	3	......	......	......	......	+ 1·1	− 11	......	......	......	......
207	5038	7798	1	......	......	......	......	− 1·0	0	......	......	−0·34	−10·3
208	5046	7814	3	......	......	......	......	− 0·2	+ 10	......	......	+0·04	+ 0·2
209	5050	7820	2 1	......	......	......	......	+ 0·2	+ 12	......	......	......	......

*) My observations refer to the following nucleus.

CONTINUED.

No.	Schönfeld II.		Schultz.		Auwers.		Vogel I., II.		Engelmann.		Engelhardt I., II.		Porter.	
	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.	Δ α cos δ.	Δ δ.
	s	″	s	″	s	″	s	″	s	″	s	″	s	″
181	−0·06	− 2·5	......	......	......	......	......	......	......	......	......	......	......	......
182	−0·05	+ 0·6	+0·01	+ 3·5	......	......	0·00	+ 3·4	−0·05	+ 3·4	+0·07	+ 1·6	......	......
183	.. ...	......	......	......	......	......	(+0·22)	(−38·5)	......	......	(−0·70)	(−19·8)	(−0·92)	−17·2)
184	......	......	+0·59	+ 1·5	......	......	{ +0·18 *+0·72*	+1·3 *+2·1* }	+1·02	+ 6·4	*+0·88*	+ *1·8*	......	......
185	......	......	(+0·06)	(+ 4·2)	......	...... {	(+0·26) (*+0·24*)	(+ 6·5) (+ *5·8*)	(+0·21)	(+ 2·1)	(+0·34)	(+ 2·5)	(+0·56)	(+ 9·7)
186	......	......	+0·87	+ 2·9	+0·54	+ 2·7	+0·55	+ 4·4	+0·56	+ 5·5	*+0·85*	+ *5·8*	......	......
187	......	......	+0·38	+ 5·1	......	......	−0·04	+ 4·8	+0·51	+11·9	*+0·60*	+ *6·1*	......	......
188	+0·70	− 1·0	+0·71	+ 0·7	......	......	+0·22	+ 2·8	+1·06	+ 3·4	......	......	......	......
189	+0·34	− 1·7	+0·37	− 1·1	......	......	......	......	+0·54	− 0·5	......	......	......	......
190	+0·35	+ 3·7	+0·30	+ 0·4	......	......	......	......	+0·32	+ 3·3	*+0·53*	+ *0·7*	......	......
191	+0·08	− 0·1	......	......	......	......	......	......	......	......	......	......	......	......
192	−0·26	+ 9·3	−0·25	+ 8·6	......	......	*−0·44*	*+29·2*	......	......	*+0·60*	+ *7·4*	......	......
193	......	......	+0·56	+ 3·9	......	......	{ −0·26 *+0·78*	+4·8 *+3·1* }	+0·94	+ 2·5	......	......	......	......
194	+0·71	+ 2·3	+0·47	+ 0·6	......	......	......	......	......	......	......	......	......	......
195	−0·08	+ 4·9	+0·09	+ 2·4	......	......	*−0·81*	+ *7·7*	......	......	......	......	......	......
196	−0·09	− 1·4	−0·36	+ 1·3	......	......	......	......	......	......	......	......	......	......
197	(−0·18)	(+ 0·9)	......	......	......	......	......	......	......	......	......	......	(−0·17)	(− 4·6)
198	+0·60	+ 5·8	+0·43	+ 3·0	......	......	+0·28	+ 4·0	+0·92	− 7·6	......	......	......	......
199	(+0·58)	(+ 5·0)	+0·18	+ 0·6	......	......	+0·12	+ 0·3	......	......	......	......	......	......
200	−0·08	+ 1·1	+0·19	+ 1·2	......	......	+0·03	− 0·7	+0·01	+ 2·2	+0·16	+ 1·1	......	......
201	−0·11	+ 4·9	+0·08	+ 0·8	.. ...	......	......	......	......	......	......	......	......	......
202	(−0·30)	(+ 2·2)	......	......	......	......	......	......	......	......	......	......	(−0·13)	(+ 0·3)
203	+0·15	+ 4·4	+0·54	+ 2·1	......	......	*+0·21*	+ 0·7	+0·56	− 3·0	......	......	......	......
204	−0·06	+ 0·2	+0·38	− 0·9	......	......	*+0·35*	− 1·2	......	......	......	......	......	......
205	+0·04	+ 7·1	......	......	......	......	......	......	......	......	......	......	......	......
206	+0·28	− 1·6	......	......	......	......	......	......	......	......	......	......	+0·07	−12·4
207	−0·38	− 3·9	−0·28	− 2·8	......	......	......	......	......	......	......	......	......	......
208	......	......	+0·17	0·0	......	......	*+0·37*	+ 0·5	+0·50	− 1·3	......	......	......	......
209	+1·10	+11·2	......	......	......	......	......	......	......	......	......	......	......	......

TABLE III.

G.C.	N.G.C.	Right Ascension, 1890·0.	Number of Observations.		Precession, 1890.	Declination, 1890·0.	Number of Observations.		Precession, 1890.
			Eye-Ear.	Chrono-graphed.			Eye-Ear.	Chrono-graphed.	
		h m s			s	° ′ ″			″
8	16	0 3 23·81	3	2	+3·083	+27 6 58·3	3	3	+20·05
53	108	0 20 12·70	..	3	+3·137	+28 36 8·2	..	2	+19·97
62	128	0 23 36·14	..	2	+3·078	+ 2 15 21·5	..	2	+19·95
90	185	0 32 52·27	..	4	+3·283	+47 43 48·0	..	4	+19·85
105	205	0 34 22·61	..	2	+3·247	+41 4 44·2	..	2	+19·83
117	221	0 36 41·77	5	3	+3·253	+40 15 41 2	5	3	+19·80
116	224	0 36 43·45	1	..	+3·256	+40 39 53·1	1	..	+19·80
149	266	0 43 50·59	1	4	+3·229	+31 40 40·6	..	4	+19·69
158	278	0 45 51·69	3	3	+3·357	+46 57 0·2	3	3	+19·65
176	315	0 51 50·20	..	3	+3·244	+29 45 24·8	..	5	+19·54
218	404	1 3 18·85	6	2	+3·329	+35 7 44·0	6	3	+19·29
264	470	1 14 4·12	..	2	+3·093	+ 2 49 59·1	..	1	+19·01
269	474	1 14 25·02	..	2	+3·094	+ 2 50 15·6	..	2	+19·00
307	524	1 19 1·20	4	2	+3·144	+ 8 57 51·0	4	3	+18·87
342	584	1 25 49·33	3	3	+3·009	− 7 26 8·1	3	3	+18·66
385	650	1 35 22·57	3	2	+3·740	+51 0 47·7	3	2	+18·34
386	651	1 35 26·39	2	4	+3·741	+51 1 23·0	2	4	+18·34
430	718	1 47 30·04	..	7	+3·111	+ 3 39 6·8	..	6	+17·89
463	772	1 53 17·71	1	..	+3·284	+18 28 18·7	1	..	+17·65
526	890	2 15 28·46	2	6	+3·552	+32 45 42·2	1	6	+16·65
544	936	2 22 1·20	1	5	+3·050	− 1 39 1·6	1	5	+16·32
549	949	2 24 3·80	..	4	+3·657	+36 38 46·0	..	4	+16·22
574	1022	2 33 5·35	..	2	+2·969	− 7 9 8·7	..	2	+15·74
575	1023	2 33 30·86	2	3	+3·735	+38 35 19·7	2	3	+15·72
600	1068	2 37 2·80	..	2	+3·065	− 0 28 58·5	..	2	+15·53
604	1084	2 40 34·90	..	3	+2·951	− 8 2 36·0	..	3	+15·33
628 } 634 }	1161	2 53 54·06	..	4	+3·975	+44 27 34·0	..	4	+14·55
648	1209	3 0 54·37	1	1	+2·800	−16 2 15·3	1	1	+14·12
675	1275	3 12 31·98	3	4	+3·941	+41 6 32·3	2	4	+13·38
709	1332	3 21 23·73	..	2	+2·662	−21 43 17·8	..	2	+12·80
752	1407	3 35 14·05	1	5	+2·702	−18 56 12·6	1	5	+11·84
778	1453	3 40 57·33	..	2	+2·990	− 4 18 45·7	..	2	+11·43
801	1501	3 57 31·07	..	3	+5·116	+60 37 19·5	..	2	+10·21
826	1535	4 9 7·43	3	5	+2·799	−13 1 6·9	2	5	+ 9·33
847	1569	4 20 24·70	1	5	+5·627	+64 36 12·4	1	6	+ 8·44
866	1600	4 26 14·18	..	2	+2·958	− 5 19 21·1	..	2	+ 7·98
888	1637	4 35 57·27	..	4	+3·005	− 3 4 21·5	..	4	+ 7·19
932	1700	4 51 30·23	..	7	+2·960	− 5 2 20·5	..	7	+ 5·90
1005	1788	5 1 25·95	1	6	+2·993	− 3 30 7·8	1	6	+ 5·07
1137	1931	5 24 9·63	3	5	+3·969	+34 9 41·2	3	5	+ 3·12
1157	1952	5 27 53·24	1	3	+3·606	+21 56 25·3	..	3	+ 2·80
1185	1982	5 30 6·88	..	1	+2·949	− 5 20 23·6	..	1	+ 2·61
1202	1999	5 31 4·47	..	3	+2·915	− 6 47 9·1	..	3	+ 2·52
1225	2022	5 36 4·68	2	2	+3·284	+ 9 1 47·3	2	2	+ 2·09
1267	2068	5 41 5·03	3	..	+3·073	+ 0 1 57·0	3	..	+ 1·65
1270	2071	5 41 29·07	2	..	+3·078	+ 0 15 27·7	2	..	+ 1·62
1337	2142	5 56 39·91	2	..	+2·822	−10 35 59·2	2	..	+ 0·29
1362	2170	6 2 10·20	3	4	+2·923	− 6 23 11·9	..	4	− 0·19
1375	2185	6 5 44·06	.	5	+2·928	− 6 11 18·7	..	5	− 0·50

TABLE III.—Continued.

G.C.	N.G.C.	Right Ascension, 1890·0.	Number of Observations.		Precession, 1890.	Declination, 1890·0.	Number of Observations.		Precession, 1890.
			Eye-Ear.	Chrono-graphed.			Eye-Ear.	Chrono-graphed.	
		h m s			s	° ′ ″			″
1425	2245	6 26 37·32	5	5	+3·312	+10 14 17·9	5	5	− 2·32
1437	2261	6 33 9·19	3	6	+3·278	+ 8 49 58·7	3	6	− 2·89
1500	2346	7 3 46·06	4	5	+3·058	− 0 37 50·8	4	5	− 5·51
1519	2371	7 18 37·20	2	5	+3·791	+29 42 8·6	2	5	− 6·74
1532	2392	7 22 40·09	3	2	+3·556	+21 8 6·8	3	2	− 7·08
1546	2415	7 29 42·27	3	4	+3·953	+35 29 2·7	2	4	− 7·65
1548	2419	7 30 40·72	2	4	+4·076	+39 7 33·6	1	5	− 7·73
1567	2440	7 37 0·49	2	4	+2·678	−17 57 2·3	2	4	− 8·24
1596	2481	7 50 38·14	1	4	+3·601	+24 3 34·6	2	4	− 9·31
1626	2532	8 3 11·32	1	2	+3·855	+34 16 45·0	..	2	−10·27
1629	2537	8 5 25·96	1	3	+4·268	+46 19 9·2	1	3	−10·43
1632	2542	8 6 6·70	1	2	+2·818	−12 36 4·4	1	2	−10·49
1660	2592	8 20 31·13	2	3	+3·614	+26 19 46·2	2	3	−11·54
1673	2612	8 28 38·56	2	3	+2·831	−12 47 40·7	2	1	−12·11
1684	2639	8 35 42·85	4	6	+4·339	+50 35 55·7	4	7	−12·60
1691	2655	8 41 6·82	1	2	+8·147	+78 38 6·3	1	2	−12·96
1704	2672	8 43 5·22	1	3	+3·431	+19 28 43·7	1	3	−13·09
1711	2681	8 45 37·38	1	3	+4·341	+51 43 38·6	..	3	−13·26
1713	2683	8 45 51·16	1	3	+3·744	+33 49 53·2	1	3	−13·28
1720	2693	8 49 5·82	2	..	+4·328	+51 46 2·3	2	2	−13·49
1728	2701	8 50 58·21	1	3	+4·434	+54 11 49·0	1	3	−13·61
1765	2768	9 3 3·04	3	4	+4·720	+60 29 11·1	3	4	−14·37
1771	2775	9 4 29·14	2	3	+3·194	+ 7 29 4·4	1	3	−14·45
1781	2787	9 9 20·60	..	3	+5·518	+69 39 53·2	..	2	−14·75
1811	2830	9 13 3·40	2	2	+3·678	+34 12 45·2	1	2	−14·96
1823	2841	9 14 25·82	..	2	+4·181	+51 26 30·8	..	2	−15·04
1848	2880	9 20 57·90	3	2	+4·748	+62 58 12·6	3	2	−15·42
1861	2903	9 25 55·98	4	2	+3·409	+21 59 5·5	4	2	−15·69
1896	2964	9 36 22·79	3	4	+3·569	+32 20 51·3	3	4	−16·24
1904	2974	9 37 0·46	..	1	+3·029	− 3 11 45·0	..	1	−16·27
1944	3021	9 44 24·99	1	3	+3·577	+34 4 2·1	1	3	−16·65
1949	3031	9 46 27·12	..	2	+5·049	+69 35 1·5	..	2	−16·74
1973	3067	9 51 54·27	1	2	+3·531	+32 53 40·5	2	3	−17·00
2008	3115	9 59 45·27	4	4	+2·988	− 7 11 6·7	3	4	−17·36
2024	3147	10 7 26·85	..	2	+5·264	+73 56 53·0	..	2	−17·68
2041	3169	10 8 32·36	2	2	+3·116	+ 4 0 42·1	2	2	−17·73
2058	3190	10 12 1·31	2	2	+3·322	+22 22 58·2	2	2	−17·87
2102	3242	10 19 28·13	1	2	+2·887	−18 5 5·2	1	2	−18·15
2104	3245	10 21 7·43	2	3	+3·383	+29 4 2·8	2	3	−18·21
2112	3254	10 23 8·05	1	2	+3·390	+30 3 12·0	1	3	−18·29
2134	3277	10 26 46·07	..	4	+3·367	+29 4 37·8	..	4	−18·42
2145	3294	10 29 55·13	2	1	+3·471	+37 53 36·4	1	1	−18·52
2150	3301	10 30 56·06	2	2	+3·282	+22 27 5·2	2	2	−18·56
2158	3310	10 31 54·74	2	3	+3·764	+54 4 24·4	2	4	−18·59
2184	3351	10 38 9·02	2	2	+3·174	+12 16 46·9	2	2	−18·79
2182	3348	10 38 52·62	1	1	+4·629	+73 25 12·0	1	1	−18·81
2201	3377	10 41 51·87	1	2	+3·189	+14 33 53·7	1	2	−18·90
2203	3379	10 42 0·84	..	3	+3·177	+13 9 40·8	..	3	−18·90
2231	3423	10 45 34·40	..	1	+3·120	+ 6 24 35·5	..	1	−19·00
2274	3486	10 54 23·03	2	3	+3·287	+29 33 54·9	2	3	−19·24

TABLE III.—Continued.

G.C.	N.G.C.	Right Ascension, 1890·0.	Number of Observations.		Precession, 1890.	Declination, 1890·0.	Number of Observations.		Precession, 1890.
			Eye-Ear.	Chrono-graphed.			Eye-Ear.	Chrono-graphed.	
		h m s			s	° ′ ″			″
2276	3489	10 54 30·44	3	4	+3·170	+14 29 26·9	3	4	−19·24
2287	3504	10 57 12·58	..	4	+3·269	+28 33 51·3	..	4	−19·30
2301	3521	11 0 10·27	..	3	+3·076	+ 0 33 30·9	..	3	−19·37
2343	3587	11 8 23·36	..	2	+3·509	+55 37 7·7	..	2	−19·55
2347	3593	11 8 51·48	1	2	+3·143	+13 25 4·3	1	3	−19·56
2358	3607	11 11 6·76	2	2	+3·168	+18 39 3·7	2	2	−19·60
2373	3623	11 13 10·84	..	2	+3·139	+13 41 39·0	..	2	−19·64
2376	3626	11 14 16·23	1	2	+3·163	+18 57 25·7	..	2	−19·65
2396	3655	11 17 8·83	1	1	+3·149	+17 11 39·5	1	1	−19·70
2404	3665	11 18 44·81	..	2	+3·269	+39 21 57·5	..	2	−19·73
2413	3675	11 20 6·95	2	3	+3·297	+44 11 21·2	2	3	−19·75
2421	3683	11 21 17·24	..	2	+3·425	+57 28 52·4	..	2	−19·77
2426	3689	11 22 21·61	..	1	+3·180	+26 15 48·0	..	2	−19·78
2445	3726	11 27 20·91	..	4	+3·281	+47 38 8·6	..	3	−19·85
2454	3738	11 29 44·02	..	3	+3·325	+55 7 55·7	..	4	−19·88
2499	3810	11 35 18·08	2	3	+3·103	+12 4 53·2	2	2	−19·94
2501	3813	11 35 28·91	..	2	+3·181	+37 9 21·3	..	2	−19·94
2520	3838	11 38 15·79	2	3	+3·280	+58 33 33·7	2	3	−19·96
2545	3877	11 40 16·44	..	4	+3·201	+48 6 8·7	..	5	−19·98
2564	3898	11 43 22·75	..	2	+3·220	+56 41 42·3	..	2	−20·00
2566	3900	11 43 26·64	1	2	+3·123	+27 37 57·3	1	2	−20·00
2597	3938	11 47 3·88	..	3	+3·147	+44 43 57·0	..	4	−20·02
2600	3941	11 47 12·42	3	2	+3·130	+37 35 52·9	3	3	−20·02
2635	3992	11 51 52·67	1	2	+3·138	+53 59 10·5	1	3	−20·04
2660	4026	11 53 43·72	2	2	+3·119	+51 34 24·3	2	2	−20·04
2663	4030	11 54 45·65	2	2	+3·072	− 0 29 16·4	2	2	−20·05
2680	4051	11 57 31·09	1	2	+3·087	+45 8 41·5	2	2	−20·05
2717	4102	12 0 47·51	..	3	+3·066	+53 19 24·0	..	3	−20·05
2723	4111	12 1 27·29	1	2	+3·064	+43 40 42·8	1	2	−20·05
2752	4147	12 4 29·84	1	4	+3·063	+19 9 12·0	..	4	−20·05
2765	4162	12 6 16·85	..	2	+3·056	+24 44 9·8	..	2	−20·04
2786	4192	12 8 11·80	2	2	+3·059	+15 30 39·6	2	2	−20·04
2796	4203	12 9 32·22	1	2	+3·035	+33 48 31·9	1	2	−20·04
2806	4216	12 10 18·37	1	3	+3·058	+13 45 40·6	..	3	−20·03
2841	4258	12 13 30·61	1	4	+2·985	+47 54 51·9	1	6	−20·02
2904	4339	12 17 58·20	..	2	+3·060	+ 6 41 30·0	..	2	−19·99
2921	4365	12 18 52·10	2	1	+3·057	+ 7 55 36·5	2	2	−19·98
2924	4369	12 19 9·15	..	2	+2·979	+39 59 39·5	..	2	−19·98
2961	4406	12 20 37·32	..	4	+3·044	+13 33 18·0	..	4	−19·97
3025	4477	12 24 28·50	..	2	+3·036	+14 14 37·7	..	3	−19·94
3042	4490	12 25 13·52	..	2	+2·939	+42 14 56·3	..	2	−19·93
3035	4486	12 25 15·34	1	2	+3·039	+12 59 55·1	1	2	−19·93
3075	4526	12 28 27·53	3	2	+3·048	+ 8 18 18·5	2	2	−19·90
3106	4565	12 30 53·22	..	3	+2·983	+26 35 33·0	..	3	−19·87
3121	4579	12 32 9·81	..	3	+3·031	+12 25 24·4	..	3	−19·86
3132	4594	12 34 16·43	1	1	+3·111	−11 1 8·5	1	1	−19·83
3165	4631	12 36 43·79	..	4	+2·933	+33 9 8·2	..	4	−19·80
3169-70	4636	12 37 13·04	..	2	+3·060	+ 3 17 25·2	..	2	−19·79
3182	4649	12 38 7·50	..	2	+3·025	+12 9 19·5	..	2	−19·78
3193	4660	12 38 59·18	1	3	+3·025	+11 47 33·8	1	3	−19·76

TABLE III.—Continued.

G.C.	N.G.C.	Right Ascension, 1890·0.	Number of Observations.		Precession, 1890.	Declination, 1890·0.	Number of Observations.		Precession, 1890.	
			Eye-Ear.	Chrono-graphed.			Eye-Ear.	Chrono-graphed.		
		h m s			s	° ′ ″			″	
3198	4666	12 39 29·38	..	1	+3·072	+ 0 8 11·8	..	1	−19·76	
3227	4697	12 42 54·84	..	3	+3·095	− 5 12 1·3	..	3	−19·70	
3249	4725	12 45 3·23	..	2	+2·945	+26 6 2·5	..	2	−19·67	
3258	4736	12 45 41·84	1	3	+2·836	+41 43 7·2	..	4	−19·66	
3274	4754	12 46 45·43	..	2	+3·015	+11 54 39·2	..	2	−19·64	
3300	4793	12 49 20·82	1	2	+2·911	+29 31 59·6	1	2	−19·59	
3315	4814	12 50 36·42	..	1	+2·586	+58 56 22·6	..	1	−19·57	
3321	4826	12 51 19·29	2	3	+2·951	+22 16 39·6	2	3	−19·55	
3342	4866	12 53 58·37	..	2	+2·990	+14 45 47·4	..	3	−19·50	
3356	4900	12 55 4·68	..	2	+3·055	+ 3 5 5·9	..	2	−19·48	
3395	4956	12 59 50·98	..	3	+2·824	+35 46 4·4	..	4	−19·37	
3397	4958	13 0 5·12	2	2	+3·118	− 7 25 52·7	1	2	−19·37	
3437	5005	13 5 50·78	2	4	+2·780	+37 38 37·5	2	4	−19·23	
3453	5024	13 7 31·75	..	2	+2·941	+18 45 2·7	..	2	−19·19	
3459	5033	13 8 22·93	1	2	+2·774	+37 10 35·2	1	3	−19·17	
3474	5055	13 10 52·69	..	4	+2·698	+42 36 38·1	..	4	−19·10	
3505	5103	13 15 37·38	..	2	+2·659	+43 39 33·3	..	3	−18·97	
3524	5127	13 18 38·12	1	2	+2·790	+32 8 28·4	1	3	−18·88	
3572	5194	13 25 14·05	..	4	+2·537	+47 45 45·1	..	5	−18·68	
3592	5218	13 28 23·83	..	3	+2·071	+63 20 9·0	..	3	−18·58	
3615	5248	13 32 3·38	2	2	+2·986	+ 9 26 47·8	2	2	−18·46	
3636	5272	13 37 6·69	2	4	+2·769	+28 55 52·5	..	4	−18·28	
3637	5273	13 37 14·93	1	3	+2·670	+36 12 32·8	1	3	−18·27	
3645	5290	13 40 38·15	..	1	+2·556	+42 15 49·4	1	2	−18·15	
3671	5322	13 45 34·74	..	3	+2·012	+60 44 15·9	..	4	−17·96	
3702	5363	13 50 35·77	2	2	+3·010	+ 5 47 35·5	2	2	−17·76	
3716	5377	13 51 54·49	..	4	+2·381	+47 46 27·4	..	3	−17·71	
3770	5457	13 59 16·29	..	1	+2·127	+54 52 20·1	..	1	−17·40	
*nr 3770	....	13 59 20·13	1	5	+2·126	+54 53 57·8	1	5	−17·40	*11–12
3794	5485	14 3 22·06	..	4	+2·074	+55 31 25·6	..	4	−17·22	
3846	5566	14 14 47·67	1	5	+3·015	+ 4 26 21·3	..	5	−16·68	
3854	5582	14 16 14·65	..	4	+2·440	+40 11 49·3	..	5	−16·61	
3897	5631	14 23 7·02	..	3	+1·865	+57 4 34·0	1	3	−16·27	
3942	5689	14 31 35·61	..	4	+2·120	+49 13 17·7	..	4	−15·82	
3956	5701	14 33 41·95	..	4	+2·987	+ 5 50 16·0	..	4	−15·71	
3987	5746	14 39 21·09	1	3	+3·036	+ 2 25 13·4	1	3	−15·40	
4021	5806	14 54 25·54	..	4	+3·035	+ 2 19 32·7	..	4	−14·52	
4045	5846	15 0 54·45	1	4	+3·039	+ 2 1 50·9	1	4	−14·12	
4058	5866	15 3 27·90	..	4	+1·640	+56 11 11·8	..	4	−13·96	
4064	5879	15 6 54·53	1	3	+1·549	+57 24 57·4	1	3	−13·75	
4077	5899	15 11 4·40	1	2	+2·167	+42 27 19·4	1	4	−13·48	
4128	5982	15 36 25·12	1	6	+1·218	+59 42 48·6	1	6	−11·76	
4234	6210	16 39 53·03	..	5	+2·513	+24 0 14·2	..	5	− 6·87	
4256	6254	16 51 21·99	..	4	+3·160	− 3 55 19·9	..	3	− 5·92	
4625	7006	20 56 20·78	1	1	+2·802	+15 45 7·4	1	1	+13·95	
4670	7078	21 24 39·13	1	..	+2·899	+11 41 2·7	1	..	+15·62	
4678	7089	21 27 46·90	3	3	+3·091	− 1 18 41·3	3	3	+15·79	
4734	7177	21 55 26·23	4	3	+2·858	+17 12 30·1	2	4	+17·16	
4760	7217	22 2 56·40	5	3	+2·683	+30 49 15·1	4	3	+17·49	
4815	7331	22 32 2·03	3	4	+2·737	+33 50 44·2	3	4	+18·59	

TABLE III.—CONTINUED.

G.C.	N.G.C.	Right Ascension, 1890·0.	Number of Observations. Eye-Ear.	Chrono-graphed.	Precession, 1890.	Declination, 1890·0.	Number of Observations. Eye-Ear.	Chrono-graphed.	Precession, 1890.
		h m s			s	° ′ ″			″
4821	7332	22 32 9·48	1	2	+2·858	+23 13 37·2	1	2	+18·60
4827	7354	22 36 14·16	1	3	+2·221	+60 42 43·4	2	3	+18·73
4845	7385	22 44 25·48	..	2	+2·988	+11 1 26·8	..	2	+18·97
4879	7448	22 54 36·18	..	3	+2·969	+15 23 22·3	..	3	+19·24
4883	7457	22 55 43·79	3	4	+2·863	+29 33 9·9	3	3	+19·27
4903	7515	23 7 17·37	..	3	+3·007	+12 4 50·6	..	2	+19·52
4921	7562	23 10 22·75	..	2	+3·042	+ 6 5 17·3	..	1	+19·58
4928	7585	23 12 20·73	1	..	+3·098	− 5 15 3·1	1	..	+19·62
4936	7619	23 14 39·88	..	3	+3·037	+ 7 36 13·5	..	3	+19·66
4939	7625	23 15 0·14	..	1	+2·995	+16 37 25·6	..	1	+19·67
4964	7662	23 20 36·90	4	4	+2·867	+41 55 47·5	4	5	+19·76
4993	7714	23 30 36·54	..	3	+3·068	+ 1 32 45·2	..	3	+19·89
5000	7727	23 34 12·34	3	..	+3·107	−12 54 12·2	3	..	+19·93
5005	7742	23 38 39·64	..	3	+3·050	+10 9 23·0	..	3	+19·97
5006	7743	23 38 44·84	1	1	+3·052	+ 9 19 28·6	1	1	+19·97
5015	7760	23 43 38·45	..	1	+3·017	+30 22 10·2	..	1	+20·00
5029	7785	23 49 41·27	..	3	+3·067	+ 5 18 15·6	..	3	+20·03
5038	7798	23 53 48·38	..	1	+3·059	+20 8 21·9	..	1	+20·05
5046	7814	23 57 36·40	..	3	+3·069	+15 31 56·3	..	3	+20·05
5050	7820	23 58 51·17	..	2	+3·072	+ 4 35 1·8	..	1	+20·05

POSITIONS OF STARS WITHIN ONE DEGREE FROM THE NORTH POLE, AND OF THREE FUNDAMENTAL POLE-STARS.

By L. BECKER, Ph.D.,

Late Assistant Astronomer at Lord Crawford's Observatory, Dunecht.

In the winter months of 1886-87 I measured, by means of the micrometers of the Transit Circle, the rectangular co-ordinates of most of the stars of BD 89°, in connection with α and λ Ursæ Minoris and B.A.C. 4165. While it was originally my intention to secure at least four observations both at upper and lower culmination, I found myself soon obliged to abandon the plan owing to the small number of clear nights and to the prior claim of other meridian work on my time. The series of observations was finally broken off at the end of the season. However incomplete it is, the results may perhaps be deemed useful in connection with the photographic survey of that region, and they may also perhaps induce other observers to make similar observations, which, if extended over a longer period, will give the means of determining the absolute positions of Pole-stars, the motion of the Pole, and the constant of aberration independent of the change of latitude.

The Transit Circle, which was made by Troughton & Simms in 1874, is well suited for work of this kind. The object-glass has an aperture of 215^{mm}, and, although badly stained, it enabled me to observe the faint stars of the zone in an illuminated field. The screws of the two micrometers at the eye-end are excellent, and the mounting is solid.

The screw value of the collimation micrometer was determined from transits of α and λ Ursæ Minoris, the mean value being—

$$R = 19''{\cdot}2873 \pm 0''{\cdot}0004.$$

As the extremes of temperature did not differ more than 18° Fahr., no correction for temperature was introduced. In the observations of the stars, about 30 revolutions of the screw were used; and in the few instances where the distance from the centre of the field exceeded this range, I employed, instead of the middle wire, the outside wire of the movable system. The progressive errors of the screw can be represented by a regular curve. The corrections were: + 0·015 R at

$-$ 20 R (East), 0·000 at 0 R (centre of field), $+$ 0·014 R at $+$ 13 R (West). The periodic errors did not exceed $\pm$ 0·002 R. The micrometer wire was slightly inclined to the vertical, the correction being 0·0032 R for every revolution of the declination screw.

The screw value of the declination micrometer, 38″·279, was found by means of the circle and collimator. Only a few revolutions were employed. The periodic errors fell within $\pm$ 0·002 R. From September 27th to November 30th the correction for the inclination of the declination wire was $-$ 0″·021 for every revolution of the collimation screw, and $-$ 0″·016 after that date—a new wire having been inserted in December. These quantities are based respectively on 52 and 139 observations of clock-stars which had been three times bisected at each transit.

In correcting the circle readings I made use of Mr. Lohse's investigation of the periodic division errors which he derived in 1878. These errors, which are the mean of the corrections of four microscopes, vary about 0″·03 from the Pole-point to a polar-distance of 4°. As to the accidental errors of division, no special observations were made; but as the Runs which correspond to the several divisions near the Pole-point agree satisfactorily with each other, it may be assumed that they must be very small. It may be added that the value of the Runs is practically the same for Pole- and equatorial stars, upwards of 700 observations being available for this comparison.

Method of Observing.—Particular stress was laid on making all the observations of the stars with *field illumination*, although the faintness of most of the stars made this a troublesome condition.

Each star was bisected about ten times with the vertical wire, and after the first half-set of these measurements the horizontal wire was brought three or four times into coincidence with the star by means of the micrometer screw. The time was noted to about a half-second. In order to avoid systematic errors in the bisections of these faint objects I turned the screws alternately in opposite directions, after having convinced myself that there was no appreciable difference in the readings when the wire of the collimator was observed in the same manner. At the end I read the circle by the four microscopes, setting the wire on the preceding as well as on the following division.

At the beginning of each night I determined the Nadir-point of the circle conjointly with the vertical position of the middle wire, and, if the observations extended over several hours, the same determination was repeated at the end of the night. The error of collimation was found at intervals of about a fortnight by means of the two collimators; it is not used in the reductions, except for deriving the clock correction from the time-stars, and, besides, it undergoes but slight alterations.

After the first observing night I chose the star nearest the Pole, BD 89°, 37, as standard, and if possible I observed it at the beginning and end of each night.

Reductions.—In addition to my own observations I made use in the reductions only of the positions of the clock-stars, and in the first approximation also of those of the three fundamental Pole-stars. Many of my observations had been made at a considerable hour-angle, and it therefore became necessary to derive special formulæ for the reductions. The course I followed may be shortly described as follows :—

The individual settings of an observation were reduced to the mean of the times (T_o) at which the declinations were measured. From the quantities reduced to T_o the Right-Ascension and Declination were computed, and the correction for daily aberration was applied. The equatorial co-ordinates were next converted into longitude and latitude, and these were reduced to the mean equinox of 1887.0. Finally, I derived the corrections of the adopted values of the instrumental errors, and corrected the positions accordingly.

Although the reductions to mean place are somewhat shorter for rectangular co-ordinates if Bessel's constants of reduction to apparent place be taken from the Nautical Almanac, I was induced to discard this method owing to a misconception as to the accuracy of my observations, which I believed required more accurate data than given in the ephemerides.

1. Reductions of Settings to T_o.

Let a be the reading of the collimation micrometer, corrected for errors of the screw and inclination of wire, the reading increasing if the vertical wire be turned towards the west,

d, the reading of the circle combined with that of the declination micrometer, and corrected for error of the screw and circle, for inclination of the wire, and for refraction,

T, the clock time of observation, and ΔT its correction,

τ, the hour-angle, positive towards the East,

D, the Pole-point of the circle, c, m, n, the errors of the Transit Circle,

T_o, the mean of the times at which the star was bisected with the horizontal wire,

and let the quantities corresponding to T_o have the suffix o; then the spherical triangle, Pole, Star, West end of axis, gives (for upper and lower culmination),

$$\text{(A.)}\quad \begin{aligned} \sin(a+c) &= \sin p \cos n \sin(\tau - m) - \sin n \cos p \\ \cos(a+c)\sin(d-D) &= \sin p \cos(\tau - m) \\ \cos(a+c)\cos(d-D) &= \cos p \cos n + \sin n \sin p \sin(\tau - m), \end{aligned}$$

and similar equations (A_o) for the quantities corresponding to time T_o. $a_o - a$ and $d_o - d$ can be found from the combination of (A) and (A_o). The degree of accuracy for $a_o - a$ need not exceed that with which the collimation screw has been read, viz. 0·001 R or 0″·02. Although n was in my observations smaller

than 2″, I shall assume in what follows that it does not exceed 1′. $a + c$, *i.e.* the angular distance of the star from the line of collimation exceeded 20 R (R = 19″·3) only a few times, but the third power can always be neglected.

The first equation of (A) gives by Taylor's theorem

$$a_\circ - a = -\sin p \cos(\tau_\circ - m)(T_\circ - T) + \sin p \sin(\tau_\circ - m)\frac{(T_\circ - T)^2}{2}$$

The unknown polar distance and $(\tau_\circ - m)$ can be eliminated by means of $(A_\circ)$. Considering that $(T_\circ - T)$ does not amount to 6^m and that $(d_\circ - D)$ is, in the case of the close Pole-stars, less than 1°, I get

$$(1.) \qquad a_\circ - a = -f\left(\frac{T_\circ - T}{10}\right) + g\left(\frac{T_\circ - T}{10}\right)^2$$

$$f = 13{\cdot}574\,\frac{d_\circ - D}{100} \qquad 13{\cdot}574 = \frac{10 \,.\, 60 \,.\, 15 \,.\, \sin 1''}{R}\;100 \,.\, 60$$

$$g = \frac{a_\circ + x}{1050} \qquad \frac{1}{1050} = \frac{(10 \,.\, 60 \,.\, 15 \,.\, \sin 1'')^2}{2}$$

$$x = c + n,$$

where $T_\circ - T$ is expressed in minutes of time, $d_\circ - D$ in minutes of arc, and $a_\circ - a$ in units of a screw revolution. The factors f and g are constant for the various settings made at a transit, $a_\circ + x$ can be roughly interpolated, and $d_\circ - D$ is known from the reductions of the declination readings.

The means of $a_\circ$ for each transit, $[a_\circ]$, have been compiled in Table II.

The second equation of (A) gives by Taylor's theorem

$$d_\circ - d = \frac{(a_\circ + x)}{\cos(d_\circ - D)}(T_\circ - T) + (d_\circ - D)\frac{(T_\circ - T)^2}{2}$$

But according to the definition of $T_\circ$, the first term disappears for the mean of all the readings. The correction of $[d]$, the mean of the several values of d, therefore becomes

$$(2.) \qquad d_\circ - [d] = h\left[\left(\frac{T_\circ - T}{10}\right)^2\right]$$

$$h = 5''{\cdot}71\,\frac{d_\circ - D}{100} \qquad 5{\cdot}71 = \tfrac{1}{2}(10 \,.\, 60 \,.\, 15 \sin 1'')^2\;100 \,.\, 60,$$

where $T_\circ - T$ and $d_\circ - D$ are expressed in minutes of time and arc respectively, and the square brackets indicate the mean of the quantities enclosed.

$d_\circ$ will be found in Table II.

2. Polar Distance.

The equations ($A_\circ$) can be written

$$
\text{(B.)}\quad
\begin{aligned}
\cos p &= \cos([a_\circ] + c)\cos n \cos(d_\circ - D) - \sin([a_\circ] + c)\sin n\\
\sin p \cos(\tau_\circ - m) &= \cos([a_\circ] + c)\sin(d_\circ - D)\\
\sin p \sin(\tau - m) &= \sin([a_\circ] + c)\cos n + \cos([a_\circ] + c)\sin n \cos(d_\circ - D).
\end{aligned}
$$

In the case of the fundamental stars, whose polar distance, always greater than 1°, can be taken from the ephemerides, the first formula can be developed in a series arranged according to powers of p. With regard to terms which may attain 0″·01, I find

$$
\text{(3.)}\quad p = \pm(d_\circ - D) + \frac{([a_\circ] + x)^2}{2p} + \frac{([a_\circ] + x)^4}{8p^3} \qquad \begin{pmatrix} +\text{Upper culmination} \\ -\text{Lower culmination} \end{pmatrix}
$$
$$
x = c + n,
$$

where in almost every case the third term may be neglected.

This formula served to determine an *approximate* value of the Pole-point (D) of the circle, the polar-distance being interpolated from the ephemerides. If more than one fundamental Pole-star had been observed, the mean of D was used in the reductions. The values of D have been entered in Table I.

By means of Lagrange's formula, one might rearrange the series (3) according to powers of $d_\circ - D$, to be used in the reductions of the close Pole-stars whose polar-distances were assumed to be unknown. As, however, the hour-angles at which the observations were taken are often considerable, I preferred to employ the following more convenient expression:—

$$
\text{(3*)}\quad
\begin{aligned}
p &= \pm(d_\circ - D)\sqrt{1 + \left(\frac{[a_\circ] + x}{d_\circ - D}\right)^2} \quad \text{if } d_\circ - D > a_\circ + x\\
p &= \pm([a_\circ] + x)\sqrt{1 + \left(\frac{d_\circ - D}{[a_\circ] + x}\right)^2} \quad \text{if } d_\circ - D < a_\circ + x
\end{aligned}
\qquad \begin{pmatrix} +\text{Upper culmination} \\ -\text{Lower culmination} \end{pmatrix}
$$

3. Right-ascensions.

The equations (B) give further

$$
\tan(\tau_\circ - m) = \frac{\tan([a_\circ] + c)}{\sin(d_\circ - D)}\cos n + \frac{\sin n}{\tan(d_\circ - D)}.
$$

In the reductions of a to $a_\circ$ terms reaching 0″·02 were taken into account, while the average difference of $a_\circ$ from the mean value $[a_\circ]$ amounted to 0″·50. In the computation of τ one may therefore neglect terms of the order 0″·01/ sin ($d_\circ$ − D) and may write

$$
\text{(4.)}\quad \tan\Big(a - (T_\circ + \Delta T + m)\Big) = \frac{([a_\circ] + x)''}{\big(\sin(d_\circ - D)\big)''}
$$

log (sin ($d_\circ$ $-$ D))$''$ was computed from log ($d_\circ$ $-$ D)$''$ by means of a table which gives $\mathrm{L}(a) = \log a'' - \log \frac{\sin a}{\sin 1''}$ in units of the sixth decimal of logarithms, with argument log a'' and a''.

The expression for tan ($\tau_\circ$ $-$ m) may also be developed in a series which is advantageous for the reductions of the fundamental Pole-stars.

$$(4.^*) \qquad \alpha = \mathrm{T}_\circ + (\Delta\,\mathrm{T} + m) + \frac{1}{\sin 1''}\,\frac{([a_\circ] + x)''}{(\sin(d_\circ - \mathrm{D}))''} - \frac{1}{3 \sin 1''}\left(\frac{([a_\circ] + x)''}{(\sin(d_\circ - \mathrm{D}))''}\right)^3$$

4. Error of Instrument. $x = c + n$, and clock correction $\Delta\mathrm{T} + m$.

The first formula of (A) can be developed in a series which I write thus:

$$(5.) \qquad \alpha - \left(\mathrm{T}_\circ + \frac{[a_\circ]}{\sin p} + \tfrac{1}{6}\left(\frac{[a_\circ] + c}{\sin p}\right)^3 + c \tan \tfrac{1}{2}\, p\right) = \Delta\mathrm{T} + m + \frac{x}{\tan p.}$$

For lower culmination p has to be taken negative.

A similar equation holds good for a clock-star. Let T denote the time of transit over the middle wire whose error of collimation is c, then

$$(5.^*) \qquad \alpha - \left(\mathrm{T} + c \tan \tfrac{1}{2}\,(90 - \delta)\right) = \Delta\mathrm{T} + m + x \tan \delta,$$

which may also be derived from Bessel's formula by adding and subtracting $c \tan \delta$. The combination of the two equations (5) and (5*), as corresponding to observations of Pole-stars and time-stars, determine $\Delta\mathrm{T} + m$ and x. x ought to be found at least with the same accuracy as $[a_\circ]$, as is obvious from (3) and (4), while c affects only indirectly, by $\Delta\mathrm{T} + m$, the positions of the Pole-stars. In conformity with the degree of accuracy of the observations it will be sufficient to know c and $\Delta\mathrm{T} + m$ to 0·1 $-$ 0·2 seconds of time. There is no difficulty in attaining this limit, no matter which acknowledged system of clock-stars be employed, and therefore in the final adjustment of the positions it will be sufficient to introduce corrections only of x and D. The values of x and D adopted in the first approximation have been entered in Table I., together with the observed Nadir-points of the circle.

5. Reduction to Mean Place.

The correction for daily aberration in Table II. was computed from

$$(6.) \qquad \begin{aligned} \alpha_1 &= \alpha - \frac{(9{\cdot}240)}{\sin p} \cos\left(\alpha - (\mathrm{T}_\circ + \Delta\mathrm{T} + m)\right) \\ p_1 &= p - (9{\cdot}240) \cos p \sin\left(\alpha - (\mathrm{T}_\circ + \Delta\mathrm{T} + m)\right) \end{aligned}$$

Let ϵ be the true obliquity of the ecliptic,
$\lambda_1 = 90 - l_1$, the apparent longitude,
$\beta_1 = 90 - \epsilon - e_1$, the apparent latitude,

then the spherical triangle, Pole of equator, Pole of ecliptic, and Star, gives

$$\text{(C.)} \qquad \begin{aligned} \cos(\epsilon + e_1) &= \cos p_1 \cos \epsilon - \sin p_1 \sin \epsilon \sin \alpha_1 \\ \cos l_1 \sin(\epsilon + e_1) &= \cos p_1 \sin \epsilon + \sin p_1 \cos \epsilon \sin \alpha_1 \\ \sin l_1 \sin(\epsilon_1 + e_1) &= \sin p_1 \cos \alpha_1 \end{aligned}$$

in which $l_1 \sin \epsilon$ and e_1 are small angles which do not exceed p. With the view of maintaining in the transformation the same degree of accuracy as before (0″·01), I develop e_1 in a series arranged according to powers of p, and get

$$e_1 = (p_1 \sin \alpha_1) + \frac{\cot \epsilon}{2}(p_1 \cos \alpha_1)^2 - \tfrac{1}{2}(\cot^2 \epsilon + \tfrac{1}{3})(p_1 \cos \alpha_1)^2 (p_1 \sin \alpha_1) +$$
$$\cot \epsilon \left(\frac{\cot^2 \epsilon}{2} + \frac{1}{3}\right)(p_1 \cos \alpha_1)^2 (p_1 \sin \alpha_1)^2 - \frac{\cot \epsilon}{8}(\cot^2 \epsilon + \tfrac{1}{3})(p_1 \cos \alpha_1)^4$$

The coefficients are practically constant during the short period which contains the observations. With $\epsilon = 23° \ 27'{\cdot}10$ they become, p and e being expressed in seconds of arc,

$$\text{(7.)} \qquad \begin{aligned} e_1 = (p_1 \sin \alpha_1) &+ 558''{\cdot}79 \left(\frac{p_1 \cos \alpha_1}{10000}\right)^2 - 66''{\cdot}36 \left(\frac{p_1 \cos \alpha_1}{10000}\right)^2 \left(\frac{p_1 \sin \alpha_1}{10000}\right) + \\ &7{\cdot}''85 \left(\frac{p_1 \cos \alpha_1}{10000}\right) \left(\frac{p_1 \sin \alpha_1}{10000}\right)^2 - 1''{\cdot}85 \left(\frac{p_1 \cos \alpha_1}{10000}\right)^4 \end{aligned}$$

The complement of longitude was computed from

$$\text{(8.)} \qquad \log(l_1'' - S(l_1)) = \log(p_1'' \cos \alpha_1) - L(p_1) - \log \sin(\epsilon + e_1)$$

where S (a) is the reduction in seconds of sin a to a, tabulated with $\dfrac{\sin a}{\sin 1''}$ as argument.

The corrections for Aberration, Nutation, and Precession were computed from the following formulæ :—

$$\text{(9.)} \qquad \begin{aligned} l_2 - l_1 &= -\kappa \frac{\sin(\odot + l_1)}{\sin(\epsilon + e_1)} \\ e_2 - e_1 &= \kappa \cos(\odot + l_1) \cos(\epsilon + e_1) \\ l - l_2 &= \psi_1 + \Delta\lambda + \pi \cot(\epsilon + e_2) \sin(l_2 + \mathrm{A}) \\ e - e_2 &= \qquad\qquad - \pi \cos(l_2 + \mathrm{A}) \\ l &= l_1 + (l_2 - l_1) + (l - l_2) \\ \epsilon + e &= (\epsilon + e_1) + (e_2 - e_1) + (e - e_2) \\ \left.\begin{aligned} \lambda_\circ &= 90 - l \\ \beta_\circ &= 90 - (\epsilon + e) \end{aligned}\right\} & \text{ referred to Equinox 1887·0.} \end{aligned}$$

The notation employed is the following :—

$\odot$ = longitude of the sun.
κ = 20″·454 (Harkness).
ψ_1 = general precession from 1887·0 to date of observation (from Harkness).
$\Delta\lambda$ = nutation in longitude.
ε = true obliquity of the ecliptic.
A = $\Pi + \psi_1 + \Delta\lambda$, where Π = longitude of ascending node of ecliptic at time of observation on ecliptic of 1887·0.
π = mutual inclination of the ecliptics.

ϵ and $\Delta\lambda$ were computed from Oppolzer's tables in "Bahnbestimmung der Planeten und Kometen" I. Vol., second edition, but in order to reduce them to Harkness' values, which have been adopted here, I subtracted 0″·014 from the mean obliquity, and one 596th part of its value from the principal term in $\Delta\lambda$.

Table III. gives the results as found by means of (3), (4), (6), (7), (8), and (9).

6. Adjustment of the Results.

In the foregoing the observations have been reduced on the assumption of approximate values of the instrumental error x and the Pole-point D, which were derived from the positions of the fundamental Pole-stars as given in the ephemerides. If the differences of the mean longitudes and latitudes from their mean values be formed and compiled for each day, it will be seen that the outstanding errors belonging to the same night are systematically connected. In accordance with No. 3 it is sufficient to eliminate the systematic differences by applying some corrections to the assumed values of x and D. The differentiation of (B) and (C) gives, if small quantities be neglected,

$$\begin{aligned} de &= \cos\Theta\, d\, x - \sin\Theta\, d\, \mathrm{D} \\ \sin\epsilon\, d\, l &= -\sin\Theta\, d\, x - \cos\Theta\, d\, \mathrm{D}. \\ \Theta &= \mathrm{T} + \Delta\mathrm{T} + m. \end{aligned} \tag{10.}$$

Let $[l]$ and $[\epsilon + e]$ express the mean of the co-ordinates of a star as reduced by x and D, and let $d\,[l]$ and $d\,[e]$ stand for the corrections to be applied to these mean values owing to errors of x and D, further, let n_l and n_e be the differences between mean and observed values, then

$$\begin{aligned} n_l \sin\epsilon &= -\, d\,[l] \sin\epsilon - \sin\Theta\, d\, x - \cos\Theta\, d\, \mathrm{D} \\ n_e \quad &= -\, d\,[e] \quad\quad + \cos\Theta\, d\, x - \sin\Theta\, d\, \mathrm{D}. \end{aligned} \tag{11.}$$

Before solving the equations (11), I corrected by means of (10) the ecliptic co-ordinates for change of the zero-point of the circle, for all those nights on which the Nadir-point had been determined more than once. Changes of x could not, however, be taken into account, since there was no azimuth mark in connection with the instrument.

If the errors of the mean values $d\,[l]$ and $d\,[e]$ were merely due to accidental errors in the observations, or to accidental errors in x and D on the various nights,

it would be legitimate to neglect these quantities in first approximation, and to determine the corrections of x and D by successive approximations. However, it is obvious from the formulæ given above, that an error in the assumed Right-ascension and Declination of the fundamental Pole-stars will affect all the observations of the same star in the same manner, unless fundamental star and comparison star have been observed both together at upper or lower culmination, as well as one of them at upper and the other at lower culmination. In the series under consideration most of the stars do not fulfil this condition. Nevertheless, the great number of observations taken of BD 89°, 37, at different hour-angles, combined with those of the fundamental Pole-stars, gives the means of entirely eliminating the assumed positions taken from the ephemerides.

To this end I eliminate $d\,x$ and d D from the equations (11) which I assume to refer to observations of a fundamental Pole-star and of DM 89°, 37 taken on the same night. The result is

$$n_l \sin\epsilon - n_l' \sin\epsilon \sin(\Theta'-\Theta) - n'_e \sin(\Theta'-\Theta) = -d[l]\sin\epsilon + d[l']\sin\epsilon\cos(\Theta'-\Theta) + d\,e'\sin(\Theta'-\Theta)$$
$$n_e \quad + n_l' \sin\epsilon \sin(\Theta'-\Theta) - n'_e \cos(\Theta'-\Theta = -d[e] \quad - d[l']\sin\epsilon\sin(\Theta'-\Theta) + d\,e'\cos(\Theta'-\Theta)$$

where the accented letters refer to the fundamental star.

The normal equations become

α Ursæ Min.		λ Ursæ Min.		BD 89°, 37.		
$d\,[l']\sin\epsilon$	$d\,[e']$	$d\,[l'']\sin\epsilon$	$d\,[e'']$	$d\,[l]\sin\epsilon$	$d\,[e]$	n
						″
+ 19·00	0	0	0	− 13·87	− 0·82	+ 3·59
0	+ 19·00	0	0	+ 0·82	− 13·87	− 1·80
0	0	+ 9·00	0	− 6·62	+ 1·19	+ 1·61
0	0	0	+ 9·00	− 1·19	− 6·62	+ 0·74
− 13·87	+ 0·82	− 6·62	− 1·19	+ 28·00	0	− 5·22
− 0·82	− 13·87	+ 1·19	− 6·62	0	+ 28·00	+ 0·35

	BD 89°, 37	λ Ursæ Min.	α Ursæ Min.
	″	″	″
$d\,[l]\sin\epsilon$	− 0·10	+ 0·11	+ 0·12
$d\,e$	− 0·04	+ 0·04	− 0·12

The corrected positions are

	BD 89°, 37.			λ Ursæ Min.			α Ursæ Min.		
	°	′	″	°	′	″	°	′	″
$90 - \lambda$	− 0	0	7·24	+ 1	6	48·28	+ 3	0	49·29
$90 - \beta$	23	21	34·71	22	30	33·04	23	54	42·88

In order to refer all the stars to the system determined by these co-ordinates I computed $d\,x$ and d D for each day, and, employing the mean where more than one of these stars had been observed, I corrected the individual positions of all the stars accordingly. By this method, undue weight is given to the two fundamental stars and to BD 89°, 37, and the accidental errors of these observations will necessarily affect all the stars observed on the same night. To avoid this, I further solved for each night the differential equations (11) by approximation, neglecting the corrections of the means of the co-ordinates. Two approximations proved to be sufficient.

The sum of the corrections of the original x and D are given in Table I., and the corrections of the co-ordinates as due to them have been compiled in the 9th and 10th columns of Table III.

It may be inferred that the final positions reduce the outstanding errors to a minimum, and that they refer to a system which is demanded by the observations themselves. The residuals are not so small as I was led to expect from the agreement of the settings, and the differential equations clearly show that the Pole-point underwent irregular changes during a night, which I was not able to take into account.

7. Probable Error.

The probable errors of *one* observation of a close Pole-star were found

$$\Delta\,\lambda \cos\beta = \pm\,0''{\cdot}29 \qquad\qquad \Delta\,\beta = \pm\,0''{\cdot}29,$$

and for the fundamental Pole-stars

$$\Delta\,\lambda \cos\beta = \pm\,0''{\cdot}23 \qquad\qquad \Delta\,\beta = \pm\,0''{\cdot}19.$$

The average differences of the observations from the final mean are 0″·30 and 0″·27 for the close Pole-stars, and 0″·25 and 0″·23 for the fundamental Pole-stars.

8. Final Results expressed in Rectangular Co-ordinates.

The connection between the ecliptic and rectangular co-ordinates, with the pole of the heavens as origin, is given by the spherical triangle Pole of ecliptic, Pole of equator at zero epoch, and Star. The formulæ are

$$\sin p_\circ \sin \alpha_\circ = -\sin \beta_\circ \sin \epsilon_\circ + \cos \beta_\circ \cos \epsilon_\circ \sin \lambda_\circ$$
$$\sin p_\circ \cos \alpha_\circ = \cos \beta_\circ \cos \lambda_\circ$$

where $\epsilon_\circ$ is the mean obliquity at the zero epoch.

Let

$$\beta_\circ = 90 - \epsilon_\circ - e_\circ$$
$$\lambda_\circ = 90 - l_\circ$$

thence

$$e_\circ = [\epsilon + e] - \epsilon_\circ \qquad\qquad l_\circ = [l]$$

$$x = \frac{\sin p_\circ \cos \alpha_\circ}{\sin 1''} = \left\{ l''_\circ - \text{A}\,(l) \right\} \sin [\epsilon + e]$$

$$y = \frac{\sin p_\circ \sin \alpha_\circ}{\sin 1''} = \left\{ e''_\circ - \text{A}\,[e_\circ] \right\} - 2 \cos \epsilon_\circ \sin 1'' \sin [\epsilon + e] \left((\tfrac{1}{2}\, l''_\circ - \text{A}\,(\tfrac{1}{2}\, l_\circ) \right)^2$$

where $[\epsilon + e]$ and $[l]$ designate mean values and A (a) is the reduction of arc to sine in seconds, which was tabulated with argument a.

The precessions of x and y, as computed with O. Struve's constant, were obtained from the following formulæ:—

$$d\,x = -\,(6{\cdot}349154 - 10)\,y - (1{\cdot}302183) \cos p.$$
$$d\,y = \quad (6{\cdot}349154 - 10)\,x$$

$$100\, d^2 x = -\,0''{\cdot}00594 \frac{x}{1000} - 0''{\cdot}00014 \frac{y}{1000} + 0''{\cdot}00863 \cos p$$

$$100\, d^2 y = +\,0{\cdot}00014 \frac{x}{1000} - 0{\cdot}00499 \frac{y}{1000} - 0{\cdot}44806 \cos p$$

$$100^2 d^3 x = -\,0{\cdot}00001 \frac{x}{1000} + 0{\cdot}00013 \frac{y}{1000} + 0{\cdot}01190 \cos p$$

$$100^2 d^3 y = -\,0{\cdot}00013 \frac{x}{1000} - 0{\cdot}00001 \frac{y}{1000} - 0{\cdot}00036 \cos p.$$

The results are contained in Table IV.

9. Comparison with Carrington's Series.

The co-ordinates published in the Redhill Catalogue of Circumpolar Stars (p. 63) were converted into rectangular co-ordinates, and then brought forward to the mean equinox of 1887·0. The differences between this series and my own are given in Table IV. The numerical means of the differences are 0″·64 and 0″·81 in x and y respectively. This is in excess of what might be expected from the average errors of a single observation, which in Carrington's series amounts to 0″·74 and 0″·52, and in my own to 0″·30 and 0″·27, and it may probably be traced to proper motions of some of the stars.

I have annexed a comparison of my results for the fundamental stars with the Nautical Almanac, Berlin Jahrbuch, and the Connaissance des Temps, for 1887,

as well as with Dr. Elkin's observations, α Ursæ Minoris and λ Ursæ Minoris having been corrected for the proper motions adopted in the Berlin Jahrbuch. The distance of α and λ Ursæ Minoris as resulting from my meridian observations is $0''\cdot13$ in excess of Dr. Elkin's measurements with the heliometer.

TABLE I.

Date.		Sid. Time.	Nadir.	c + n = x.	Pole-point D.	dx.	dD.
		h	° ′ ″	″	° ′ ″	″	″
1886	Sept. 8	22·5	152 33 23·81	−3·54	299 42 58·58	−0·67	+0·11
1886	Sept. 10	21·5	23·34	−4·02	57·94	−0·04	+0·37
1886	Sept. 15	......	.....	−2·86	59·13	−1·04	−0·64
1886	Sept. 16	21·5	23·33	−3·85	57·61	−0·49	−0·49
1886	Oct. 13	3·2	23·89	−5·24	58·77	−0·69	−0·25
1886	Oct. 14	22·4	23·77	−4·06	59·83	−0·12	−1·82
1886	Oct. 14	2·7	23·01				
1886	Nov. 2	22·8	24·66	−4·92	60·05	−0·60	−0·63
1886	Nov. 2	2 3	24·49				
1886	Nov. 3	22·0	25·25	−4·77	59·61	−0·62	−0·72
1886	Nov. 3	2·5	24·31				
1886	Nov. 4	22·9	23·86	−4·49	58·64	−0·40	−0·02
1886	Nov. 4	4·0	23·89				
1886	Nov. 21	2·0	24·86	−5·15	60·01	+0·01	−0·71
1886	Nov. 22	2·0	23·98	−5·16	58·77	−0·27	−0·49
1886	Nov. 23	5·2	24·04	−5·58	59·00	−0·32	−0·40
1886	Nov. 24	......	.....	−5·28	58·14	−0·28	−0·43
1886	Dec. 4	0·9	24·17	−2·47	58·93	−0·11	−0·39
1886	Dec. 4	3·6	23·79				
1886	Dec. 6	1·1	24·03	−2·70	58·71	+0·20	−0·34
1886	Dec. 6	5·5	24·07				
1886	Dec. 7	22·5	24·00	−2·20	59·21	−0·25	−0·59
1886	Dec. 7	5·5	24·07				
1886	Dec. 10	0·6	23·79	−2·80	58·95	+0·18	−0·45
1886	Dec. 10	2·5	23·97				
1886	Dec. 10	8·2	24·17				
1887	Jan. 10*	1·4	152 34 2·92	−3·10	299 43 37·10	−0·34	+0·27
1887	Jan. 10	4·3	3·02				
1887	Jan. 12	5·4	3·03	−2·58	37·05	−0·56	+0·10
1887	Jan. 13	7·2	3·46	−2·70	36·78	−0·49	+0·22
1887	Jan. 13	9·5	2·48				
1887	Jan. 14	1·2	3·46	−3·31	37·35	−0·04	+0·37
1887	Jan. 14	9·8	3·07				
1887	Jan. 29	2·9	4·02	−3·60	37·89	−0·69	−0·82
1887	Jan. 29	4·0	3·73				
1887	Jan. 31	4·9	3·05	−3·40	36·84	−0·85	+0·06
1887	Jan. 31	6·2	2·58				
1887	Feb. 9	3·1	2·21	−2·91	36·12	−0·59	+0·30
1887	Feb. 9	10·8	2·00				
1887	Mar. 3	6·4	1·38	−4·50	35·51	+0·06	−0·18
1887	Mar. 3	8·7	1·54				
1887	Mar. 5	9·1	0·99	−3·96	36·56	+0·20	−0·54
1887	Mar. 5	11·9	0·10				

* Micrometer supplied with a new system of wires.

TABLE II.

Date.	$T_\circ + \Delta T + m.$	$[a_\circ]$	$d_\circ$	Daily Aberration. B.A.	Daily Aberration. Polar Distance.	$l_1 = 90 - \lambda_1$	$\epsilon + e_1 = 90 - \beta_1$
	h m s	s	° ′ ″	″	″	° ′ ″	° ′ ″
1886 Sept. 8							
B.D. 89°, 36	19 57 19·62	− 0·069	300 4 22·42	− 27·9	0·00	+0 26 29·12	23 8 26·77
89°, 12	20 21 15·61	+ 9·356	299 17 31·40	+ 23·2	−0·02	−0 30 28·36	23 49 37·60
89°, 16	21 54 55·58	− 0·541	299 13 48·66	+ 20·5	0·00	−1 2 19·05	23 42 14·97
89°, 17	22 59 11·48	+ 4·883	299 5 3·31	+ 15·8	−0·01	−1 30 15·00	23 38 56·36
88°, 135	23 24 37·47	−19·841	300 40 19·56	− 10·3	+0·02	+2 21 16·50	23 12 58·46
89°, 38	23 43 49·45	−17·584	300 31 29·99	− 12·2	+0·02	+2 1 18·60	23 18 45·59
89°, 20	0 4 6·43	+ 3·030	298 58 45·31	+ 13·5	0·00	−1 51 5·83	23 27 52·79
B.A.C. 4165	0 24 42·42	+15·677	298 2 54·95	+ 6·0	−0·01	−4 11 55·04	23 24 39·41
1886 Sept. 10							
λ Ursæ Min.	19 36 23·41	+ 1·712	300 45 2·93	− 9·6	0·00	+1 7 20·59	22 30 51·69
B.D. 89°, 36	19 56 44·40	+ 0·087	300 4 22·52	− 27·9	0·00	+0 26 28·36	23 8 25·83
89°, 12	20 23 2·37	+ 9·949	299 17 31·74	+ 23·2	−0·02	−0 30 30·77	23 49 37·51
89°, 37	20 49 39·35	−10·371	299 46 55·99	− 86·8	+0·11	+0 0 24·66	23 21 52·86
1886 Sept. 15							
B.D. 89°, 17	23 1 5·49	+ 5·860	299 5 0·81	+ 15·7	−0·01	−1 30 23·83	23 38 58·23
89°, 19	23 26 14·48	+ 2·323	299 5 44·19	+ 16·1	0·00	−1 31 55·66	23 33 42·81
89°, 18	23 48 30·47	+ 8·235	299 16 36·52	+ 22·4	−0·02	−1 5 41·28	23 31 15·15
89°, 20	0 9 42·46	+ 6·322	298 58 44·97	+ 13·5	−0·01	−1 51 13·35	23 27 52·26
B.A.C. 4165	0 25 29·45	+16·730	298 2 53·71	+ 6·0	−0·01	−4 12 2·22	23 24 39·60
B.D. 89°, 26	0 46 13·45	− 2·746	299 29 15·68	+ 43·4	+0·01	−0 33 23·81	23 23 30·16
89°, 21	1 4 31·44	+12·066	299 1 16·13	+ 14·2	−0·02	−1 43 53·55	23 19 45·91
α Ursæ Min.	1 25 19·43	− 7·156	301 0 54·89	− 7·7	+0·01	+3 1 13·88	23 55 2·33
1886 Sept. 16							
λ Ursæ Min.	19 42 55·11	− 3·886	300 45 0·81	− 9·6	0·00	+1 7 15·33	22 30 51·97
B.D. 89°, 36	20 13 6·10	− 4·753	300 4 16·61	− 27·9	+0·01	+0 26 19·89	23 8 28·08
89°, 12	20 27 35·10	+11·468	299 17 33·47	+ 23·1	−0·03	−0 30 35·14	23 49 38·72
1886 Oct. 13							
B.A.C. 4165	0 19 58·72	+ 9·343	298 2 37·09	+ 6·0	0·00	−4 12 28·01	23 24 38·97
B.D. 89°, 21	0 47 58·66	+ 2·883	299 0 56·03	+ 14·2	0·00	−1 44 18·38	23 19 45·69
89°, 23	1 5 19·62	+ 2·864	299 5 31·78	+ 15·9	0·00	−1 31 28·15	23 17 48·83
α Ursæ Min.	1 18 31·59	+ 0·279	....	− 7·7	0·00	+3 0 47·25	23 55 0·33
B.D. 89°, 2	1 40 50·54	− 2·021	300 33 4·55	− 11·9	0·00	+1 53 4·75	23 48 27·78
89°, 25	2 14 8·47	+ 3·751	299 15 52·52	+ 22·0	−0·01	−0 58 52·00	23 13 14·67
89°, 4	2 37 5·42	+ 0·371	300 10 43·00	− 21·6	0·00	+0 53 16·58	23 44 50·33
1886 Oct. 14							
B.D. 88°, 126	21 15 54·54	− 1·679	300 37 7·60	− 11·1	0·00	+1 44 5·80	22 51 40·76
89°, 16	21 52 19·51	− 1·980	299 13 36·94	+ 20·3	0·00	−1 2 58·07	23 42 15·15
89°, 1	0 41 43·35	− 1·947	299 56 6·71	− 45·5	0·00	+0 32 41·23	23 28 51·09
89°, 23	1 2 46·33	+ 1·474	299 5 29·57	+ 15·9	0·00	−1 31 34·75	23 17 47·53
89°, 2	1 40 47·29	− 2·056	300 33 3·28	− 11·9	0·00	+1 52 59·80	23 48 26·49
1886 Nov. 2							
B.D. 89°, 37	21 41 7·30	−13·488	299 45 51·74	− 61·7	+0·15	−0 0 25·08	23 21 50·20
89°, 16	22 17 35·07	+ 7·884	299 13 38·59	+ 20·2	−0·02	−1 3 13·61	23 42 12·61
89°, 17	22 59 40·11	+ 4·711	299 4 45·52	+ 15·6	−0·01	−1 31 8·20	23 38 52·55
88°, 135	23 21 37·12	−17·772	300 40 6·06	− 10·4	+0·02	+2 20 24·65	23 12 54·18
89°, 38	23 40 41·14	−15·720	300 31 16·91	− 12·2	+0·02	+2 0 30·08	23 18 40·97
89°, 20	0 6 36·16	+ 4·398	298 58 26·76	+ 13·4	0·00	−1 51 58·01	23 27 48·65
B.A.C. 4165	0 21 18·18	+11·019	298 2 32·66	+ 5·9	−0·01	−4 12 45·17	23 24 36·28
α Ursæ Min.	1 28 33·23	−10·314	301 0 34·83	− 7·7	+0·01	+3 0 29·37	23 54 56·63

TABLE II.—Continued.

Date.	$T_{\circ} + \Delta T + m.$	$[a_{\circ}]$	$d_{\circ}$	Daily Aberration. R.A.	Daily Aberration. Polar Distance.	$l_1 -$ $90 - \lambda_1$	$\epsilon + e_1 =$ $90 - \beta_1$
	h m s	R	° ′ ″	″	″	° ′ ″	° ′ ″
1886 Nov. 3							
B.D. 89°, 16	21 59 44·25	+ 0·759	299 13 31·99	+ 20·3	0·00	−1 3 13·48	23 42 12·82
89°, 37	22 27 42·25	−15·020	299 44 54·16	− 41·1	+0·16	−0 0 25·45	23 21 50·04
88°, 135	23 10 57·25	− 9·546	300 40 17·68	− 10·4	+0·01	+2 20 23·24	23 12 53·34
89°, 19	23 30 19·25	+ 4·129	299 5 29·73	+ 15·9	−0·01	−1 32 37·96	23 33 38·07
89°, 17	23 42 59·25	+26·974	299 5 42·67	+ 15·2	−0·04	−1 31 6·70	23 38 52·46
89°, 20	0 6 11·25	+ 4·140	298 58 25·78	+ 13·4	0·00	−1 51 59·02	23 27 48·68
B.A.C. 4165	0 21 23·25	+11·088	298 2 31·38	+ 5·9	−0·01	−4 12 47·33	23 24 35·52
B.D. 89°, 1	0 49 6·25	− 3·287	299 55 59·63	− 45·6	+0·02	+0 32 29·48	23 28 48·45
α Ursæ Min.	1 17 38·25	+ 1·182	301 0 38·87	− 7·7	0·00	+3 0 29·45	23 54 56·79
1886 Nov. 4							
B.D. 89°, 37	21 5 21·53	−11·966	299 46 30·26	− 75·7	+0·13	−0 0 23·87	23 21 49·58
88°, 126	21 29 27·58	−11·979	300 36 56·33	− 11·0	+0·01	+1 43 50·20	22 51 35·91
89°, 16	21 56 47·64	− 0·478	299 13 32·32	+ 20·3	0·00	−1 3 11·93	23 42 11·42
89°, 17	23 6·56·80	+ 8·431	299 4 47·98	+ 15·6	−0·01	−1 31 8·64	23 38 51·97
89°, 19	23 25 28·84	+ 1·629	299 5 27·21	+ 15·9	0·00	−1 32 39·34	23 33 37·77
89°, 18	23 49 40·90	+ 8·432	299 16 18·20	+ 22·2	−0·02	−1 6 28·86	23 31 9·98
88°, 1	0 11 3·94	− 1·782	300 40 31·87	− 10·4	0·00	+2 24 14·96	23 30 20·13
B.A.C. 4165	0 27 16·98	+19·093	298 2 37·54	+ 5·9	−0·01	−4 12 47·84	23 24 35·23
B.D. 89°, 1	0 49 1·03	− 3·310	299 55 59·32	− 45·6	+0·02	+0 32 31·28	23 28 48·15
89°, 23	1 8 20·08	+ 4·292	299 5 23·54	+ 15·9	−0·01	−1 31 49·15	23 17 44·92
α Ursæ Min.	1 20 29·10	− 1·856	301 0 37·36	− 7·7	0·00	+3 0 28·93	23 54 56·23
B.D. 89°, 37	1 44 25·16	−14·278	299 40 30·06	+ 53·0	+0·15	−0 0 25·74	23 21 49·19
1886 Nov. 21							
B.D. 89°, 38	23 27 5·82	− 7·077	300 31 23·40	− 12·3	0·00	+2 0 11·73	23 18 35·82
89°, 18	23 46 21·83	+ 6·998	299 16 12·12	+ 22·2	−0·01	−1 6 43·19	23 31 5·06
88°, 1	0 7 21·84	+ 0·915	300 40 27·86	− 10·4	0·00	+2 24 1·88	23 30 15·11
89°, 1	0 30 11·85	− 0·111	299 55 57·18	− 46·2	0·00	+0 32 16·24	23 28 43·83
89°, 21	0 52 39·86	+ 5·258	299 0 45·89	+ 14·2	−0·01	−1 44 47·49	23 19 36·98
89°, 23	1 6 27·87	+ 3·204	299 5 18·36	+ 15·8	0·00	−1 32 1·66	23 17 40·00
α Ursæ Min.	1 23 21·87	− 4·987	301 0 32·05	− 7·7	0·00	+3 0 18·32	23 54 51·36
B.D. 89°, 37	2 16 5·90	−13·114	299 39 47·45	+ 66·5	+0·14	−0 0 38·03	23 21 44·02
1886 Nov. 22							
B.D. 89°, 37	23 8 15·70	−16·141	299 43 54·73	− 19·4	+0·17	−0 0 41·20	23 21 44·65
89°, 18	23 46 37·84	+ 7·114	299 16 11·78	+ 22·2	−0·01	−1 6 41·22	23 31 5·33
B.A.C. 4165	0 12 40·94	− 1·000	298 2 21·83	+ 5·9	0·00	−4 13 0·32	23 24 30·85
B.D. 89°, 1	0 44 53·06	− 2·714	299 55 53·97	− 46·0	+0·01	+0 32 16·75	23 28 43·37
B.D. 89°, 37	1 6 41·14	−15·471	299 41 12·10	+ 36·9	+0·16	−0 0 37·97	23 21 44·23
α Ursæ Min.	1 24 58·21	− 6·659	301 0 29·45	− 7·7	0·00	+3 0 16·80	23 54 51·43
1886 Nov. 23							
B.D. 89°, 37	23 10 17·27	−16·161	299 43 53·21	− 18·7	+0·17	−0 0 38·80	23 21 44·06
89°, 18	23 51 28·29	+ 8·855	299 16 14·66	+ 22·1	−0·02	−1 6 42·60	23 31 4·50
89°, 20	0 6 53·30	+ 4·330	298 58 19·90	+ 13·4	−0·01	−1 52 12·85	23 27 43·09
89°, 1	0 43 18·31	− 2·433	299 55 54·41	− 46·0	+0·01	+0 32 16·51	23 28 42·96
α Ursæ Min.	1 19 28·32	− 0·877	301 0 31·52	− 7·7	0·00	+3 0 17·17	23 54 50·93
B.D. 89°, 2	1 37 24·33	+ 0·230	300 32 49·49	− 12·0	0·00	+1 52 32·20	23 48 18·49
89°, 37	3 37 25·38	− 8·776	299 38 28·15	+ 93·4	+0·09	−0 0 39·36	23 21 43·63
1886 Nov. 24							
B.D. 89°, 37	0 24 50·63	−16·225	299 42 7·57	+ 17·4	+0·17	−0 0 39·93	23 21 43·73
α Ursæ Min.	1 12 47·71	+ 6·131	301 0 29·05	− 7·7	0·00	+3 0 16·60	23 54 50·72
1886 Dec. 4							
α Ursæ Min.	1 16 57·73	+ 1·487	....	− 7·7	0·00	+3 0 11·90	23 54 47·56
B.D. 89°, 37	1 45 52·69	−14·545	299 40 17·77	+ 54·4	+0·15	−0 0 45·52	23 21 40·32
89°, 25	2 3 22·67	− 0·507	299 15 32·85	+ 21·8	0·00	−0 59 28·95	23 13 2·20

TABLE II.—Continued.

Date.	$T_{\circ}+\Delta T+m.$	$[a_{\circ}]$	$d_{\circ}$	Daily Aberration. R.A.	Daily Aberration. Polar Distance.	$l_1 = 90-\lambda_1$	$\epsilon + e_1 = 90-\beta_1$
	h m s	R	° ′ ″	″	″	° ′ ″	° ′ ″
1886 Dec. 6							
α Ursæ Min.	1 21 46·70	− 3·586	301 0 27·07	− 7·7	0·00	+3 0 11·30	23 54 47·00
B.D. 89°, 37	1 49 14·66	−14·451	299 40 13·02	+ 55·6	+0·15	−0 0 45·44	23 21 39·40
89°, 25	2 10 46·63	+ 2·233	299 15 32·67	+ 21·8	0·00	−0 59 29·14	23 13 1·51
89°, 5	2 28 39·60	− 0·994	300 20 28·32	− 15·9	0·00	+1 14 30·51	23 49 45·01
89°, 3	2 48 8·57	− 0·228	300 4 48·62	− 27·4	0·00	+0 40 33·09	23 41 42·61
89°, 7	3 56 17·46	− 3·022	300 32 11·01	− 12·1	0·00	+1 3 57·04	24 9 0·11
89°, 29	4 16 4·42	+ 7·153	299 24 6·59	+ 31·2	−0·02	−0 26 8·13	23 11 9·26
89°, 28	4 35 54·39	+ 5·521	298 58 28·92	+ 13·4	−0·01	−0 45 24·38	22 46 17·12
89°, 37	5 3 51·35	− 3·065	299 37 38·25	+107·9	+0·03	−0 0 44·33	23 21 39·70
1886 Dec. 7							
B.D. 89°, 37	22 43 16·65	−16·135	299 44 29·05	− 30·2	+0·17	−0 0 45·81	23 21 39·91
88°, 135	23 3 42·63	− 4·592	300 40 15·56	− 10·4	0·00	+2 20 2·83	23 12 43·76
89°, 38	23 16 46·62	− 0·713	300 31 25·97	− 12·3	0·00	+2 0 9·57	23 18 30·50
89°, 19	23 35 44·60	+ 6·231	299 5 24·33	+ 15·9	−0·01	−1 32 59·62	23 33 28·61
89°, 1	0 36 29·54	− 1·589	299 55 51·50	− 46·4	+0·01	+0 32 7·19	23 28 38·74
89°, 21	0 58 32·52	+ 8·296	299 0 43·37	+ 14·1	−0·01	−1 44 57·68	23 19 32·32
89°, 23	1 7 49·51	+ 3·579	299 5 13·16	+ 15·8	−0·01	−1 32 11·09	23 17 35·35
α Ursæ Min.	1 28 50·49	−11·047	301 0 22·93	− 7·7	+0·01	+3 0 11·05	23 54 46·70
B.D. 89°, 2	1 50 53·46	− 9·215	300 32 40·16	− 12·0	+0·01	+1 52 26·54	23 48 13·69
89°, 25	2 8 30·45	+ 1·382	299 15 32·06	+ 21·8	0·00	−0 59 31·37	23 13 1·39
89°, 4	2 51 15·40	− 5·062	300 10 20·87	− 21·8	+0·01	+0 52 39·74	23 44 36·88
89°, 29	4 11 28·32	+ 5·985	299 24 3·75	+ 31·3	−0·02	−0 26 10·78	23 11 8·80
89°, 37	4 47 4·28	− 4·295	299 37 42·65	+105·7	+0·04	−0 0 45·52	23 21 38·27
1886 Dec. 10							
B.D. 89°, 37	1 9 17·07	−15·769	299 41 4·09	+ 38·3	+0·16	−0 0 45·95	23 21 38·11
α Ursæ Min.	1 27 18·04	− 9·443	301 0 23·59	− 7·7	+0·01	+3 0 11·07	23 54 45·61
B.D. 89°, 1	1 54 4·00	−14·823	299 54 57·00	− 43·0	+0·06	+0 32 9·92	23 28 37·77
89°, 7	3 59 40·81	− 5·285	300 32 8·93	− 12·1	+0·01	+1 3 55·95	24 8 58·95
89°, 29	4 22 40·77	+ 8·850	299 24 9·57	+ 31·1	−0·03	−0 26 10·01	23 11 7·66
89°, 37	7 8 36·52	+ 6·048	299 37 50·85	+102·5	−0·06	−0 0 44·53	23 21 37·59
λ Ursæ Min.	7 35 7·48	− 0·762	298 40 52·28	+ 9·6	0·00	+1 6 7·54	22 30 35·84
1887 Jan. 10							
α Ursæ Min.	1 27 6·70	− 9·681	301 0 56·57	− 7·7	+0·01	+3 0 8·04	23 54 35·67
B.D. 89°, 37	1 51 58·66	−14·781	299 40 42·65	+ 55·0	+0·15	−0 0 47·14	23 21 29·18
89°, 3	3 29 53·61	−12·759	300 4 53·78	− 27·1	+0·03	+0 40 27·52	23 41 31·55
1887 Jan. 12							
B.D. 89°, 37	3 44 8·43	− 8·772	299 38 45·19	+ 91·2	+0·09	−0 0 51·64	23 21 27·62
7	4 11 2·39	−13·033	300 32 29·32	− 12·1	+0·02	+1 3 53·29	24 8 49·90
29	4 31 7·36	+10·883	299 24 44·38	+ 30·6	−0·03	−0 26 15·16	23 10 57·74
28	4 50 8·34	+14·015	298 59 7·30	+ 13·3	−0·02	−0 45 29·00	22 46 5·89
1887 Jan. 13							
B.D. 89°, 37	7 31 56·06	+ 7·965	299 38 32·98	+ 92·6	−0·08	−0 0 50·78	23 21 27·31
λ Ursæ Min.	7 46 2·04	+ 8·741	298 41 23·97	+ 9·6	−0·01	+1 6 5·03	22 30 24·92
B.D. 89°, 36	8 7 7·01	+ 5·313	299 22 8·52	+ 27·7	−0·01	+0 25 11·44	23 8 0·27
89°, 12	8 34 28·98	−11·459	300 8 50·25	− 23·2	+0·03	−0 31 45·95	23 49 11·33
89°, 37	8 58 10·95	+13·206	299 39 48·77	+ 71·0	−0·13	−0 0 49·30	23 21 26·70

TABLE II.—CONTINUED.

Date.	$T_{\circ}+\Delta T+m.$	$[a_{\circ}]$	$d_{\circ}$	Daily Aberration. R.A.	Daily Aberration. Polar Distance.	$l_1 =$ $90-\lambda_1$	$\epsilon+e_1=$ $90-\beta_1$
	h m s	R	° ′ ″	″	″	° ′ ″	° ′ ″
1887 Jan. 14							
α Ursæ Min.	1 17 7·94	+ 0·710	301 1 0·64	− 7·7	0·00	+3 0 9·60	23 54 34·07
B.D. 89°, 37	1 35 31·92	−15·505	299 41 1·87	+ 48·2	+0·15	−0 0 49·65	23 21 26·30
89°, 26	2 2 9·90	+10·704	299 29 43·37	+ 40·6	−0·04	−0 34 29·71	23 23 0·67
89°, 5	2 24 12·88	+ 0·826	300 20 58·22	− 16·0	0·00	+1 14 26·97	23 49 32·66
89°, 4	3 10 35·84	−12·549	300 10 34·78	− 21·7	+0·03	+0 52 37·29	23 44 24·73
89°, 3	3 27 53·82	−12·216	300 4 55·39	− 27·1	+0·03	+0 40 28·18	23 41 30·57
89°, 29	4 19 50·77	+ 7·966	299 24 36·65	+ 30·9	−0·02	−0 26 11·94	23 10 58·10
89°, 28	4 46 3·75	+11·567	298 59 3·34	+ 13·3	−0·01	−0 45 28·87	22 46 5·86
89°, 35	5 1 38·74	+ 0·331	299 31 28·40	+ 49·2	0·00	−0 7 52·12	23 15 21·35
89°, 31	5 21 28·72	− 2·848	299 23 9·36	+ 29·2	+0·01	−0 6 16·61	23 6 45·54
89°, 10	6 38 18·65	− 4·663	300 37 59·67	− 11·0	0·00	−0 18 13·02	24 20 59·37
89°, 11	6 51 16·63	− 6·438	300 40 59·49	− 10·4	+0·01	−0 25 47·83	24 23 32·75
89°, 32	7 11 43·62	+15·094	299 0 11·27	+ 13·6	−0·02	+0 22 47·82	22 44 19·41
89°, 34	7 19 47·61	+ 9·346	299 2 34·90	+ 14·7	+0·01	+0 28 25·28	22 48 9·36
λ Ursæ Min.	7 40 40·59	+ 4·239	298 41 21·51	+ 9·6	0·00	+1 6 5·55	22 30 24·88
B.D. 89°, 36	8 4 24·57	+ 4·547	299 22 7·03	+ 27·7	−0·01	+0 25 13·07	23 7 59·55
89°, 12	8 24 2·55	− 7·867	300 8 59·78	− 23·3	+0·02	−0 31 45·43	23 49 11·90
89°, 37	9 14 8·50	+14·015	299 40 7·45	+ 65·5	−0·14	−0 0 48·79	23 21 26·99
1887 Jan. 29							
B.D. 89°, 4	3 7 35·68	−11·649	300 10 35·02	− 21·7	+0·02	+0 52 39·23	23 44 19·74
89°, 37	3 31 43·65	− 9·784	299 38 52·78	+ 86·4	+0·10	−0 0 46·07	23 21 22·66
1887 Jan. 31							
B.D. 89°, 37	5 10 57·70	− 2·707	299 37 56·70	+102·7	+0·03	−0 0 44·68	23 21 21·96
89°, 35	5 30 49·68	+ 5·021	299 31 28·26	+ 48·4	−0·02	−0 7 48·56	23 15 15·54
1887 Feb. 9							
B.D. 89°, 37	3 23 18·17	−10·555	299 38 59·28	+ 83·2	+0·10	−0 0 36·98	23 21 21·27
89°, 5	3 47 51·16	−41·073	300 18 30·93	− 15·0	+0·06	+1 14 38·14	23 49 26·96
89°, 3	4 5 13·15	−23·187	300 3 54·51	− 25·9	+0·06	+0 40 39·14	23 41 24·28
89°, 7	4 25 4·14	−22·631	300 32 1·13	− 12·1	+0·03	+1 4 2·14	24 8 42·24
89°, 28	4 48 10·13	+12·499	298 58 58·28	+ 13·3	−0·02	−0 45 18·58	22 45 58·79
89°, 35	5 35 43·11	+ 5·699	299 31 27·40	+ 48·2	−0·03	−0 7 44·25	23 15 13·28
89°, 31	5 53 44·11	+ 5·897	299 23 4·55	+ 28·9	−0·02	−0 6 7·54	23 6 38·46
89°, 30	6 11 52·10	+11·901	298 54 0·08	+ 12·0	−0·01	−0 3 7·43	22 37 22·67
89°, 9	6 27 1·09	− 7·691	300 14 49·52	− 19·0	+0·01	−0 2 52·20	23 58 24·65
89°, 10	6 43 32·08	− 8·646	300 37 47·25	− 11·0	+0·01	−0 18 4·36	24 20 52·22
89°, 11	6 56 56·08	−11·003	300 40 46·25	− 10·4	+0·01	−0 25 36·89	24 23 26·36
89°, 36	8 26 9·04	+10·884	299 22 11·03	+ 27·2	−0·03	+0 25 24·51	23 7 52·80
89°, 12	8 47 57·03	−15·935	300 8 25·51	− 23·1	+0·04	−0 31 35·92	23 49 4·37
89°, 37	9 16 30·02	+14·273	299 40 1·87	+ 64·1	−0·14	−0 0 40·76	23 21 20·36
1887 Mar. 3							
B.D. 89°, 37	6 46 17·94	+ 4·334	299 37 54·11	+ 99·7	−0·04	−0 0 22·22	23 21 16·43
89°, 32	7 5 52·92	+11·265	298 59 47·95	+ 13·6	−0·01	+0 23 20·05	22 44 8·39
89°, 34	7 17 38·91	+ 7·907	299 2 55·09	+ 14·6	−0·01	+0 28 53·30	22 47 58·48
λ Ursæ Min.	7 40 41·88	+ 4·053	298 41 5·01	+ 9·6	0·00	+1 6 34·43	22 30 14·81
B.D. 89°, 36	7 58 12·86	+ 2·604	299 21 49·03	+ 27·4	−0·01	+0 25 39·55	23 7 49·82

TABLE II.—CONTINUED.

Date.	$T_\circ + \Delta T + m$.	$[a_\circ]$	$d_\circ$	Daily Aberration.		$l_1 = 90 - \lambda_1$	$\epsilon + e_1 = 90 - \beta_1$
				R.A.	Polar Distance.		
	h m s	R	° ′ ″	″	″	° ′ ″	′ ″
1887 Mar. 5							
B.D. 89°, 37	5 46 39·67	− 0·423	299 37 44·60	+101·8	+0·01	−0 0 20·69	23 21 14·66
89°, 35	6 4 30·65	+10·042	299 31 41·87	+ 46·9	−0·04	−0 7 24·40	23 15 8·64
89°, 31	6 23 22·62	+13·763	299 23 23·41	+ 28·3	−0·04	−0 5 47·56	23 6 33·41
89°, 10	6 42 52·60	− 8·437	300 37 43·98	− 11·0	+0·01	−0 17 47·37	24 20 49·59
89°, 11	6 58 27·57	−12·437	300 40 39·77	− 10·4	+0·01	−0 25 19·84	24 23 22·87
89°, 33	7 22 8·54	+21·017	299 14 51·39	+ 19·7	−0·04	+0 9 48·06	22 57 50·75
λ Ursæ Min.	7 37 5·52	+ 0·955	298 41 4·93	+ 9·6	0·00	+1 6 35·42	22 30 14·52
B.D. 89°, 12	7 59 7·49	+ 0·573	300 8 50·73	− 23·7	0·00	−0 31 17·30	23 49 0·67
89°, 37	8 23 10·45	+11·184	299 38 57·16	+ 81·7	−0·10	−0 0 20·70	23 21 16·36
89°, 16	10 20 39·28	− 7·057	300 12 37·95	− 20·5	+0·01	−1 3 6·18	23 41 38·28
89°, 17	10 45 37·25	+ 4·651	300 21 39·06	− 15·7	−0·01	−1 31 3·82	23 38 20·10
88°, 135	11 12 33·21	+12·924	298 46 9·80	+ 10·4	−0·01	+2 20 34·61	23 12 20·69

TABLE III.

B.D.	Date.	Aberration.		Reduction to 1887·0.		Corr. for Change of Nadir.		Corr. for dx and dD.		1887·0.	
		$l_2 - l_1$	$e_2 - e_1$	$l - l_2$	$e - e_2$	Δ .	Δ *e.*	Δ *l.*	Δ *e.*	$90 - \lambda_0$	$90 - \beta_0$
		″	″	″	″	″	″	″	″	° ′ ″	° ′ ″
89°, 1	Oct. 14	+19·30	−17·38	−19·48	− 0·10	−0·25	−0·02	+4·51	+0·21	+0 32 45·3	23 28 33·8
	Nov. 3	+34·35	−13·94	−16·97	− 0·08	−0·24	−0·02	+2·10	−0·46	48·7	33·9
	„ 4	+35·01	−13·72	−16·87	− 0·07	......	......	+0·26	−0·39	49·7	34·0
	„ 21	+44·49	− 9·37	−14·78	− 0·05	......	......	+1·76	+0·10	47·7	34·5
	„ 22	+44·94	− 9·07	−14·70	− 0·05	......	......	+1·33	−0·17	48·3	34·1
	„ 23	+45·37	− 8·78	−14·60	− 0·05	......	......	+1·13	−0·23	48·4	33·9
	Dec. 7	+49·87	− 4·46	−12·45	− 0·03	......	......	+1·57	−0·16	46·2	34·1
	„ 10	+50·46	− 3·47	−11·93	− 0·03	−0·07	−0·01	+0·77	+0·38	49·1	34·6
89°, 2	Oct. 13	+19·38	−17·29	−19·53	− 0·10	......	......	+1·30	−0·51	+1 53 5·9	23 48 9·9
	„ 14	+20·18	−17·17	−19·48	− 0·10	+0·18	+0·04	+4·22	+0·67	4·9	9·9
	Nov. 23	+45·34	− 8·36	−14·59	− 0·05	......	......	+1·24	−0·13	4·2	9·9
	Dec. 7	+49·50	− 4·01	−12·44	− 0·03	−0·02	......	+1·61	+0·05	5·2	9·7
89°, 3	Dec. 6	+49·27	− 4·70	−12·54	− 0·03	−0·04	−0·01	+0·30	+0·38	+0 41 10·1	23 41 38·2
	Jan. 10	+47·51	+ 6·72	− 6·38	− 0·01	−0·10	−0·06	+0·26	−0·42	8·8	37·8
	„ 14	+46·10	+ 7·94	− 5·86	+ 0·02	+0·14	+0·07	−0·49	−0·31	8·1	38·3
	Feb. 9	+31·73	+14·65	− 2·18	+ 0·05	−0·10	−0·08	+0·94	−0·54	9·5	38·4
89°, 4	Oct. 13	+18·63	−17·42	−19·53	− 0·10	......	......	+1·56	−0·37	+0 53 17·2	23 44 32·4
	Dec. 7	+49·43	− 4·32	−12·43	− 0·03	−0·04	−0·01	+1·50	+0·22	18·2	32·7
	Jan. 14	+45·94	+ 8·00	− 5·86	+ 0·02	+0·13	+0·06	−0·55	−0·30	17·0	32·5
	„ 29	+38·64	+12·16	− 3·78	+ 0·04	+0·18	+0·08	+2·65	+0·13	16·9	32·2
89°, 5	Dec. 6	+49·13	− 4·52	−12·54	− 0·03	−0·04	−0·01	+0·38	+0·36	+1 15 7·4	23 49 40·8
	Jan. 14	+45·66	+ 8·09	− 5·86	+ 0·02	+0·10	+0·03	−0·69	−0·25	6·2	40·6
	Feb. 9	+31·17	+14·75	− 2·18	+ 0·05	−0·13	−0·08	+0·84	−0·57	7·8	41·1
89°, 7	Dec. 6	+48·49	− 4·55	−12·54	− 0·03	−0·04	−0·02	......	+0·39	+1 4 33·0	24 8 55·9
	„ 10	+49·23	− 3·26	−11·91	− 0·03	−0·17	−0·12	+0·17	+0·48	33·3	56·0
	Jan. 12	+45·86	+ 7·44	− 6·04	+ 0·02	......	......	+1·14	−0·35	34·2	57·0
	Feb. 9	+30·88	+14·68	− 2·18	+ 0·05	−0·08	−0·08	+1·06	−0·52	31·8	56·4
89°, 9	Feb. 9	+31·80	+14·49	− 2·17	+ 0·05	+0·01	−0·03	+1·55	−0·23	−0 2 21·0	23 58 38·9
89°, 10	Jan. 14	+45·23	+ 7·65	− 5·85	+ 0·02	−0·09	+0·24	+0·26	−0·36	−0 17 33·5	24 21 6·9
	Feb. 9	+31·50	+14·40	− 2·17	+ 0·05	+0·01	−0·02	+1·59	−0·18	33·4	6·5
	Mar. 5	+13·12	+17·97	+ 0·12	+ 0·08	......	......	−0·71	+0·50	34·8	8·1
89°, 11	Jan. 14	+45·20	+ 7·61	− 5·85	+ 0·02	−0·13	+0·25	+0·30	−0·35	−0 25 8·3	24 23 40·3
	Feb. 9	+31·53	+14·37	− 2·17	+ 0·05	+0·01	−0·02	+1·63	−0·14	5·9	40·6
	Mar. 5	+13·19	+17·96	+ 0·12	+ 0·08	......	......	−0·82	+0·47	7·3	41·4
89°, 12	Sept. 8	−12·59	−18·12	−22·08	− 0·15	......	......	−1·54	−0·30	−0 31 4·6	23 49 19·0
	„ 10	−10·92	−18·27	−21·91	− 0·14	......	......	−0·61	+0·28	4·2	19·4
	„ 16	− 5·84	−18·58	−21·75	− 0·13	......	......	−0·24	−0·68	3·0	19·3
	Jan. 13	+46·59	+ 7·34	− 5·91	+ 0·02	−0·52	+0·27	+1·29	+0·14	4·5	19·1
	„ 14	+46·23	+ 7·64	− 5·84	+ 0·02	−0·46	+0·25	+0·62	−0·28	4·9	19·5
	Feb. 9	+32·24	+14·43	− 2·16	+ 0·05	−0·05	+0·02	+1·60	+0·18	4·3	19·1
	Mar. 5	+13·54	+18·03	+ 0·13	+ 0·08	......	......	−1·10	+0·37	4·7	19·2
89°, 16	Sept. 8	−13·05	−18·10	−22·08	− 0·15	......	......	−1·11	−0·51	−1 2 55·3	23 41 56·2
	Oct. 14	+17·72	−17·56	−19·49	− 0·10	−1·25	+0·34	+3·71	−1·06	57·4	56·8
	Nov. 2	+32·22	−14·49	−17·12	− 0·08	−0·33	+0·07	+0·78	−0·81	58·1	57·3
	„ 3	+32·90	−14·29	−16·99	− 0·07	−1·47	+0·35	+0·80	−0·90	58·2	57·9
	„ 4	+33·57	−14·08	−16·88	− 0·07	......	......	−0·47	−0·35	55·7	56·9
	Mar. 5	+13·97	+18·01	+ 0·15	+ 0·08	−1·08	+0·21	−1·44	+0·05	54·6	56·6

TABLE III.—CONTINUED.

B.D.	Date.	Aberration.		Reduction to 1887·0.		Corr. for Change of Nadir.		Corr. for dx and dD.		1887·0.	
		$l_2 - l_1$	$e_2 - e_1$	$l -$	$e - e_2$	Δ .	$\Delta\, e.$	$\Delta\, l.$	$\Delta\, e.$	$90 - \lambda_0$	$90 - \beta_0$
		″	″	″	″	″	″	″	″	° ′ ″	° ′ ″
89°, 17	Sept. 8	−13·45	−18·07	−22·07	− 0·15	......	......	−0·71	−0·61	−1 30 51·2	23 38 37·5
	,, 15	− 7·53	−18·53	−21·78	− 0·14	......	......	+0·90	−1·17	52·2	38·4
	Nov. 2	+31·99	−14·59	−17·12	− 0·08	−0·29	+0·03	+1·14	−0·75	52·5	37·2
	,, 3	+32·71	−14·38	−16·98	− 0·07	−0·82	+0·03	+1·69	−0·67	50·1	37·4
	,, 4	+33·36	−14·17	−16·88	− 0·07	......	......	−0·18	−0·39	52·3	37·3
	Mar. 5	+14·39	+17·98	+ 0·15	+ 0·08	−1·43	+0·21	−1·44	−0·02	52·1	38·3
89°, 18	Sept. 15	− 7·18	−18·57	−21·77	− 0·14	......	......	+1·47	−1·07	−1 6 8·8	23 30 55·4
	Nov. 4	+33·83	−14·09	−16·88	− 0·07	......	......	......	−0·40	11·9	55·4
	,, 21	+43·65	− 9·84	−14·79	− 0·05	......	......	+1·78	−0·03	12·5	55·1
	,, 22	+44·11	− 9·55	−14·71	− 0·05	......	......	+1·18	−0·30	10·6	55·4
	,, 23	+44·57	− 9·26	−14·60	− 0·05	......	......	+0·97	−0·34	11·7	54·9
89°, 19	Sept. 15	− 7·57	−18·54	−21·77	− 0·14	......	......	+1·19	−1·13	−1 32 23·8	23 33 23·0
	Nov. 3	+32·80	−14·39	−16·99	− 0·07	−0·93	+0·06	+1·58	−0·70	21·5	23·0
	,, 4	+33·47	−14·18	−16·88	− 0·07	......	......	−0·10	−0·40	22·8	23·1
	Dec. 7	+49·22	− 5·14	−12·45	− 0·03	+0·02	......	+1·40	−0·31	21·4	23·1
89°, 20	Sept. 8	−13·81	−18·07	−22·07	− 0·15	......	......	−0·25	−0·67	−1 51 42·0	23 27 33·9
	,, 15	− 7·85	−18·54	−21·78	− 0·14	......	......	+1·70	−1·01	41·3	32·6
	Nov. 2	+32·02	−14·67	−17·11	− 0·08	−0·15	......	+1·62	−0·58	41·6	33·3
	,, 3	+32·72	−14·47	−16·98	− 0·07	−0·63	......	+1·85	−0·60	42·1	33·5
	,, 23	+44·33	− 9·48	−14·60	− 0·05	......	......	+1·02	−0·31	42·1	33·3
89°, 21	Sept. 15	− 7·75	−18·57	−21·78	− 0·14	......	......	+2·27	−0·82	−1 44 20·8	23 19 26·4
	Oct. 13	+16·66	−17·78	−19·54	− 0·10	......	......	+0·97	−0·63	20·3	27·2
	Nov. 21	+43·70	−10·01	−14·78	− 0·05	......	......	+1·73	+0·17	16·8	27·1
	Dec. 7	+49·65	− 5·19	−12·45	− 0·03	......	......	+1·59	−0·09	18·9	27·0
89°, 23	Oct. 13	+16·88	−17·76	−19·54	− 0·10	......	......	+1·08	−0·59	−1 31 29·7	23 17 30·4
	,, 14	+17·72	−17·65	−19·49	− 0·10	−0·12	−0·01	+4·45	+0·37	32·2	30·1
	Nov. 4	+33·89	−14·19	−16·87	− 0·07	......	......	+0·34	−0·37	31·8	30·3
	,, 21	+43·87	− 9·95	−14·79	− 0·05	......	......	+1·70	+0·22	30·9	30·2
	Dec. 7	+49·77	− 5·12	−12·45	− 0·03	.....	......	+1·60	−0·07	32·2	30·1
89°, 25	Oct. 13	+17·43	−17·70	−19·54	− 0·10	......	......	+1·47	−0·43	−0 58 52·6	23 12 56·4
	Dec. 4	+49·27	− 5·89	−12·72	− 0·04	+0·24	+0·05	+0·98	+0·11	51·2	56·4
	,, 6	+49·82	− 5·26	−12·55	− 0·03	−0·02	......	+0·44	+0·35	51·5	56·6
	,, 7	+50·06	− 4·94	−12·44	− 0·03	−0·03	......	+1·58	+0·10	52·2	56·5
89°, 26	Sept. 15	− 6·70	−18·61	−21·77	− 0·14	......	......	+2·09	−0·89	−0 33 50·2	23 23 10·5
	Jan. 14	+47·16	+ 7·57	− 5·86	+ 0·02	+0·06	+0·02	−0·74	−0·22	49·1	8·1
89°, 28	Dec. 6	+50·82	− 5·17	−12·54	− 0·03	−0·04	−0·04	−0·15	+0·39	−0 44 46·3	22 46 12·3
	Jan. 12	+49·11	+ 6·97	− 6·03	+ 0·01	......	......	+1·26	−0·26	44·7	12·6
	,, 14	+48·39	+ 7·58	− 5·85	+ 0·02	+0·13	+0·14	−0·19	−0·36	46·4	13·2
	Feb. 9	+33·94	+14·46	− 2·17	+ 0·05	−0·06	−0·07	+1·18	−0·46	45·7	12·8
89°, 29	Dec. 6	+50·04	− 5·06	−12·54	− 0·03	−0·03	−0·03	−0·08	+0·39	−0 25 30·7	23 11 4·5
	,, 7	+50·28	− 4·74	−12·43	− 0·03	−0·03	−0·02	+1·23	+0·42	31·7	4·4
	,, 10	+50·90	− 3·76	−11·91	− 0·03	−0·17	−0·15	+0·05	+0·48	31·1	4·2
	Jan. 12	+48·18	+ 7·04	− 6·03	+ 0·02	......	......	+1·20	−0·30	31·8	4·5
	,, 14	+47·46	+ 7·65	− 5·85	+ 0·02	+0·14	+0·12	−0·30	−0·36	30·5	5·5
89°, 30	Feb. 9	+33·60	+14·63	− 2·17	+ 0·05	+0·01	−0·04	+1·52	−0·27	−0 2 34·5	22 37 37·0
89°, 31	Jan. 14	+47·45	+ 7·77	− 5·85	+ 0·02	+0·08	+0·18	−0·05	−0·38	−0 5 35·0	23 6 53·1
	Feb. 9	+32·97	+14·57	− 2·17	+ 0·05	......	−0·05	+1·46	−0·32	35·3	52·7
	Mar. 5	+13·61	+18·16	+ 0·12	+ 0·08	......	......	−0·64	+0·51	34·5	52·2

TABLE III.—CONTINUED.

B.D.	Date.	Aberration.		Reduction to 1887·0.		Corr. for Change of Nadir.		Corr. for *dx.* and *dD.*		1887·0.	
		$l_2 - l_1$	$e_2 - e_1$	$l - l_2$	$e - e_2$	$\Delta\, l.$	$\Delta\, e.$	$\Delta\, l.$	$\Delta\, e.$	$90 - \lambda_0$	$90 - \beta_0$
		″	″	″	″	″	″	″	″	° ′ ″	° ′ ″
89°, 32	Jan. 14	+47·97	+ 7·96	− 5·85	+ 0·02	−0·22	+0·26	+0·39	−0·34	+0 23 30·1	22 44 27·3
	Mar. 3	+15·14	+18·08	− 0·23	+ 0·08	−0·02	+0·03	−0·27	+0·15	34·7	26·7
89°, 33	Mar. 5	+13·43	+18·20	+ 0·12	+ 0·08	......	......	−0·94	+0·44	+0 10 0·7	22 58 9·5
89°, 34	Jan. 14	+47·81	+ 7·98	− 5·84	+ 0·02	−0·24	+0·25	+0·40	−0·34	+0 29 7·4	22 48 17·3
	Mar. 3	+15·01	+18·08	− 0·23	+ 0·08	−0·01	+0·01	−0·28	+0·15	7·8	16·8
89°, 35	Jan. 14	+47·19	+ 7·75	− 5·85	+ 0·02	+0·11	+0·16	−0·13	−0·37	−0 7 10·8	23 15 28·9
	„ 31	+38·73	+12·48	− 3·65	+ 0·04	+0·10	+0·29	+2·08	−0·16	11·3	28·2
	Feb. 9	+32·81	+14·54	− 2·17	+ 0·05	−0·01	−0·05	+1·39	−0·36	12·2	27·5
	Mar. 5	+13·57	+18·14	+ 0·12	+ 0·08	......	......	−0·53	+0·54	11·2	27·4
89°, 36	Sept. 8	−12·12	−18·29	−22·08	− 0·15	......	......	−1·60	−0·23	+0 25 53·3	23 8 8·1
	„ 10	−10·40	−18·43	−21·91	− 0·14	......	......	−0·54	+0·30	55·5	7·6
	„ 16	− 5·16	−18·72	−21·74	− 0·13	......	......	−0·35	−0·68	52·6	8·5
	Jan. 13	+47·56	+ 7·66	− 5·92	+ 0·02	−0·23	+0·15	+1·33	+0·07	54·2	8·2
	„ 14	+47·17	+ 7·96	− 5·84	+ 0·02	−0·40	+0·25	+0·57	−0·30	54·6	7·5
	Feb. 9	+32·50	+14·70	− 2·17	+ 0·05	−0·03	+0·02	+1·62	+0·11	56·4	7·7
	Mar. 3	+14·83	+18·03	− 0·22	+ 0·08	+0·03	−0·02	−0·35	+0·13	53·8	8·0
89°, 38	Sept. 8	−10·51	−18·39	−22·05	− 0·15	......	......	−0·40	−0·66	+2 0 45·6	23 18 26·4
	Nov. 2	+34·86	−13·87	−17·10	− 0·08	−0·23	......	+1·46	−0·65	49·1	26·4
	„ 21	+45·42	− 8·97	−14·78	− 0·05	......	......	+1·76	−0·09	44·1	26·7
	Dec. 7	+50·49	− 4·02	−12·45	− 0·03	+0·03	......	+1·33	−0·36	49·0	26·1
88°, 1	Nov. 4	+36·16	−13·30	−16·87	− 0·07	......	......	+0·10	−0·40	+2 24 34·3	23 30 6·4
	„ 21	+45·24	− 8·84	−14·78	− 0·05	......	......	+1·78	+0·03	34·1	6·2
88°, 126	Oct. 14	+20·69	−17·33	−19·48	− 0·10	−1·20	+0·38	+3·21	−1·29	+1 44 9·0	22 51 22·4
	Nov. 4	+36·61	−13·55	−16·88	− 0·07	......	......	−0·57	−0·33	9·4	22·0
88°, 135	Sept. 8	−10·27	−18·43	−22·05	− 0·15	......	......	−0·53	−0·64	+2 20 43·6	23 12 39·2
	Nov. 2	+35·21	−13·81	−17·10	− 0·08	−0·25	+0·01	+1·31	−0·70	43·8	39·6
	„ 3	+35·87	−13·59	−16·97	− 0·07	−1·10	+0·08	+1·44	−0·76	42·5	39·0
	Dec. 7	+50·76	− 3·92	−12·45	− 0·03	+0·04	......	+1·29	−0·38	42·5	39·4
	Mar. 5	+11·24	+18·35	+ 0·15	+ 0·08	−1·92	+0·13	−1·42	−0·09	42·6	39·2
89°, 37	Sept. 10	−10·66	−18·37	−21·91	− 0·14	......	......	−0·69	+0·24	−0 0 8·6	23 21 34·6
	Nov. 2	+33·37	−14·32	−17·12	− 0·08	−0·35	+0·10	+0·45	−0·85	8·7	35·1
	„ 3	+34·08	−14·10	−16·99	− 0·07	−1·42	+0·24	+1·06	−0·85	8·7	35·3
	„ 4	+34·71	−13·89	−16·88	− 0·07	......	......	−0·65	−0·30	6·7	35·3
	„ 4	+34·84	−13·85	−16·87	− 0·07	......	......	+0·48	−0·35	7·3	34·9
	„ 21	+44·49	− 9·51	−14·78	− 0·05	......	......	+1·47	+0·41	6·9	34·9
	„ 22	+44·87	− 9·26	−14·71	− 0·05	......	......	+1·03	−0·37	10·0	35·0
	„ 22	+44·91	− 9·23	−14·70	− 0·05	......	......	+1·37	−0·12	6·4	34·8
	„ 23	+45·32	− 8·97	−14·61	− 0·05	......	......	+0·81	−0·39	7·3	34·6
	„ 23	+45·40	− 8·91	−14·58	− 0·05	......	......	+1·23	+0·13	7·3	34·8
	„ 24	+45·77	− 8·66	−14·46	− 0·05	......	......	+1·15	−0·23	7·5	34·8
	Dec. 4	+49·25	− 5·59	−12·72	− 0·03	+0·16	+0·03	+0·99	+0·08	7·8	34·8
	„ 6	+49·76	− 4·95	−12·55	− 0·03	−0·02	......	+0·53	+0·34	7·7	34·8
	„ 6	+49·80	− 4·90	−12·54	− 0·03	−0·02	−0·04	−0·28	+0·38	7·4	35·1
	„ 7	+49·96	− 4·67	−12·46	− 0·03	+0·05	−0·01	+1·19	−0·43	7·1	34·8
	„ 7	+50·02	− 4·59	−12·43	− 0·03	−0·03	−0·04	+1·05	+0·48	6·9	34·1
	„ 10	+50·60	− 3·66	−11·94	− 0·03	+0·07	+0·01	+0·91	+0·30	6·3	34·7
	„ 10	+50·64	− 3·58	−11·88	− 0·03	+0·18	−0·24	−0·75	+0·38	6·3	34·1

TABLE III.—CONTINUED.

B.D.	Date.	Aberration.		Reduction to 1887·0.		Corr. for Change of Nadir.		Corr. for *dx.* and *dD.*		1887·0.	
		$l_2 - l_1$	$e_2 - e_1$	$l - l_2$	$e - e_2$	Δ *l.*	Δ *e.*	Δ *l.*	Δ *e.*	$90 - \lambda_\circ$	$90 - \beta_\circ$
		″	″	″	″	″	″	″	″	° ′ ″	° ′ ″
89°, 37	Jan. 10	+48·39	+ 6·51	− 6·39	+ 0·01	−0·05	−0·01	−0·19	−0·43	−0 0 5·4	23 21 35·3
	„ 12	+47·71	+ 7·15	− 6·04	+ 0·01	......	......	+1·02	−0·39	8·9	34·4
	„ 13	+47·29	+ 7·50	− 5·92	+ 0·02	+0·09	−0·08	+1·34	−0·01	8·0	34·7
	„ 13	+47·27	+ 7·52	− 5·91	+ 0·02	−0·90	+0·36	+1·25	+0·20	7·6	34·8
	„ 14	+47·01	+ 7·73	− 5·86	+ 0·02	+0·02	......	−0·80	−0·19	9.3	33·9
	„ 14	+46·89	+ 7·83	− 5·84	+ 0·02	−0·68	+0·23	+0·76	−0·21	7·7	34·9
	„ 29	+39·74	+11·97	− 3·78	+ 0·04	+0·32	+0·17	+2·60	+0·25	7·2	35·1
	„ 31	+38·51	+12·50	− 3·65	+ 0·04	+0·09	+0·18	+2·04	−0·23	7·7	34·5
	Feb. 9	+32·66	+14·54	− 2·18	+ 0·05	−0·19	−0·09	+0·70	−0·60	6·0	35·2
	„ 9	+32·48	+14·59	− 2·16	+ 0·05	−0·08	+0·03	+1·53	+0·25	9·0	35·3
	Mar. 3	+15·11	+17·95	− 0·23	+ 0·08	−0·02	+0·05	−0·24	+0·17	7·6	34·7
	„ 5	+13·42	+18·13	+ 0·11	+ 0·08	......	......	−0·42	+0·55	7·6	33·4
	„ 5	+13·33	+18·14	+ 0·13	+ 0·08	......	......	−1·20	+0·32	8·4	34·9
α Ursæ Min.	Sept. 15	− 3·40	−18·66	−21·75	− 0·14	......	......	+2·43	−0·74	+3 0 51·2	23 54 42·8
	Oct. 13	+20·20	−17·14	−19·52	− 0·10	......	......	+1·16	−0·57	49·1	42·5
	Nov. 2	+34·72	−13·57	−17·09	− 0·08	......	......	+2·04	−0·32	49·0	42·7
	„ 3	+35·36	−13·34	−16·96	− 0·08	......	......	+2·25	−0·34	50·1	43·0
	„ 4	+35·98	−13·11	−16·86	− 0·07	......	......	+0·39	−0·36	48·4	42·7
	„ 21	+44·78	− 8·62	−14·77	− 0·05	......	......	+1·65	+0·27	50·0	43·0
	„ 22	+45·19	− 8·32	−14·70	− 0·05	......	......	+1·37	−0·07	48·7	43·0
	„ 23	+45·57	− 8·03	−14·59	− 0·05	......	......	+1·21	−0·16	49·4	42·7
	„ 24	+45·94	− 7·73	−14·44	− 0·05	......	......	+1·25	−0·14	49·3	42·8
	Dec. 4	+48·90	− 4·62	−12·72	− 0·04	......	......	+1·01	+0·03	49·1	42·9
	„ 6	+49·30	− 3·98	−12·55	− 0·03	......	......	+0·63	+0·31	48·7	43·3
	„ 7	+49·49	− 3·65	−12·44	− 0·03	......	......	+1·62	−0·01	49·7	43·0
	„ 10	+49·95	− 2·67	−11·93	− 0·03	......	......	+0·88	+0·34	50·0	43·2
	Jan. 10	+46·36	+ 7·39	− 6·40	+ 0·01	......	......	−0·31	−0·42	47·7	42·7
	„ 14	+44·83	+ 8·58	− 5·86	+ 0·02	......	......	−0·83	−0·16	47·7	42·5
λ Ursæ Min.	Sept. 10	−10·06	−18·56	−21·91	− 0·14	......	......	−0·47	+0·31	+1 6 48·1	22 30 33·3
	„ 16	− 4·68	−18·82	−21·74	− 0·14	......	......	−0·58	−0·65	48·3	32·4
	Dec. 10	+52·64	− 3·24	−11·87	− 0·03	+0·30	−0·24	−0·86	+0·34	47·8	32·7
	Jan. 13	+48·55	+ 7·89	− 5·92	+ 0·02	......	......	+1·34	+0·02	49·0	32·9
	„ 14	+48·15	+ 8·19	− 5·84	+ 0·02	−0·32	+0·25	+0·49	−0·31	48·0	33·0
	Mar. 3	+14·66	+18·18	− 0·23	+ 0·08	+0·01	−0·01	−0·33	+0·13	48·5	33·2
	„ 5	+12·82	+18·35	+ 0·12	+ 0·08	......	......	−1·00	+0·41	47·4	33·4
B.A.C. 4165	Sept. 8	−15·85	−17·86	−22·08	− 0·14	......	......	−0·10	−0·67	−4 12 33·1	23 24 20·7
	„ 15	− 9·94	−18·42	−21·79	− 0·13	......	......	+1·87	−0·96	32·1	20·1
	Oct. 13	+14·48	−18·01	−19·55	− 0·10	......	......	+0·77	−0·67	32·3	20·2
	Nov. 2	+30·42	−15·14	−17·12	− 0·08	−0·12	......	+1·71	−0·54	30·3	20·5
	„ 3	+31·14	−14·95	−16·99	− 0·07	−0·48	−0·01	+1·94	−0·56	31·7	19·9
	„ 4	+31·86	−14·75	−16·88	− 0·07	......	......	+0·17	−0·39	32·7	20·0
	„ 22	+42·83	−10·42	−14·72	− 0·05	......	......	+1·26	−0·24	30·9	20·1

TABLE IV.

Star.	Mag. B.D.	No. of Obs.	1887·0.		Precession.						B − Carrington.	
			x.	y.	$d\,x$.	$100d^2x$.	100^2d^3x.	$d\,y$.	$100d^2y$.	100^2d^3y.	$\Delta\,x$.	$\Delta\,y$.
			″	″	″	″	″	″	″	″	″	″
B.D. 89°, 1	9·5	8	+ 783·95	+ 76·48	−20·0701	+0·0040	+0·0119	+0·1752	−0·4483	−0·0005	−0·7	−0·5
2	9·2	4	+2737·87	+1214·35	−20·3224	+0·0068	+0·0120	+0·6117	−0·4537	−0·0007	+0·7	−0·2
3	8·8	4	+ 992·20	+ 858·51	−20·2446	+0·0026	+0·0120	+0·2217	−0·4522	−0·0005	−0·6	−1·4
4	9·4	4	+1287·27	+1029·10	−20·2825	+0·0008	+0·0120	+0·2876	−0·4530	−0·0005	+1·7	+0·2
5	9·5	3	+1820·71	+1328·35	−20·3488	+0·0075	+0·0120	+0·4068	−0·4544	−0·0006	+0·2	−0·1
7	9·3	4	+1584·42	+2488·40	−20·6072	−0·0011	+0·0122	+0·3540	−0·4602	−0·0006	+0·4	+0·5
9	9·1	1	− 57·3	+1884·5	−20·4735	+0·0087	+0·0117	−0·0128	−0·4575	−0·0004	+1·1	−0·5
10	9·5	3	− 434·57	+3231·82	−20·7728	+0·0107	+0·0123	−0·0971	−0·4642	−0·0003	+0·3	−1·9
11	9·5	3	− 622·49	+3384·31	−20·8066	+0·0118	+0·0123	−0·1391	−0·4650	−0·0003	−0·3	+1·8
12	9·1	7	− 752·98	+1321·88	−20·3480	+0·0129	+0·0121	−0·1682	−0·4548	−0·0003	+0·3	+0·2
16	9·5	6	−1517·83	+ 869·99	−20·2468	+0·0175	+0·0120	−0·3391	−0·4526	−0·0002	−0·4	−0·2
17	9·0	6	−2186·17	+ 656·97	−20·1987	+0·0215	+0·0120	−0·4885	−0·4516	−0·0001	+0·9	+3·2
18	8·9	5	−1584·35	+ 207·04	−20·0989	+0·0180	+0·0119	−0·3540	−0·4493	−0·0001	+0·2	+0·7
19	9·5	4	−2214·77	+ 341·54	−20·1283	+0·0217	+0·0120	−0·4949	−0·4500	−0·0001	..	..
20	9·5	5	−2667·51	− 20·65	−20·0469	+0·0245	+0·0119	−0·5960	−0·4483	−0·0000	..	..
21	8·8	4	−2477·84	− 501·79	−19·9395	+0·0234	+0·0119	−0·5536	−0·4459	−0·0000	−0·4	+0·8
23	9·4	5	−2171·10	− 610·49	−19·9156	+0·0216	+0·0118	−0·4851	−0·4453	−0·0001	−1·5	−0·6
25	9·0	4	−1392·17	− 868·65	−19·8585	+0·0170	+0·0118	−0·3111	−0·4439	−0·0002	−0·8	−0·9
26	9·5	2	− 805·60	− 248·56	−19·9975	+0·0134	+0·0119	−0·1800	−0·4469	−0·0003	−0·8	−0·6
28	8·7	4	−1039·45	−2467·64	−19·5001	+0·0151	+0·0116	−0·2323	−0·4359	−0·0002	+0·7	0·0
29	9·4	5	− 602·81	− 971·61	−19·8358	+0·0123	+0·0118	−0·1347	−0·4433	−0·0002	+0·5	+0·5
30	9·3	1	− 59·4	−2977·1	−19·3859	+0·0094	+0·0115	−0·0133	−0·4332	−0·0003	+2·0	+1·0
31	9·5	3	− 131·48	−1221·63	−19·7799	+0·0096	+0·0117	−0·0294	−0·4420	−0·0003	+0·2	−1·9
32	9·5	2	+ 546·0	−2568·8	−19·4776	+0·0057	+0·0116	+0·1220	−0·4351	−0·0004	−0·6	−0·9
33	9·5	1	+ 234·4	−1745·0	−19·6626	+0·0075	+0·0117	+0·0524	−0·4393	−0·0004	+0·3	+1·7
34	9·3	2	+ 677·35	−2339·76	−19·5290	+0·0049	+0·0116	+0·1513	−0·4363	−0·0004	+0·2	+0·1
35	9·3	4	− 170·34	− 706·38	−19·8952	+0·0097	+0·0118	−0·0381	−0·4446	−0·0003	+0·3	+0·1
36	9·5	7	+ 610·72	−1148·37	−19·7962	+0·0052	+0·0117	+0·1365	−0·4422	−0·0004	+0·5	+1·2
37	9·3	31	− 3·01	− 339·46	−19·9773	+0·0087	+0·0119	−0·0007	−0·4464	−0·0004	+0·3	+1·2
38	9·0	4	+2866·76	− 574·02	−19·9228	−0·0083	+0·0118	+0·6405	−0·4448	−0·0007	+0·3	+0·5
88°, 1	9·5	2	+3458·05	+ 105·38	−20·0739	−0·0119	+0·0119	+0·7727	−0·4480	−0·0008	+1·8	0·0
126	9·5	2	+2426·94	−2185·71	−19·5623	−0·0055	+0·0116	+0·5423	−0·4368	−0·0007	..	..
135	9·5	5	+3326·60	− 937·38	−19·8410	−0·0110	+0·0117	+0·7433	−0·4429	−0·0001	+0·1	+0·1
α Ursæ Min.	2·0	15	+4395·50	+1542·56	−20·3924	−0·0177	+0·0121	+0·9821	−0·4550	−0·0009	+0·1	+1·3*
λ Ursæ Min.	6·5	7	+1534·35	−3414·76	−19·2870	0·0000	+0·0114	+0·3428	−0·4307	−0·0005	−0·6	−0·5*
B.A.C. 4165	6·5	7	−6013·50	− 376·69	−19·9605	+0·0444	+0·0119	−1·3436	−0·4468	+0·0004	+1·1	+1·3

* Corrected for proper motion.

TABLE V.

	B−N.A.		B−B.J.		B−C. *d.* T.	
	Δ *x*.	Δ *y*.	Δ *x*.	Δ *y*.	Δ *x*.	Δ *y*.
	″	″	″	″	″	″
α Ursæ Min.	+0·25	+0·63	+0·42	−0·07	+0·57	+0·25
λ Ursæ Min.	+0·17	−0·32	−0·29	−0·03	−0·07	−0·74
B.A.C. 4165.	......	......	......	......	−0·80	−0·82

	1887·0.						B−N.A.		B−B.J.		B−C. *d.* T.		B−Elkin.	
	α.			δ.			Δ α.	Δ δ.	Δ α.	Δ δ.	Δ α.	Δ δ.	Δ α.	Δ δ.
	h	m	s	°	′	″	s	″	s	″	s	″	s	″
α Ursæ Min.	1	17	21·13	88	42	21·28	+1·50	−0·45	−0·60	−0·38	+0·14	−0·6	−0·66	−0·23
λ Ursæ Min.	19	36	46·97	88	57	36·15	+0·09	−0·37	−1·03	+0·09	−1·37	−0·6	−0·62	+0·15
B.A.C. 4165.	12	14	20·25	88	19	33·87	......	......	......	......	+1·75	−0·8	−0·79	−0·73

CONTRIBUTIONS TO THE THEORY OF THE SUN.

BY J. HALM, Ph.D.

CONTENTS.

CONTRIBUTIONS TO THE THEORY OF THE SUN.

Natura nihil agit frustra, et frustra sit per plura quod fieri potest per pauciora. Natura enim simplex est, et rerum causis superfluis non luxuriat. Newton—"Regulæ Philosophandi."

INTRODUCTION.

It is a strange fact that in the numerous theories which have been propounded in explanation of the periodic changes of the solar phenomena, no account has yet been taken of so important an element as the light- and heat-absorbing envelope surrounding the photosphere. The attention which this so-called solar atmosphere has hitherto received, on the part even of our most eminent investigators, in connection with the economy of radiant energy on our luminary, is utterly disproportioned to the importance of the subject. It is no doubt well-known that this atmosphere prevents a great part of the solar energies from radiating into space, and we are even in possession of measurements which enable us to compute pretty accurately the enormous quantities of light and heat which, by the interference of this envelope, are preserved to the stock of energy in the sun, instead of being dissipated and lost to it by radiation.

One would have thought that such an important fact as this quite enormous light- and heat-absorbing faculty of the solar atmosphere would have led solar physicists to inquire whether, in view of the stupendous changes going on incessantly at the sun's surface, we can possibly rely on the absolute constancy of the solar envelope, or whether the density of the absorbing matter might not rather be exposed to variations which would lead to serious consequences as regards the maintenance of the thermal equilibrium in the solar body.

So far as we know, however, such an attempt has never been made. In spite of the fact which was first reliably established by Langley's observations, and was afterwards confirmed by others, that the sun, if deprived suddenly of this protecting screen, would radiate into space as much as double its present amount of energy, solar physicists failed to perceive that changes in the density of the absorbing envelope must entail consequences of the most far-reaching character with respect to the thermal conditions on and in the sun. That such changes—and these, too, of no inconsiderable magnitude—must inevitably occur, is a conclusion which it is hardly possible to evade when it is remembered that the supreme control over the dispensation of solar energy depends entirely on a thin, shallow layer, the matter of which is constantly tossed about by vehement eruptions, and acted upon by a most complicated system of convection currents to and from the sun's centre.

The possibility of such variations was indeed strongly urged, more than twenty years ago, by one of the greatest authorities on this question. Langley, in 1875, shortly after his well-known researches into the absorbing faculty of the solar envelope, pointed out the decisive influence on the sun's radiation into space of these changes in the transmissive power of the atmosphere. But, unfortunately, his attention was at the time solely directed towards their probable effects on the temperature of our own planet, and it did not occur to him to take into consideration the disastrous results which such variations of density as he assumes must produce on the sun itself. What he says is, however, so well adapted to impress upon our minds the importance of the line of research on which it is proposed to enter, that it may be well to quote the passage referred to in the author's own words :—

"If it be true, then, that the sun is surrounded by an atmosphere whose principal action in obscuring the heat radiation is due to a thin stratum which cuts off one-half of the heat which should reach us, and in whose absence this radiation should be doubled—an atmosphere not independent of the interior of its globe in such a degree as our own, but one to and from which matter is constantly being added and withdrawn—it follows that any change in the ratio of supply and withdrawal, or other cause which should increase its absorption by so much as 25 per cent., would diminish the mean surface temperature of our globe by 100° Fahrenheit, whilst a like diminution in the envelope would produce a corresponding change in the opposite direction." (Langley, "The Solar Atmosphere, an Introduction to an Account of Researches made at the Alleghany Observatory." American Journal of Science and Art, 1875.)

Now, if the influence of a change in the density of the solar atmosphere is so enormous in the case of a planet at a distance of almost a hundred millions of miles, of what inconceivable importance must it not be for the sun itself? Drawing the very natural inference that a *deficit* of outside radiation means a *surplus* of energies working on the sun, and *vice versa*, we are forcibly led to conclude that even slight changes, such as would elude our most refined observations, are bound to greatly influence the state of equilibrium on our luminary.

The following considerations may give some idea of the enormous quantity of radiation which by the interference of the solar atmosphere is prevented from escaping into space, and which thereby naturally reduces the rate of the sun's secular cooling. It can be shown by computations founded on sound reasoning that the incessant loss of energy by radiation would gradually lower the temperature of the whole sun at the rate of about 3° Cels. per annum,* unless there be some force which would compensate for the loss by means of a continuous generation of heat.

As we go back, however, in the history of our planet, no reliable traces are discernible of any change in the amount of solar heat received by it. It would, therefore, seem certain that such a compensating force must actually be at work in the sun, and that, moreover, this force is sufficiently intense to fully make good

* The numerical data are taken from a paper by Professor Scheiner—"The Temperature of the Sun II.": Publication of the Astronomical Society of the Pacific, Vol. X.–No. 65.

the loss by radiation under existing conditions. We may assume, then, with confidence that since the days of the Pharaohs up to the present moment there has existed what, within the limits of human perception, we may call a complete equilibrium of these heat-dispensing and heat-creating forces. But what, on the other hand, would have been the present state of matters if the sun, some 4000 years ago, had been suddenly deprived of its protecting envelope, and thus had been forced to throw off during all these centuries, day by day, and year after year, twice the present amount of energy?

Let us, according to the latest researches, assume the temperature of the photosphere to be about 7000°-8000° Cels., and let us also keep in mind the fact that, *without* the atmosphere, each element of the whole solar body must lose every year about 3° Cels. of its temperature, which can no longer be compensated for by the heat-generating force. Then, according to Scheiner, the temperature of the radiating photosphere would have fallen, so early as the time of Christ, to less than *half* its original intensity. And even if we assume (what is all but impossible) that, at the beginning of the period under consideration, organic nature on the earth might have escaped annihilation from the fierce heat produced by such an excessive solar radiation, there cannot at any rate be the slightest doubt that the store of energy in the sun, and along with it the temperature of our planet, would by this time have sunk so low as no longer to afford the conditions required for the existence of organic life. Thus, without the heat-conserving power of its atmosphere, the sun, within a few thousand years, would have cooled down into a body devoid of light and heat, and with it the whole solar system would have been plunged into night and cold.

Speculations such as these must tend to convince the investigator that the solar atmosphere has to be looked upon as a most important regulator of the continuous and enormous output of energy which the sun is forced to transmit to outside space. Moreover, they give us an *a priori* feeling of certainty that a research into the causes of the variability of the forces which we see acting at the solar surface, if not identical with, is yet at least closely akin to, the investigation of the origin and the physical properties of this protecting envelope. To attempt a solution of the solar problem from such a point of view is the main object of this paper, which is published in the hope that the simplicity of the principles expounded, and the relative completeness of the results derived therefrom, may justify our boldness in attempting the solution of so difficult a question.

SECTION I.

The Solar Atmosphere, and its Importance for the Maintenance of the Sun's Radiation.

Helmholtz's theory.

1. There is perfect unanimity amongst astronomers as regards the nature of the force which by a continuous generation of heat compensates for the loss of energy into space. Helmholtz's theory, which attributes this heat-generation to the progressive contraction of the sun's mass as a consequence of gravitation, may certainly be regarded

as one of the most probable hypotheses ever propounded in the history of physical science. The theory is so well known that we may limit ourselves to a brief exposition of it, sufficient for the purpose of our investigations.

Observations show that over the whole surface of the sun there are scattered thousands of minute, grain-like, bright patches seemingly floating in a darker medium. The general opinion respecting the physical nature of these peculiar formations is that they represent the products of condensation in gaseous currents, which, starting from the interior of the solar body, rise to the surface and there begin to cool down. This cooling process is sufficient to cause the liquefaction of some of the gaseous elements. Since, however, the radiating power of these liquefied elements becomes very considerable, they rapidly throw out their heat into space, and consequently soon become less luminous and at the same time denser. This increase in density is shared by the gases in which they are suspended, and this cooler and denser surface matter must eventually fall towards the sun's centre, thereby generating return currents in that direction.

But such a system of continuous interchange between the hot matter of the interior and the cooled elements at the surface—quite familiar to us in the phenomenon of boiling water—however strong and efficient it may be, does not yet explain the unvarying constancy of the radiating power of the sun. The quantity of heat at present existing in the solar body could by no means suffice to maintain the thermal equilibrium for any length of time. Hence, unless there be a force which, whilst heat is radiated into space, by some means or other generates a new supply, there would be no possibility whatever of the sun's keeping up its present temperature and radiating power. Without such a force the matter lowered in temperature at the surface, in sinking towards the centre, would more and more tend to cool the interior layers, and thereby weaken the intensity of the ascending currents. The heat supply to the surface would in this way steadily become less, thus entailing the gradual cooling of the whole solar body.

That our sun, however, is not hurried with such speed towards its final extinction is due, according to Helmholtz, to gravitation—to the same force which maintains and governs the stability of the universe. The influence of this force by which every single element of the whole solar mass is moved towards the centre causes a continuous contraction of the solar body, and it is the vast amount of heat generated by the impacts between these elements which prevents the exhaustion of the store of radiant energy.

Changes in the relation between the radiating and contracting forces during a star's life.

2. Now, if it were possible to assume that this heat is sufficient to neutralise fully the loss of energy caused by radiation, our question as to the stability of the sun's radiating power would of course be answered. But whether the amount of heat generated in this way *is* really equal to the loss sustained by radiation, or whether the conditions of contraction peculiar to the sun may not perhaps produce *more* or *less* heat than is just required for compensation, are questions to which Helmholtz's theory gives no immediate answer.*

* For a most lucid and scientific exposition of Helmholtz's theory the reader is referred to Sir Robert

It is indeed scarcely conceivable that the conditions of contraction remain the same throughout the whole life of a star. The Spectroscope has revealed the fact that the photospheres of different stars exhibit widely different stages as regards temperature. No doubt, there are suns hotter than ours, and others considerably cooler. Further, if, as we may confidently assume, the various conditions of temperature now recognised in the different types of star-spectra represent the phases which successively appear in the evolution of each of these bodies from its origin as a far-extended nebula down to its complete obscuration, we are forced to conclude that the ratio between radiation and contraction can by no means be a constant, but must vary very greatly during the lifetime of a celestial body.

As long as the temperature of a star is rising, we may be sure that more heat is being generated by its condensation than the amount necessary to compensate for loss into space. If, on the other hand, its temperature is decreasing, then the loss by radiation must be greater than the gravitational force is able to replace.

Position of our sun in the curve representing the evolution of temperature in a star.

3. Here we find ourselves at once face to face with the question—What place does the sun occupy at present in this scale of star-temperatures?

The tendency of modern astronomical research has been to bring about an almost general consensus of opinion to the effect that our luminary, notwithstanding its still vast store of potential energy, has already passed its maximum temperature, and is now gradually though slowly approaching extinction.

Objections to Dr. See's theory that the sun is growing hotter.

In the following we adopt this hypothesis, which is, as will presently be shown, supported by the existence of a cooler absorbing atmosphere above the photospheric layers. It has, however, been recently attacked by Dr. See, who, from theoretical considerations, arrives at a contrary conclusion. But at least one of the inferences brought forward by this eminent astronomer, and indeed the inference which has the most important bearing on the question under consideration, appears to me highly improbable, if not altogether impossible.

Dr. See contends that, as long as a star is gaseous, *free* contraction goes on in every particle at a rate proportional to the attracting mass and inversely proportional to the square of the distance of the element from the centre. Under this assumption the author deduces his law that the temperature of a gaseous star is inversely proportional to its radius. Thus, the more a celestial body shrinks under the influence of gravitation, the higher will be its temperature.

Ball's "Story of the Sun," chap. XIV. One passage relating to the important question raised above may be here quoted:—"It appears from this discussion of the problem that the temperature of the sun may remain absolutely constant during vast time periods. This is, however, a matter as to which we have no assurance, either from observation or from theory. If the heat generated by contraction in the way we have described were exactly the necessary amount to compensate for the losses incurred by daily radiation, then, of course, the solar temperature would remain invariable. If, however, the quantity of heat generated by the augmentation of the molecular velocity due to contraction should exceed the loss of heat due to radiation, then the temperature must on the whole rise. Of course it might also happen that though the production of heat by the transformation of mechanical effect was partly sufficient to neutralise the loss incurred in radiation, it might not be completely sufficient. If this be the case, then, of course, the solar temperature must be, on the whole, declining." (Pp. 275–76.)

Clearly, however, this process cannot go on indefinitely, otherwise the star's temperature would rise to infinity. There must then be a certain limit to it, and Dr. See argues that, as condensation goes on, a point is at last reached at which the whole mass becomes so dense that it begins to liquefy. "After the mass has condensed so far that liquefaction sets in, free contraction is obstructed by molecular forces, or practically ceases; the temperature falls, and the body eventually cools down to obscurity." (A. N., No. 3540, p. 180.)

Dr. See consequently supposes a star which has already passed its state of highest temperature to be in a condition of liquefaction, and that therefore, since "all observational evidence goes to show that our sun is wholly gaseous, it is difficult to resist the conclusion that he is growing hotter from age to age." (p. 181.)

Now, quite apart from the disputable point that "obstruction by molecular forces" does not begin until the body becomes "liquefied," it is difficult to understand, according to our knowledge of the present temperature of the solar matter, that any such gaseous body could ever reach the state of liquefaction by an increase of density, *so long as this increased density is*, as Dr. See asserts, *accompanied by an increase of temperature.*

In his book on "The Sun" (2nd edition, p. 329), Professor Young remarks on this very point that "the researches of Andrews and his successors have shown that to liquefy a gas two things must go together—*increase of pressure* and *diminution of temperature.* For each gas there is a so-called 'critical temperature,' and so long as the temperature does not fall below this point no pressure whatever can reduce the gas to the liquid form. Now, on or in the sun the temperature can not be supposed to be below the 'critical temperature' of several of the gases found there, and hence their liquefaction is out of the question."

On these grounds it seems impossible to imagine that the sun, or in fact any gaseous star at the sun's temperature, can ever be liquefied by increase of pressure if the temperature increases at the same time. Thus Dr. See's argument that the gaseous state of the sun at present is to be considered a proof of its being still on the ascending branch of the temperature curve appears to be untenable. This theory, coming into conflict, as it does, with one of the fundamental laws of nature, leads to no result which can be adduced against the generally adopted opinion that our sun, although gaseous, has already passed the point of culmination, and belongs to what may properly be called the class of "cooling" stars—viz., to those celestial bodies in which, according to our former explanation, the heat-generating power of contraction is no longer sufficient to fully compensate for the loss of energy by radiation into space.

The solar atmosphere—its nature, and effects on solar heat radiation.

4. Having thus adopted a definite assumption regarding the present stage of temperature-development in the sun, let us revert to a closer consideration of the so-called solar "atmosphere."

It must be clearly understood that by this term we comprehend those superficial layers where the *general* absorption of light and heat takes place. For some time this atmosphere was identified with what is known as the "reversing layer"; but Professor

Hastings in 1881 gave strong reasons for refuting this idea,* and for assuming that the general absorption originates in a stratum composed of dust-like matter formed, by cooling, from the luminous clouds of the photosphere. That this substance cannot be gaseous is, in his opinion, absolutely certain, inasmuch as no gas at the sun's high temperature can possibly absorb otherwise than *selectively*. What we actually observe, however, is a general weakening of all the rays throughout the whole spectrum, a fact which undoubtedly points to the presence of solid or liquid absorbing particles.

Indeed, it seems obvious that these particles, which, while ascending in the currents from the interior to the surface, are precipitated so as to form the luminous clouds of the photosphere, must "rapidly cool on account of their great radiating power, and form a fog or smoke which settles slowly through the spaces between the granules"; and that "it is this smoke which produces the general absorption at the limb."

Analogy between terrestrial and solar atmospheres as heat-conserving agents.

5. Such an envelope encircling the photosphere must act on the sun's radiation in the same way as do our atmosphere and its clouds on the radiation from the soil. We are quite familiar with the fact that clear nights are, as a rule, cooler than cloudy ones, and we explain this phenomenon by the hypothesis that on clear nights radiation from the soil into space goes on more freely than when clouds offer an effective impediment to the dissipation of radiant energy. How all-important the protection from excessive radiation afforded by our atmosphere is may be inferred from Professor Langley's result that, without this screening envelope, the temperature at the earth's surface would probably be not much above the absolute zero point, and that therefore the existence of organic life on our globe is made possible only by the absorptive faculty of its atmosphere.

It must, then, be accepted that the cooled, dark envelope surrounding the sun is of the greatest moment for the preservation of energy in the solar body, in so far as it immensely reduces the rate of dissipation of energy into space. Whatever fraction of the total radiation, which originally left the photosphere, is thus stopped in its outward progress, will be in part absorbed by, and in part reflected from, the obstructing atmosphere, and there can be no doubt whatever that some at least of this arrested energy will ultimately be thrown back towards the layers from which it originated.

Alterations in the absorbing power of the atmosphere during the star's life-time.

6. Adopting this explanation of the nature and effect of the absorbing atmosphere on a body like our sun, it becomes of the utmost moment to inquire whether the absorptive power of this layer may not be subject to alterations during the lifetime of the star.

Let us assume that the generation of heat by contraction in a star is inferior to the dissipation of energy by outward radiation.

If radiation were to go on at a *constant* rate, that is to say, if it were not hindered by any absorptive impediment, the temperature of every layer in the star's mass would decrease *uniformly* with time. In particular, the *surface* of the star must be subject to pronounced secular cooling; for whenever contraction is not sufficient to compensate for the *whole* loss of outward radiation, the currents circulating between the centre and

* "A Theory of the Constitution of the Sun," Americ. Jour. of Science—Third Series, Vol. XXI., pp. 33–44.

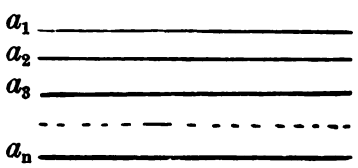
the surface of the sun can no longer be strong enough to carry back to the interior all the matter as soon as it has cooled by exposure at the surface. Part of this cooled matter must in that case collect above the photosphere, and spread out so as to form a cooler and darker envelope above the radiating layers.

But the very production of such an envelope, together with its gradually increasing opacity, gives the conditions of outward radiation an entirely different complexion, and renders altogether untenable the supposition that its amount can remain constant. The thicker this "cloak" of opaque elements round the radiating layers, the more effectively will photospheric radiation be prevented from escaping into space. Such an increasing augmentation of absorbing elements must cause a progressive reduction in the amount of outward radiation from the photospheric layers beneath, and must therefore tend to diminish the original disproportion between the heat-dispensing and heat-creating forces. Finally, a moment will arrive when the heat-preserving power of the ever-increasing absorbing envelope becomes sufficiently great to reduce the total amount of heat dissipated from the photosphere into space to such an extent as to admit of its being exactly compensated for by the amount generated by the contractile force. All disproportion then vanishes, and the two forces completely counterbalance each other.

Proof of the sequence of alterations in the absorbing power of the atmosphere. Ultimate establishment of a state of thermal equilibrium in a certain layer.

7. This may be very simply demonstrated. Let a_1 be the level of the photosphere—or, as it is more properly expressed, the level of maximum incandescence, and therefore also of maximum radiation—at a certain epoch t_1 of the star's life. In consequence of deficient contraction the temperature of this layer must decrease, and the materials composing it must cool down, so that, at a subsequent epoch t_2, the level of maximum incandescence will have shifted towards a_2 where the temperature is still sufficiently high to maintain the liquid state of all the precipitated matter.

The space between a_1 and a_2 will then be occupied by those particles which, being no longer luminous, act as an absorbing screen on the radiation emanating from a_2. As a result of the heat-conserving property of this layer, a_2 will now radiate less heat into space than did a_1 at the former epoch t_1. But if, notwithstanding this diminution, the amount actually radiated into space by a_2 still exceeds what is produced by contraction, then a_2 also will be progressively cooled, and, at a later epoch t_3, the level of maximum incandescence will be found at a_3. The absorptive layer has now increased in



thickness, and extends from a_1 to a_3: hence a_3 can dissipate still less energy than a_1 and a_2 did at the earlier stages. Should this amount, however, still be greater than that supplied by contraction, there will be a repetition of the whole phenomenon. Thus the opacity of the cooled atmosphere gradually increases as time goes on, and hence the rate of radiation from the underlying photosphere becomes less and less. Since no force is present to interfere with the cooling of the outside layers, a moment t_n must eventually be reached, at which the net radiation into space from the photosphere at a_n is so far reduced by the increase of absorbing particles in the superincumbent atmosphere that it is exactly compensated for by the heat produced through the contraction of the sun's mass.

The increasing reflective power of the atmosphere cannot prevent, but only retards, the star's final extinction.

8. From these considerations we see clearly the important part played by the dark absorbing envelope as regards the conservation of energy in the sun. It would, however, be against reason to assume that the action of this atmosphere could altogether prevent the star's final extinction; for whatever may be the economising effect of the protecting screen, the star by continually dissipating its energy through radiation and producing a fresh supply by contraction must eventually exhaust its stock of potential energy. But the approach of such a consummation would be immensely hastened, did the atmosphere not possess this inestimable heat-conserving property.

We have to conclude, then, that on a cooling star, such as our sun is supposed to be, the rate of progressive cooling must decrease from the surface towards the centre, owing to the interference of the atmospheric particles; that the layer a_1 must cool more rapidly than a_2, this again more rapidly than a_3, and so on, till finally the layer a_n is reached at a certain distance from a_1 where the total dissipation of energy towards space is completely counteracted by the combined effects due to contraction and to the heat-reflecting property of the overlying envelope.

State of thermal equilibrium not permanent. Subsequent overheating of the photosphere.

9. We are now confronted with a question the answer to which is of fundamental importance for the following research. Can this state of thermal equilibrium, eventually attained by the layer a_n, be permanent? Obviously this could only be so, provided that from the moment t_n onward no alteration took place in the heat-conserving power of the layers outside a_n. But, as we have seen, these layers are progressively cooling, and this process is bound to go on. Consequently, the ability of the atmosphere to absorb and reflect heat must still further increase even after the establishment of thermal equilibrium at a_n.

Owing to this greater amount of reflection towards it the layer a_n will now dissipate *less* energy than is required for the maintenance of thermal equilibrium, and therefore will become *overheated.* It thus comes to pass that, while the function of the absorbing envelope is that of reducing as much as possible the waste of energy from the photospheric layers beneath, it is, by the very nature of the process, compelled to *overdo* its work, and to finally preserve too much energy within the star.

It is this peculiarity of the absorbing envelope on which is based the development of the subsequent theory of solar eruptions and spots, and which we believe to be the prime cause of all those marvellous phenomena exhibiting their periodic evolutions on the solar surface.

SECTION II.

The Sun's Atmosphere and the Periodic Changes of Solar Activity.

A.—*The Eleven Years' Period.*

Anticipation of some results of

10. With a view to the clearest possible discussion of the problems before us we shall first consider analytically the periodicity of the chief solar phenomena, although this

course necessitates the anticipation of some results derived in later parts of this paper. What is required for our present purpose is a precise explanation both of the nature and of the origin of solar "eruptions" and "spots"—the chief surface manifestations of the forces at work in the sun. As will afterwards be found, the gradual overheating of the photosphere consequent on the increased reflective power of the overlying atmospheric layers, by causing an unstable equilibrium, must lead to dynamical displays at the solar surface which are exhibited in the form of eruptions. An attempt shall also be made to demonstrate that such eruptions are the prime cause of solar spots, and that these, in accordance with Faye's theory, represent vortex motions generated by the interaction of oppositely directed photospheric currents.

Section III. required for the analytical deduction of periodicity.

Postulating meantime these preliminary statements, we may proceed to formulate the following propositions :—

1. The outbreak of eruptions and formation of spots are the consequence of an unstable equilibrium in the photospheric layers, and take place whenever the supply of heat from the interior is so supplemented by the continuous reflection of heat from the overlying atmosphere that the photospheric layers receive more heat than is required for the maintenance of their thermal equilibrium.
2. The number and the intensity of eruptions and spots are proportional to the quantity of reflected heat.
3. The function of eruptions, consisting, as they do, in the ejection of overheated photospheric matter, is to produce a general heating and clearing up of the cooled absorbing layers of the solar atmosphere.
4. The action of spots consists in drawing down the cooled portions of this atmosphere into the hotter regions of the photosphere.

Of these propositions, the second is the direct consequence of the first, and therefore needs no special proof. The third is so obvious that it may be accepted *a priori*. Thus only the first and the last remain to be proved, and will be fully discussed later.

Antagonism between the heat-reflecting force of the absorbing envelope, and the action of eruptions and spots.

11. Now, it will be noticed that from the moment when the overheating of the photosphere causes unstable equilibrium to set in, a decided antagonism must arise between the acting forces. For, according to the third and fourth propositions, the action of the eruptions and spots is directed towards heating and clearing up the atmosphere. Thus we have, on the one hand, a gradual increase of heat-reflection towards the photosphere, in consequence of the progressive cooling of the absorptive atmospheric layers above it; and, on the other, the very opposite phenomenon—viz., an increase of heat-radiation into space from the photosphere, due to the simultaneous "clarifying" action both of eruptions and spots.

This very singular antagonism may, however, be accounted for by a simple mathematical demonstration. All that is required is that an analytical expression shall be found for the alteration which the solar radiation into space undergoes through each of these two factors—the absorption of the atmosphere, and the action of eruptions and spots.

Mathematical expression for this antagonism. *Fundamental equation of the problem.*

12. Let A denote the quantity of radiation which, during a certain unit of time, is emitted from the photosphere, and let I be the quantity of radiation which, during the same interval, leaves the upper limit of the atmosphere. Then we have, according to Bouguer's equation,

$$I = Ae^{-as}, \qquad (1)$$

where a represents a certain constant peculiar to the solar atmosphere, and where s is proportional to its density.

Now, as this density increases with time, in consequence of progressive cooling, we may assume

$$s = s_0 + bt, \qquad (2)$$

s_0 and b being constants, and t representing the time. By substituting this value in (1), and by differentiating the equation with regard to t, we obtain

$$dI = -ab \,.\, Ae^{-as}\, dt \,. \quad = -ab \,.\, I\, dt. \qquad (3)$$

This relation represents the change during the infinitely small interval dt, to which the energy radiated from the sun to any point in the universe is subjected in consequence of the increasing reflective power of the solar atmosphere. The expression on the right of (3), taken with the opposite sign, then indicates the increase of radiation prevented from leaving the sun.

This increase of reflected energy, or decrease of radiated energy, is counteracted by the action of eruptions and spots in clearing up the atmosphere and thereby strengthening the power of outward radiation. If we denote by S the amount of radiation which by this action is allowed to escape into space during a unit of time, then the *total change* in the amount of energy radiated by the sun towards space may be expressed by the equation:

$$dI = \quad -\alpha I dt + S dt, \qquad (4)$$

where ab in (3) is replaced by α.

It now only remains to discover how S may be represented as a function of I. According to our second proposition, the number and intensity of eruptions and spots, and therefore the quantity of radiation, S, may be taken to be proportional to the amount of reflected heat. Let us take two moments, t_0 and t, and let S_0 be the special value of S at the moment t_0. The heat reflected during the interval $t_0 \sim t$ will then be represented by the integral

$$\alpha \int_{t_0} I dt. \qquad (5)$$

Consequently the total action of eruptions and spots during the same time must likewise be proportional to this integral. Now the difference between the amount of energy S_0 radiated away in unit time at the beginning of the interval and the amount S radiated away in the same unit time at the end of the interval must be proportional to the sum

of the forces which have acted upon the atmosphere during the whole space of time $t_o \sim t$. This difference is therefore also proportional to the integral (5), and thus we have

$$S = S_o + \alpha\beta \int_{t_o}^{t} I dt. \tag{6}$$

There is only one objection which may perhaps be urged against this consideration. As long as the interval of time $t_o \sim t$ is supposed to be finite, it may be difficult to perceive how the whole amount of radiation expressed by the integral (5) could have been reflected towards the photosphere. For it is obvious that during this finite interval the reflection is continuously being modified by the "clarifying" process going on at the same time in the atmosphere. But any doubts on this score may be removed at once by making the interval $t_o \sim t$ infinitesimally small. $S - S_o$ also becomes infinitesimally small, and thus the above equation assumes the form

$$dS = \alpha\beta \, I dt.$$

Now by differentiating equation (4) according to t we obtain

$$\frac{d^2 I}{dt^2} = - \alpha \frac{dI}{dt} + \frac{dS}{dt},$$

whence

$$\frac{d^2 I}{dt^2} + \alpha \frac{dI}{dt} - \alpha\beta \, . \, I = o, \tag{7}$$

which in this form may be taken to represent *the fundamental equation* of our problem.

Analytical formula for the variation of solar radiation into space.

13. The integration of (7) leads to the following result: we obtain

$$I = A \, . \, e^{\lambda_1 t} + B \, . \, e^{\lambda_2 t}, \tag{8}$$

where λ_1, λ_2 are the roots of the quadratic equation

$$\lambda^2 + \alpha\lambda - \alpha\beta = o,$$

whence

$$\lambda_1 = - \frac{\alpha}{2} \quad \sqrt{\frac{\alpha^2}{4} + \alpha\beta},$$

$$\lambda_2 = - \frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} + \alpha\beta}.$$

For a certain initial instant of time, $t = o$, we have

$$I_o = A + B, \tag{9}$$

and by subtracting (9) from (8) our integral becomes

$$I - I_0 = A\left(e^{\lambda_1 t} - 1\right) + B\left(e^{\lambda_2 t} - 1\right) \qquad (10)$$

Let us now look somewhat more closely at the constants in this equation. From the fact that α and β are certainly *positive* quantities, we infer that λ_1 must invariably be positive and λ_2 negative. Hence the function $(e^{\lambda_1 t} - 1)$ is always positive, whereas $(e^{\lambda_2 t} - 1)$ is clearly negative. Thus as t increases the first of these functions gradually attains larger positive values, while the second attains larger negative values. The two functions therefore tend to neutralise each other, thus expressing the action of just such an antagonism as was to be expected. We may conclude, then, that the arbitrary constants A and B have the same sign, and from (9) it follows at once that this sign must be positive.

Proof that this formula must have a *minimum* at a *finite* value for t.

14. From equation (10) a very important conclusion may be drawn, inasmuch as it can be demonstrated that the curve represented by it has a *minimum* for a *finite* value of t. By forming the first two differential quotients, we obtain

$$\frac{dI}{dt} = A\,.\,\lambda_1 e^{\lambda_1 t} + B\,.\,\lambda_2 e^{\lambda_2 t}$$

$$\frac{d^2 I}{dt^2} = A\,.\,\lambda_1^2 e^{\lambda_1 t} + B\,.\,\lambda_2^2 e^{\lambda_2 t}$$

By putting $\frac{dI}{dt} = o$, the time, (t_m), either of a minimum or of a maximum, is found from the relation

$$e^{(\lambda_1 - \lambda_2)\, t_m} = \frac{B}{A}\,\frac{\sqrt{\alpha^2/_4 + \alpha\beta} + \alpha/_2}{\sqrt{\alpha^2/_4 + \alpha\beta} - \alpha/_2}$$

Now $(\lambda_1 - \lambda_2)$ is positive under all circumstances: hence, in order that t_m shall be infinite, it is necessary that the expression on the right in the above equation should also be infinite. But this can never occur so long as $\beta > 0$; consequently t_m must have a *finite* value. Since, moreover, the expression $A\lambda_1^2\, e^{\lambda_1 t} + B\lambda_2^2\, e^{\lambda_2 t}$ is *positive*, whatever be the value of t, it is clear that the curve under discussion has a *minimum* at a *finite* value of t.

So far, then, the following peculiarity of the curve represented by (10) appears to be established:—

Whatever be the values of the constants A, B, α, β, this curve, starting from zero for $t = o$, gradually descends into the negative region of the system of co-ordinates, where, at a certain point, it attains a minimum. The presence of a minimum of course involves the return of the curve, from this moment, towards the positive side, so as again to approach zero.

The solar radiation must revert

15. Considerable interest attaches to the question as to whether the curve does actually vanish again, or, in other words, as to whether the outward radiation from the

sun, after having passed through a certain minimum, will ultimately attain the same value from which it started. Equation (10) gives a clear answer to this question.

to its *initial* value after a *finite* space of time.

It has already been shown that the expression $(e^{\lambda_1 t}-1)$ increases towards the *positive* side with increasing values of t, and becomes infinite when t is infinite.

On the other hand, the function $(e^{\lambda_2 t}-1)$ attains larger and larger *negative* values as t increases, and since $e^{\lambda_2 t}=0$ when $t = +\infty$, the function has the finite value -1 when t is infinitely great.

Hence, while the first of these functions tends towards *positive infinity*, the other can never be otherwise than a *finite negative* quantity. Thus it follows that our curve, which is the component of these two branches, must attain its original value, zero, *before* t becomes *infinitely great*.

The solar radiation into space cannot surpass a certain maximum.

16. Before we can decide whether or not the curve eventually *passes* through zero to positive values, we must define more precisely what we assume to be the initial instant of time t_0 adopted in derivation of equation (10). This initial moment is to be understood as that at which the atmosphere is just sufficiently absorptive to produce the state of thermal equilibrium in the photospheric layers. According to the first of our propositions, the outbreak of eruptions and formation of spots can only occur as long as the absorptive power of the atmosphere is still further increased. Hence the existence of the function S in equation (4) is possible only when $I < I_0$. So long, then, as I is equal to or greater than I_0, it naturally follows that $S=o$, and our differential equation, in these circumstances, reduces to

$$dI = -\alpha I dt; \qquad I - I_0 = A\,(e^{-at} - 1).$$

Hence, since S disappears as soon as $I \geqq I_0$, equation (10) is applicable only for the time during which $I < I_0$. The curve, therefore, represented by this equation can never extend so far as to pass over into the positive region; for, were it to do so, we should then have, instead of (10), the relation

$$I - I_0 = A\,(e^{-at} - 1),$$

from which it must be inferred that the curve must immediately begin to descend again.

Thus, as soon as the expression on the right hand side of equation (10) reverts to its original value *zero*, the applicability of the equation to the problem before us ceases, owing to the disappearance of S. That is to say, the action of eruptions and spots dies out, and for some time thereafter the absorption of the outside layers of the atmosphere is the only force affecting the intensity of the solar radiation. This absorption, as has been shown, increases with the time, and consequently there will again be a greater reflection of heat towards the photosphere, leading in time to a recurrence of the phenomenon which has been described as an overheating of the photospheric layers. As soon as this fresh process of overheating sets in, it produces the conditions necessary for a renewed outbreak of eruptions and spots. The

antagonism of forces is repeated, and equation (10) again becomes applicable for another period, lasting until the limiting condition $I = I_0$ is once more attained. In this way, changes in solar radiation, apparently partaking of a periodic character, will follow each other in infinite succession.

Fundamental formula not *periodic* in the proper sense.

17. It must, then, be clearly understood that this investigation does not furnish us with a strictly periodic function, in the proper sense of that term, but rather with a series of single branches of curves immediately succeeding one another, each of which starts from a certain initial value, and after a certain lapse of time returns to the same value. Thus, instead of one and the same cycle constantly and endlessly repeating itself, we observe an apparently coherent chain of individual appearances, which must in fact be taken as perfectly separate and independent—a result, as will subsequently subsequently be found, in perfect accordance with observational evidence.

The spot-variation proportional to the inverted curve of solar radiation.

18. Equation (10) represents the successive changes of solar radiation into space; and since, as a matter of course, whatever radiation is prevented from escaping outward must be preserved to the sun itself, the right-hand expression in (10), if taken with the opposite sign, will represent the variations of radiant energy acting on the photospheric layers. But, in accordance with our second proposition, the number and intensity of eruptions and spots is directly proportional to the amount of this energy. Hence we may express the successive changes in the frequency of solar spots—which frequency may be denoted by the letter r—by the following formula:

$$r = a \, . \left(1 - e^{\lambda_1 t}\right) + b \, . \left(1 - e^{\lambda_2 t}\right) \qquad (11)$$

Observational evidence that the spot-curve cannot be periodic in the proper sense.

19. From the above remarks it will appear that the spot-curve also is not periodic in the true sense of the word—viz., that it cannot be represented by a single equation with alternate maxima and minima, but consists of a series of detached branches connected in the manner just described. We should therefore be led to expect that the last feeble manifestations of spot-life dying away at the close of a so-called "cycle," should, as far as their originating forces are concerned, have nothing in common with the first reappearance of spots at the beginning of the following cycle. Now, observation shows that the first spots of a new cycle make their appearance in high solar latitudes, whilst the last vanishing members of the previous cycle are found in close proximity to the equator. This fact seems to indicate that the period commencing must be different from that just ending, and that, therefore, each period constitutes a perfectly independent phenomenon requiring a new branch of the curve for its geometrical representation.

Theoretical proof that the *ascent* in the spot-curve must be *more rapid* than the *descent*.

20. Since the expressions on the right-hand sides of equations (10) and (11) are identical in all but sign, it follows that if the curve represented by (10) be inverted, we obtain that represented by (11). Hence, the spot-curve, starting from zero, reaches eventually a maximum value, and from that point gradually descends again to its initial value, zero, which is attained after an interval of time to be denoted by p. This quantity, being the length of the period, is determined by the equation

$$o = a\left(1-e^{\lambda_1 p}\right) + b\left(1-e^{\lambda_2 p}\right),$$

from which we obtain

$$b = -a\frac{1-e^{\lambda_1 p}}{1-e^{\lambda_2 p}}.$$

By substituting this value in (11) we find

$$r = a\left\{1-e^{\lambda_1 t} + \frac{e^{\lambda_1 p}-1}{1-e^{\lambda_2 p}}\left(1-e^{\lambda_2 t}\right)\right\} \qquad (12).$$

It has been clearly established by observation that in the spot-curve the ascent from one minimum to the next maximum takes place in a shorter time than the descent to the subsequent minimum; or, in other words, that the time which elapses between a minimum and the following maximum is less than half the period p. We shall endeavour to elicit this peculiarity also from our theoretical equation.

For this purpose, the first differential quotient $\frac{dr}{dt}$ must again be derived. If this be equated to zero, we obtain as the equation of condition for determining the time of maximum, t_m,

$$e^{(\lambda_1-\lambda_2)t_m} - \frac{\lambda_2}{\lambda_1}\cdot\frac{1-e^{\lambda_1 p}}{1-e^{\lambda_2 p}} \qquad (13)$$

But if $t_m < {}^p/_2$, as we want to prove, we should also have the inequality

$$e^{t_m} < e^{p/2}, \quad \text{or } e^{(\lambda_1-\lambda_2)t_m} < e^{(\lambda_1-\lambda_2)p/2}$$

Hence, taking (13) into account, we have to show

$$\frac{\lambda_2}{\lambda_1}\cdot\frac{1-e^{\lambda_1 p}}{1-e^{\lambda_2 p}} < e^{(\lambda_1-\lambda_2)p/2}$$

or

$$\frac{\lambda_2}{\lambda_1}\cdot\frac{e^{\frac{p\lambda_1}{2}} - e^{\frac{-p\lambda_1}{2}}}{e^{\frac{p\lambda_2}{2}} - e^{\frac{-p\lambda_2}{2}}} < 1.$$

Now, by developing the exponential functions in series, the following relation is obtained:—

$$\begin{aligned} 1 + \frac{p^2\lambda_1^2}{2^2\,3!} + \frac{p^4\lambda_1^4}{2^4\,5!} + \frac{p^6\lambda_1^6}{2\ \ 7!} + \ldots\ldots\ldots\ldots \\ < 1 + \frac{p^2\lambda_2^2}{2^2\,3!} + \frac{p^4\lambda_2^4}{2^4\,5!} + \frac{p^6\lambda_2^6}{2^6\,7!} + \ldots\ldots\ldots\ldots \end{aligned} \qquad (14)$$

this inequality being true so long as $\lambda_1^2 < \lambda_2^2$. But we know that

$$\lambda_1^2 = \frac{\alpha^2}{2} + \alpha\beta - \alpha\sqrt{\frac{\alpha^2}{4} + \alpha\beta}$$

$$\lambda_2^2 = \frac{\alpha_2}{2} + \alpha\beta + \alpha\sqrt{\frac{\alpha^2}{4} + \alpha\beta},$$

where α and β are positive quantities. The condition $\lambda_1^2 < \lambda_2^2$ will therefore be satisfied under all circumstances. Hence the conclusion that in equations (11) and (12) the maximum must always occur before the middle of the period. Thus theory leads to a curve showing the same peculiarity as that derived from observation.

Representation of the *observed* mean curve of sun-spots by the theoretical formula.

21. Unfortunately no material can be obtained from spot observations sufficient to determine *all* the constants which appear in equation (12). These constants are four in number, namely, a, p, λ_1 and λ_2; whilst observational results afford only three sufficiently accurate data, viz., the length of the period, the time of maximum, and the spot-intensity at the epoch of maximum. Owing to this insufficient determination there will be an infinite series of equations of the form of (12) capable of satisfying the three observational data. This series, however, is reduced to a single equation as soon as a definite value is assumed for one of these constants.

Now, in an investigation such as this, undertaken in absolute ignorance of the numerical constants employed, it is of importance to show this much at least, that our theoretical equation, under certain conditions regarding the constants, *can* represent the observations, though we may not be able to ascertain whether or not these assumed conditions are really in accordance with fact.

Let us, for instance, take $t_m = 3{\cdot}2$ years, $p = 9{\cdot}2$ years,* and the relative frequency of sun-spots at maximum, $r_m = 100$. Equation (13) may be re-written in the form

$$\frac{e^{-\lambda_2 t_m}\left(1 - e^{\lambda_2 p}\right)}{\lambda_2} = \frac{e^{-\lambda_1 t_m}\left(1 - e^{\lambda_1 p}\right)}{\lambda_1}$$

If in this equation we insert the above values for t_m and p_1 and then give to λ_1 the successive values $+ 0{\cdot}01$, $+ 0{\cdot}02$, + ... $+ 0{\cdot}10$, we obtain a corresponding series of values for λ_2, as subjoined in the following table:—

* This number differs from Wolf's sun-spot period inasmuch as it does not represent the duration of one complete cycle of sun-spots for the whole surface, but only the time occupied by the appearance of spots in a relatively small part of the solar surface, as will be seen further on.

λ_1	λ_2
+0·010	−0·460
+0·020	−0·472
+0·030	−0·485
+0·040	−0·498
+0·050	−0·511
+0·060	−0·524
+0·070	−0·538
+0·080	−0·551
+0·090	−0·565
+0·100	−0·579
....	....
....	....

The value of a may be found from the equation

$$100 = a \left\{ 1 - e^{\lambda_1 t_m} + \frac{e^{\lambda_1 p} - 1}{1 - e^{\lambda_2 p}} \left(1 - e^{\lambda_2 t_m}\right) \right\}$$

If for the present we restrict our attention to three out of all the possible cases, and assume

$$\left.\begin{array}{l}\lambda_1 = +\ 0{\cdot}010 \\ \lambda_2 = -\ 0{\cdot}460\end{array}\right\} \qquad \left.\begin{array}{l}\lambda_1 = +\ 0{\cdot}050 \\ \lambda_2 = -\ 0{\cdot}511\end{array}\right\} \qquad \left.\begin{array}{l}\lambda_1 = +\ 0{\cdot}100 \\ \lambda_2 = -\ 0{\cdot}579\end{array}\right\}$$

we determine for each pair respectively the values

$$a = 2342{\cdot}0\,; \qquad a = 333{\cdot}3\,; \qquad a = 111{\cdot}4$$

By introducing these numerical quantities into equation (12) the following table of values for r is obtained:

t	r		
0·0	0·0	0·0	0·0
0·5	35·3	35·8	36·6
1·0	60·8	61·5	62·3
1·5	78·7	79·2	79·8
2·0	90·4	90·6	90·9
2·5	97·1	97·3	97·3
3·0	100·0	100·1	100·0
3·5	99·6	99·9	99·8
4·0	96·9	97·2	97·3
4·5	92·2	92·6	93·0
5·0	85·8	86·4	87·2
5·5	78·3	79·1	80·2
6·0	69·6	70·6	72·1
6·5	60·1	61·3	63·0
7·0	50·0	51·3	53·1
7·5	39·3	40·5	42·4
8·0	28·1	29·2	30·8
8·5	16·7	17·3	18·4
9·0	4·8	5·0	5·5
9·2	0·0	0·0	0·0

It will be perceived that the differences between the single values are quite insignificant. For this reason our further investigation may be confined to a consideration of the curve belonging to $\lambda_1 = + 0{\cdot}050$, given as the middle series in the above table.

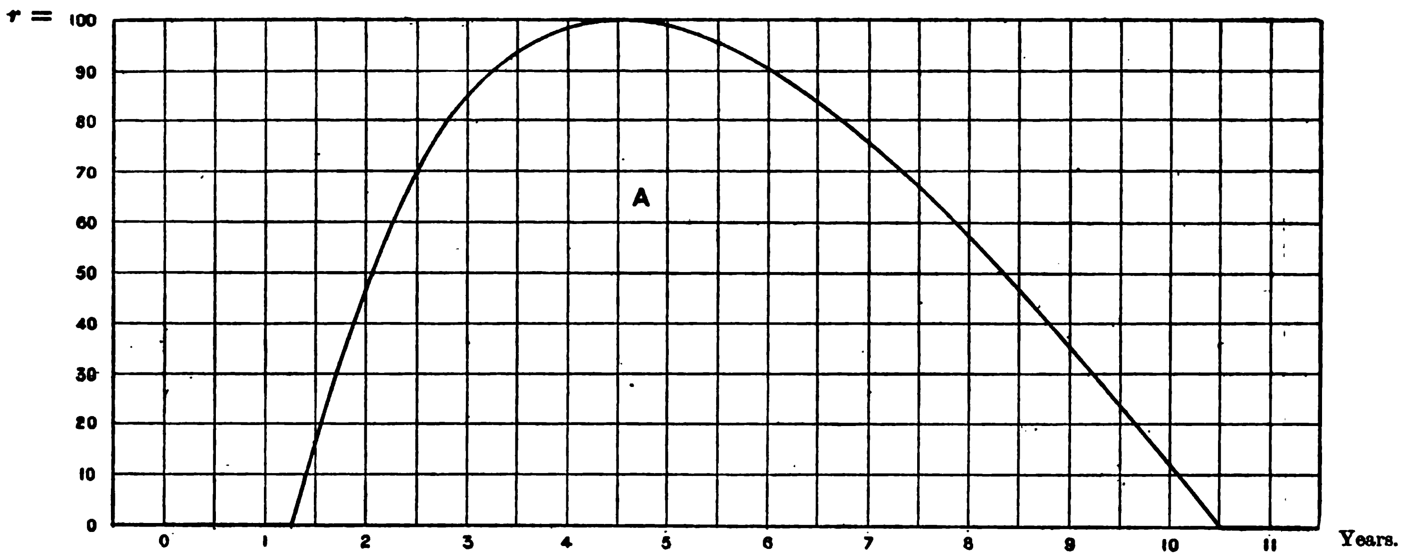
The curve representing these values cannot, however, as it stands, be taken as the true expression of the changes in the general development of spots.

For it has to be remembered that this phenomenon is spread over an entire central zone of the sun's surface extending to the parallels of $\pm 40°$. Thus it is only to be expected that at the end of a period spots situated on different parts of this vastly extended area will not disappear all at once, and that the generation of new spots at the beginning of the subsequent cycle will not take place at the same time in different localities.

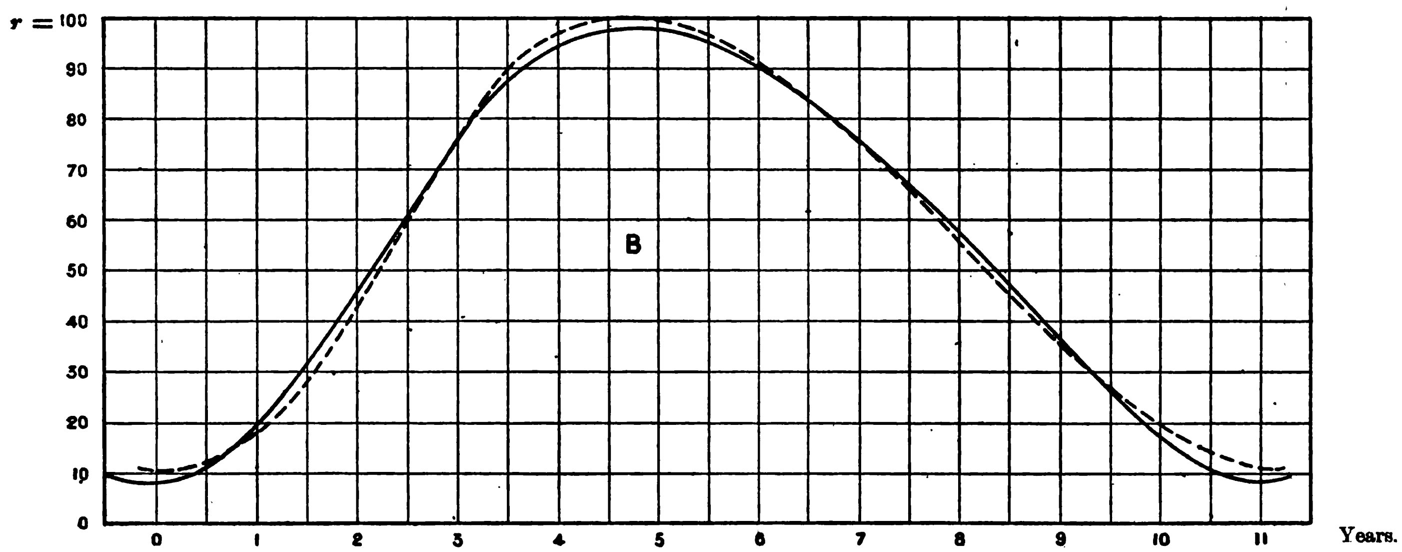
There will, as a rule, be districts where the last spots of one period have not yet vanished, when, in other places, the first signs of a fresh outbreak have already made their appearance. The general effect of this overlapping of periods will be that, even at a minimum, the mean spot-curve does not quite descend to zero, and that the transition from one cycle to the next does not take place *abruptly*, as theory would seem to require, but so gradually as to necessitate for its representation a smooth and rounded curve. The existence of such an encroachment of one period upon another is placed beyond all doubt by observational evidence. (*Cf.* Spörer, "Sonnenfleckenbeobachtungen"; Publ. d. Astrophys. Observatoriums zu Potsdam, Band IV., II. Theil, pp. 412-414.) Now, the theoretical equation deduced above takes no account of this phenomenon, but in fact represents the course of the spot development only in one particular part of the spot-zone. The mean spot-curve, on the other hand, as derived from observation, must be taken to be the component of a series of this kind of district-curves partly overlapping each other. Hence, in order to represent the actual mean spot-curve by our theoretical equation, we must make allowance for this effect of overlapping. To this end the following method has been adopted. The first curve of the annexed plate represents the values of the theoretical equation for the particular case $\lambda_1 = + 0{\cdot}050$. In order, however, to bring the time of maximum into accord with observation, the ordinates have been displaced by 1·3 years, this quantity having accordingly to be added to the times given in the above table, to reduce the middle series of values shown therein to agreement with the curve A. The zero-value consequently appears at $1^{y}{\cdot}3$ and at $10^{y}{\cdot}5$. To fill up the remaining gap of about 1·9 years between the mean period of 11·1 years and the particular district-period of 9·2 years assumed above, we assume the eruptive forces in the special district considered to be too weak during that interval of low atmospheric reflection to produce spots in any appreciable number. Such an assumption seems perfectly warranted; it simply means that, in a certain district, after the spot-generating forces have died out, some time is required before a revival of eruptive energy attains sufficient strength to produce fresh spots.

We thus take the curve A as a representation of the changes in the development of spots within a certain district of the spot-area. Now, as already stated, there will be other districts *earlier* and others again *later* in their stage of spot-development

Plate I.



Theoretical District curve of Solar Spots.



——— General Theoretical curve of Solar Spots.

...... Observed curve of Solar Spots (Spörer).

than the particular area under discussion. To represent the successive changes within these various districts, the curve A must be repeated in positions respectively in advance of and behind the central curve. To this end, the ordinates of the curve A are tabulated in column IV. of the following scheme, while these same values are arranged in columns I., II., III. at intervals of $\frac{1}{2}$, 1, and $1\frac{1}{2}$ years *before*, and in columns V., VI., VII. at intervals of the same extent *after*, the times *t* corresponding to the figures of the central column.

t	I.	II.	III.	IV.	V.	VI.	VII.	Σ	Reduction to Spörer's Scale.
0·0	24	12					16	52	8
0·5	12					16	47	75	11
1·0					16	47	70	133	20
1·5				16	47	70	85	218	32
2·0			16	47	70	85	94	312	46
2·5		16	47	70	85	94	98	410	61
3·0	16	47	70	85	94	98	100	510	76
3·5	47	70	85	94	98	100	99	593	88
4·0	70	85	94	98	100	99	95	641	95
4·5	85	94	98	100	99	95	90	661	98
5·0	94	98	100	99	95	90	83	659	98
5·5	98	100	99	95	90	83	76	641	95
6·0	100	99	95	90	83	76	67	610	90
6·5	99	95	90	83	76	67	57	567	84
7·0	95	90	83	76	67	57	47	515	76
7·5	90	83	76	67	57	47	36	456	67
8·0	83	76	67	57	47	36	24	390	58
8·5	76	67	57	47	36	24	12	319	47
9·0	67	57	47	36	24	12		243	36
9·5	57	47	36	24	12			176	26
10·0	47	36	24	12				119	18
10·5	36	24	12					72	11
11·0	24	12					16	52	8
0·5	12					16	47	75	11
1·0					16	47	70	133	20
1·5				16	47	70	85	218	32
2·0			16	47	70	85	94	312	46
2·5		16	47	70	85	94	98	410	61
3·0	16	47	70	85	94	98	100	510	76

According to Wolf, the period is $11\frac{1}{9}$ years, but to simplify our computations we may safely adopt 11·0 years, this value being within the probable error of Wolf's period.

The above scheme may thus be explained in this way. We assume *seven* eruptive districts, I., II., - - - - VII., distributed over the whole spot region of the solar surface. The district VII. is supposed to be the first on which the spot-phenomenon appears. It is followed half-a-year later by VI.; after a similar interval, V. becomes affected, and so on. Comparing I. with VII., we see that the old cycle at I. has not yet expired when a new period has already set in at VII. In fact, the phenomenon of spots in district I. does not cease till one year after the

new mean period has begun; whilst $0^{y}\cdot 3$ before the end of this cycle, a fresh development again appears at VII. Thus these two extreme districts overlap each other to the extent of 1·3 years—an amount in accordance with the observations of Carrington and Spörer. A very clear idea of this peculiar phenomenon of overlapping may be obtained from an inspection of a table published by Spörer in the "Publicationen des Astrophysicalischen Observatoriums zu Potsdam," IV. Band, II. Theil, p. 414. This interesting table shows to evidence that a sub-division into separate groups of the *observed* spot-material leads to a result similar to that embodied in the rows of figures deduced above from our *theoretical* discussion.

The choice of *seven* eruptive districts is of course an arbitrary one, as is also the interval of half-a-year at which they are supposed to follow each other as regards their development of spots. A larger number of districts might have been chosen, and also a much smaller interval of time; the only effect, however, would have been to make the final numbers somewhat more regular. But the adopted sub-divisions are sufficiently numerous to furnish accurate ordinates for the final theoretical spot-curve. These ordinates are derived at once by forming the sum of the figures in each horizontal row, as given in column Σ.

To admit of a direct comparison with Spörer's curve, as deduced in his paper "Periodicität der Sonnenflecken, Häufigkeitscurven, Epochen und mittlere Länge der Periode," (Astr. Nachr. Nr. 2335) from the material so carefully compiled by R. Wolf, the figures given under Σ have been reduced to Spörer's scale by dividing them by 6·78. A comparison of the two curves is exhibited in diagram B of the annexed plate (Fig. I.), where the full curve represents the figures given in the last column of the above table, while the dotted one is formed from Spörer's figures as deduced in the paper just referred to. The differences between the two curves are so slight that their general agreement may be taken as practically perfect.

B.—*The Great Period of Sunspots.*

Difference in the display of eruptions and spots in different cycles. Wolf's "great" period of sunspots.

22. Whilst the object of the foregoing discussion was to propound a physical explanation of the available observational facts regarding the eleven years' fluctuations of solar activity, another peculiar feature of the sunspot curve has still to be accounted for before the new theory can presume to claim a serious consideration on the part of astronomical science. There remains the question as to how far the theory can offer a sufficient explanation for that other periodic fluction generally known as the *great* period of sunspots.

An examination of the data collected by *Wolf* shows that in different cycles the development of spots may vary considerably in intensity. Thus, for example, in 1817 the maximum shows a relative number of only 39, while that of the year 1837 gives no less than 138. An investigation of the whole of the then available material led Wolf to conclude that besides the eleven-years' cycle there exists another and greater cycle

which, as he was inclined to believe, embraces about six of the ordinary periods. The reality of this greater period, first pointed out by *Wolf*, has, we think, been amply corroborated by the great differences of relative activity exhibited in the spot-development of more recent cycles. Obviously, then, while the disturbing forces at work in the sun are sometimes relatively weak, they must at other times display a vastly greater amount of energy.

Causes of the *great* sunspot cycle. Influence of temperature-changes in the superficial layers on the intensity of the circulating currents.

23. It has hitherto been tacitly assumed that the intensity of the currents circulating between the surface and the interior of the sun remains constant whatever may be the changes in the temperature and opacity of the atmosphere, or the consequent variations in the temperature of the photospheric layers. On this point, however, our theory evidently requires some modification. The continuous variations in the temperature of the superficial layers, caused, as we have tried to show, by the changing opacity of the atmosphere, cannot but exercise a certain influence upon the intensity of these currents. If a process of cooling is really going on in the layers of the solar atmosphere, and if at the same time, in consequence of this cooling, a certain quantity of heat is prevented from escaping into space, and is returned to the photospheric regions, the oscillations of temperature brought about by the antagonistic action of these two phenomena must have an appreciable influence on the convection currents to and from the sun's interior. In fact, as a consequence of this antagonism these currents must exhibit periodic changes in their intensity, quite similar to those already shown to exist in the case of outward radiation.

By no means, however, does it follow that the length of the period is the same in both cases. For as regards the convection currents, the varying forces affect a very much greater part of the solar mass than the relatively thin superficial stratum which we had under discussion in the previous investigation; and hence the cycle of convectional activity may be expected, in all probability, to be *greater* than that of eruptive activity.

As a natural consequence of the previous part of our theory, it would thus follow that the intensity of the currents circulating between the surface and the interior of the solar body must undergo periodic fluctuations. To what extent can these fluctuations affect the phenomena of eruptions and spots?

From what has been stated, these outbreaks must be expected to occur the more vigorously, the stronger the reflection of heat towards the photosphere, or, what is the same thing, the greater the rate of cooling in the absorbing envelope. Now, if the convection currents are relatively feeble, there will be a diminished heat-supply from the interior. Outward radiation must therefore be followed by a more rapid exhaustion of the store of radiant energy present in the surface layers, and, as a result, the rate of cooling must become greater.

In the case of energetic convection, on the contrary, radiation into space is more efficiently counterbalanced by heat from the interior, and thus the rate of surface-cooling must become smaller. Hence it appears that at those times when the

circulating currents are weak the intensity of eruption- and spot-phenomena must be great, and *vice versâ*.

These considerations afford a simple explanation of Wolf's so-called "great" period—*i.e.*, of the fact that the intensity of the development of eruptions and spots varies in successive cycles, the period of this variation being much greater than eleven years.

24. It will be remembered that the theoretical equation derived on p. 89 proves the existence of this peculiarity in the spot-curve, that it rises from a minimum to the next maximum in less time than it requires to descend to the subsequent minimum. We may now go a step further in the interpretation of inequality (13), for obviously this inequality becomes the greater, and consequently the maximum occurs the sooner, the larger the difference $\lambda_2^2 - \lambda_1^2$. Now, inasmuch as a weak circulation of the convection currents involves a rapid cooling of the atmosphere, it follows that in our former relation (*see* p. 84)

Theoretical proof that the time of maximum must be earlier—the greater the activity of eruptions and spots—during a cycle.

$$s = s_0 + bt$$

the co-efficient b will, in such a case, be larger than at other times when the circulating currents are more powerful. Hence the quantity a in differential equation (7), which is proportional to b, will have a *greater* value when the circulation is *weak*.

Now

$$\lambda_2^2 - \lambda_1^2 = 2a\sqrt{\frac{a^2}{4} + a\beta};$$

this difference therefore increases as a increases, and hence the result, that the maximum must occur earlier in periods which are characterised by a copious development of spots, than in those where the number of spots is small.

We have thus reached by a theoretical demonstration two very important conclusions regarding the time of maximum in a sunspot cycle.

(i.) The maximum always occurs before the middle of the period.

(ii.) It is reached the sooner, the stronger the development of eruptions and spots in the cycle under consideration.

25. The first of these results has already been shown to be amply corroborated by observational evidence. As regards the second, sufficiently strong indications of its truth may also be discerned in the long series of spot-waves which R. Wolf's tables have placed at our disposal. The simplest way in which to illustrate this result from the observations might perhaps have been to ascertain the exact times of maxima and minima in each cycle. A comparison of the ratio $\frac{\text{Time of max.} - \text{Time of preceding min.}}{\text{Time of min.} - \text{Time of preceding max.}}$ with the intensity of spot-development at the time of maximum should then yield the observational confirmation which we desire. This method, however, is not entirely free from objection, as the determination of the exact times of maxima and minima is usually a matter of some uncertainty, especially when small accidental oscillations are present. This drawback may, however, be partially avoided by adopting a somewhat different

Test by observations.

method. For this purpose let the straight lines AD, BC be drawn parallel to the axis of abscissæ, and respectively close to the points of minimum and maximum in the sun-spot curves representing the values tabulated by Spörer (Astr. Nachr., 2335). Then the smaller the quotient $\frac{AB}{CD}$ the earlier does the maximum occur relatively to the middle of the period.

A.

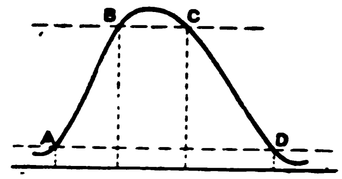


Fig. 1.

Now, if the theoretical result (*ii.*) be correct, this quotient must vary inversely with the intensity of the spot-display in the single periods. That such a remarkable law does actually disclose itself in the observations may be seen from the following figures derived from Spörer's tables. The intensity of the spot-phenomenon is here expressed by the number of spots given by Spörer for the time of maximum in each period. To facilitate the comparison of the two sets of numbers, the quotients are arranged according to the corresponding spot-intensities instead of in chronological order.

Spot-intensity r at time of maximun.	Quotient $= \dfrac{\text{Time required for ascent.}}{\text{Time required for descent.}}$
138	0·46
126	0·45
123	0·50
98	0·67
96	0·64
94	0·65
89	0·63
68	0·90
67	0·85
64	1·18
51	0·95
38	1·00

These figures would seem to leave no doubt as to the general correctness of our two conclusions. As regards the result stated in (*i.*) we find an exception at $r = 64$, where the quotient is greater than unity. Although such a value is at variance with the theory, no objection can reasonably be raised against our deductions from a single exceptional case. It ought, moreover, to be mentioned that this exception appears in the spot-cycle whose maximum occurred in 1829; and, as it happens, there are great discrepancies in the results of different observers regarding the moment of maximum in this particular period.

Derivation of the "great" sunspot period from observations.

26. The consecutive changes in the spot intensity r at the time of maximum in different cycles will give us an idea of the nature of the "great" period of solar spots. Indeed, by drawing a curve with these intensities as ordinates and the times as abscissæ we obtain a clear notion of the successive variations in the force of the circulating currents.

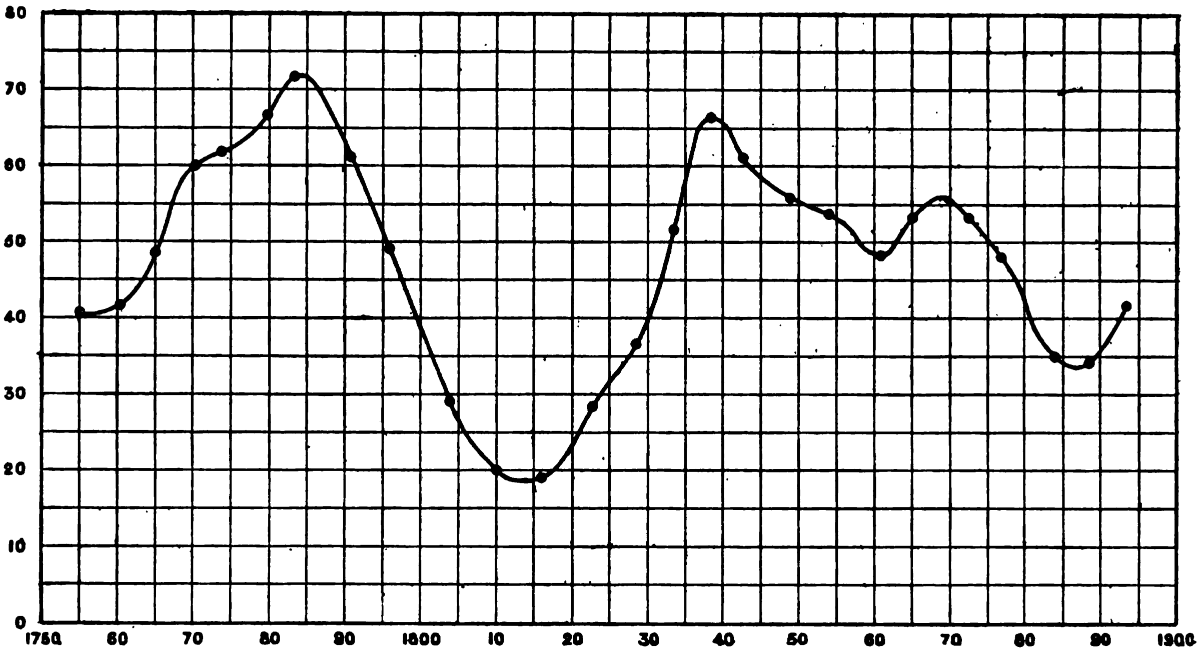
It may, however, be preferable for this purpose to adopt Wolf's plan of forming the sums of the monthly relative spot-numbers between each two neighbouring sunspot maxima on the one hand, and minima on the other. The quotients obtained by dividing

these sums by the number of months elapsed in each case, give approximate values for the differences in the intensity of the spot-development in the different eleven-years' cycles. Thus, the peculiar curve formed by these quotients (which are designated as ρ) is the representation of the "great" sunspot period—the smallest quotient corresponding to a minimum of this cycle, and the largest to a maximum.

The subjoined table contains the values of ρ from 1750 to 1898. Up to 1870, the figures are taken directly from the last two columns of Table V. in Astr. Mitth. XLI., p. 46; the figures for the years subsequent to 1870 were computed from the relative spot-numbers published by Prof. Wolf and by his successor, Prof. Wolfer, in later parts of the same valuable collection.

Epoch.	ρ	Epoch.	ρ
1750-1761	41	1823-1833	37
1755-1766	42	1829-1837	52
1761-1769	48	1833-1843	67
1766-1775	60	1837-1848	61
1769-1778	62	1843-1855	56
1775-1784	67	1848-1860	54
1778-1788	72	1855-1867	48
1784-1798	62	1860-1870	53
1788-1804	49	1867-1878	53
1798-1810	29	1870-1884	48
1804-1816	20	1878-1889	35
1810-1823	19	1884-1893	34
1816-1829	28	1889-1898	42

Fig. 2.—Great Wave of Solar Activity.



The periodic character of these figures is unmistakable. They rise from a minimum near the middle of last century to a high maximum in 1783, then rapidly descend to a low minimum in 1816, attain subsequently another high maximum in 1838, and so on.

These values are laid down in Fig. 2, and are connected by a smoothed curve, which may be accepted as indicating the changes from one value to the next.

In accordance with our theory, this curve, taken inversely, represents the changes in the intensity of the currents circulating between the surface and the interior of the sun.

Variations in the length of the sunspot period: their explanation found in the varying intensity of the circulating currents.

27. It has already been stated that it is impossible to determine all the constants of equation (12). In particular, we can obtain no theoretical information regarding the absolute length of the sunspot period. But there is one point in this connection on which we are not left entirely in the dark.

It is known from Wolf's researches that the various cycles differ considerably as regards the length of their periods, the range of which is found to be from 7 to 16 years.

The possibility of this difference being due to the influence of the long-period fluctuations of solar activity was likewise first pointed out inductively by Wolf. By a graphic process he proved the reality of a connection between the length of the period and the intensity of spot-development in the individual cycles. His investigation led him to conclude that the spot-period was shorter according as the development of spots was more vigorous. But the necessity for such an inter-relation is also clearly indicated by the foregoing theoretical considerations. It will be remembered that the cessation of eruptions and spots at the end of a cycle indicates a restoration of the state of thermal equilibrium in the photospheric layers. This condition, however, is only transitory. The deficiency of heat-supply from the interior, due to slow contraction of the sun's mass, promptly re-creates the conditions of unstable equilibrium, thereby giving rise to a fresh outbreak of eruptions and spots which signalise the commencement of a new cycle. In the light of these facts, we cannot well avoid the belief that the *time* required for the reproduction of the conditions of unstable equilibrium in the photosphere depends on the rate at which heat is supplied to the surface from the inner parts of the sun. If this supply is relatively small, the cooling of the superposed atmosphere increases rapidly, consequently the outward radiation of heat from the photosphere and from the layers immediately above it suffers greater retardation, and thus the new state of unstable equilibrium sets in at a time comparatively soon after the expiration of the old cycle.

If, on the other hand, the heat-supply from the sun's interior is relatively large, the secular cooling of the atmosphere takes place more slowly, and hence an appreciably longer time is required for the genesis of those conditions conducive to the beginning of a new era of the spot-phenomenon. We therefore conclude that at times when the convection currents are relatively feeble, the spot-cycles will follow each other in more rapid succession than at other times when these currents are more powerful.

Agreement between theory and observation.

28. Now, we have seen that the curve of Fig. 2, taken *inversely*, represents graphically the varying intensity of the convection currents: whence it follows that the curve showing the variations in the lengths of the periods must run parallel to this inverted curve.

There are, of course, two ways in which the length of the period can be determined: (1st) as the interval elapsing between two successive minima, and (2nd) as the similar interval between two successive maxima. But in the latter case it has to be borne in

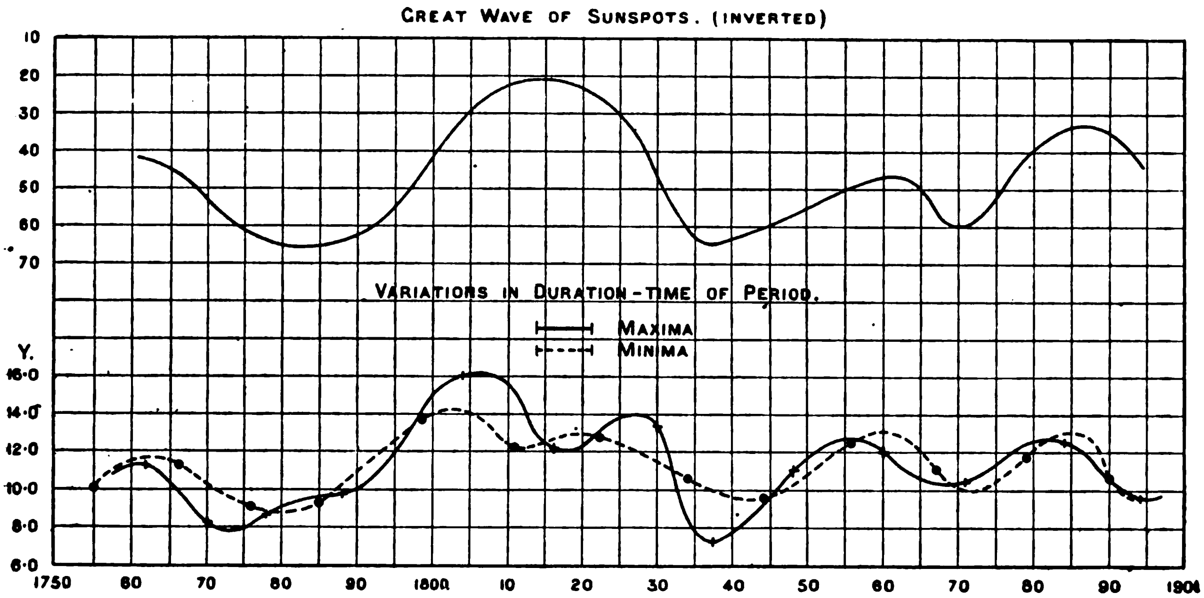
mind that the position of the maximum itself varies with respect to the middle of the period. This influence naturally tends to still further augment the differences in the lengths of the periods, so that the curve representing these has a greater range than that obtained from the corresponding differences determined by means of the minima.

The following table shows the lengths of the sunspot period for various epochs, as obtained from observation. It is transcribed from Wolf's table in Astr. Mitth., XLI., p. 40, and from later numbers of the same collection.

Epoch.	Length of period as deduced from *minima*.	Epoch.	Length of period as deduced from *maxima*.
1755	$10^{y}\cdot2$	1762	$11^{y}\cdot2$
66	11·3	70	8·2
76	9·0	78	8·7
85	9·2	88	9·7
98	13·6	1804	16·1
1811	12·3	16	12·2
23	12·7	30	13·5
34	10·6	37	7·3
44	9·6	48	10·9
56	12·5	60	12·0
67	11·2	71	10·5
79	11·7	84	12·4
90	10·7	94	9·5

The curves representing these data are given in Fig. 3 in comparison with the inverted

Fig. 3.



curve of the great wave of sunspots. The close connection between the two phenomena is thus rendered evident to the eye.

Concluding Remarks.

29. We have now arrived at the end of this investigation concerning the periodicity of solar phenomena.

It has been attempted to show how the fluctuations of the solar forces find a satisfactory explanation on the single assumption that the action of the gravitational forces within the sun is insufficient to compensate for the loss of energy by outward radiation without the aid of the light- and heat-conserving power of an absorptive atmosphere. The proposed theory does not, however, confine itself to demonstrating the law of periodicity in a merely general way; it goes farther, and enables us to theoretically derive, by means of a simple analytical deduction, the principal features peculiar to the periodic waves, which, although long known empirically from the researches of Carrington, Wolf, and Spörer, have hitherto eluded all attempts to refer them to one fundamental principle. As regards the further discussion of our problem, we have still two questions to answer:—

(*i.*) How does the heat reflected from the atmosphere towards the photospheric layers give rise to the formation of eruptions and spots?

(*ii.*) What are sunspots in the light of this theory?

A consideration of these important questions will be attempted in the remaining sections of this paper.

SECTION III.

The Causes of Solar Eruptions and Spots and their Heliographic Distribution.

Solar eruptions caused by an unstable equilibrium of the photospheric layers.

30. It is well known that the maintenance of stable equilibrium in an atmosphere is possible only so long as the temperature-gradient does not exceed a certain maximum of steepness. When this limit is reached, the state of equilibrium becomes neutral, and for a still higher gradient it is unstable.

As we have seen, the conditions engendered in the superficial layers by the sun's temperature reaching this last stage are those directly associated with the phenomenon of eruptions on the solar surface.

Analogy with terrestrial cyclones.

The causal agency in the case of these eruptions must therefore be considered as closely allied to the generally assumed origin of terrestrial cyclones, particularly of those occurring within the tropical zone. Eminent meteorologists as Ferrel, Reye, Mohn, and Blanford are agreed that "the primary cause of cyclone formation is the production and ascent of a large quantity of vapour, which is condensed, with the liberation of its latent heat, over the place of its production, instead of being carried away to some distant region."*

Most of these cyclones originate in the belt of calms, viz. in that region where, under the influence of a tropical sun, the air near the surface gets heated and saturated.

* Blanford—"The Indian Meteorologist's Vade-Mecum," Part II., p. 155.

As long as the free ascent of this heated air and its subsequent outflow towards the poles is not interfered with, there is nothing to cause a disturbance of the equilibrium. But supposing that some force were to partly obstruct the upward motion of the particles and thereby prevent the cooling which would otherwise result from their expansion, then the heated and saturated air, instead of being carried away, would be forced to accumulate over the surface, until, by the progressive increase of the vertical temperature-gradient, the equilibrium of the atmosphere in that locality eventually became unstable. The stronger the obstructing force happened to be, the more would the gradient be increased, until at last the tendency of the over-heated surface layers to break through those overlying them would become so intense that even the slightest disturbance in this direction would suffice to set the whole mass suddenly in rapid upward motion. When this motion has once begun, the amount of aqueous vapour contained in the ascending column becomes a matter of the highest importance. For it is this vapour which, by its condensation, liberates heat, and thereby tends to increase the energy developed in the ascent. The upward velocity of the air in a cyclone depends, therefore, mainly on two factors: (*i.*) on the excess of the temperature-gradient, and (*ii.*) on the amount of moisture in the ascending current.

Let us now consider the corresponding conditions prevailing on the sun.

The amount of heat supplied to the surface is continuously kept up by a powerful system of currents circulating between the interior and the superficial layers of the solar body, and the velocity of these currents must be the principal agency in regulating the momentum of the individual particles floating in them. Thus, even when the tendency of such particles to expand is strengthened through their becoming overheated by reflection, they yet cannot respond freely to this increase of molecular force, but must be irresistibly swept along in their path with just that impulse which is imparted to them by the velocity of the surrounding current. In this way these currents act upon the particles contained in them as a most powerful impediment to their increased upward tendency. Thus, in exact analogy to the conditions produced in our own atmosphere during the preparatory stages of the generation of a cyclone, the overheated particles on the sun are forcibly detained in levels inconsistent with their state of increased temperature, so that their state of equilibrium is rendered unstable.

It is obvious that the degree of steepness in the temperature-gradient thereby produced depends in great measure on the power of the circulating currents. Although this power must certainly be very great, a time must at last come when the ever-strengthening upward tendency of the overheated particles will counterbalance the action of the currents; and when even a slight disturbance will be sufficient to bring about the upward motion so long restrained, thus giving rise to a solar eruption.

A Cyclonic theory first propounded by Reye.

31. It is far from our intention to represent this explanation as entirely new. A somewhat similar theory was, for instance, suggested almost thirty years ago by Prof. Reye, in his excellent treatise "Die Wirbelstürme, Tornados und Wettersäulen in der Erd-Atmosphäre mit Berücksichtigung der Stürme in der Sonnen-Atmosphäre, 1872." But Prof. Reye at the time was unable to demonstrate definitely the existence on the

solar surface of the first and fundamental essential demanded by the cyclonic theory—viz., the frequent occurrence of a state of unstable equilibrium in certain areas. He assumed the existence of such a state as highly probable, but did not go so far as to venture a hypothesis regarding its origin. For this reason, probably, his theory did not receive sufficient attention on the part of astronomers.

Our present considerations give a different aspect to the question. The existence in the sun from time to time of regions in a state of unstable equilibrium appears to follow as an inevitable necessity in the light of our theory. Thus we are impelled to the conclusion that the natural consequences of such a state—viz., sudden and forcible exchanges of matter between the lower and higher layers—are just as certain to exist on the sun as in our own atmosphere.

This theory reconcilable with the gaseous constitution of the sun. The velocities in solar eruptions.

32. This explanation at once removes the great difficulty which has always been experienced in understanding how eruptions can take place in a body which is certainly for the most part gaseous. The fact that the velocity exhibited in these eruptions is often so stupendous as to be quite beyond the grasp of human conception, cannot be brought forward as an argument against the cyclonic theory. For the enormous amount of mechanical energy evolved even in the case of our terrestrial cyclones often far exceeds the scale to which we are accustomed in ordinary meteorological phenomena. Prof. Reye has computed the work done during the three days from the 5th to the 7th October, 1844, in the carrying of the air towards the centre of the disturbance occasioned by the great Cuban hurricane of that date. He finds that it amounts to no less than 473·5 millions of horse-power, or, as he adds:—"mindestens 15 mal so viel als alle Windmühlen, Wasserräder, Dampfmaschinen und Locomotiven, Menschen- und Thierkräfte der ganzen Erde in der gleichen Zeit leisten." (Reye, "Die Wirbelstürme," etc., p. 121.)

Now, if such vast mechanical force can be generated under the very moderate temperature-conditions prevailing on our globe, how vastly more stupendous must be the power developed in a solar outbreak, where perhaps thousands of degrees of temperature difference come into play, and where the disturbed medium is not moisture-laden air at 70° – 80° F., but glowing gases saturated with metallic vapours at a temperature absolutely unimaginable by us. In view, then, of such a gigantic scale, our incapacity to realise the extreme velocities observed in so many solar eruptions can assuredly not be used as an argument against the theory, for such an immeasurable cause must obviously give rise to an altogether immeasurable effect.

Influence of the sun's rotation on the distribution of eruptions. Explanation of the preponderance of equatorial outbreaks some time after the beginning of a cycle.

33. Having thus explained the existence of cyclonic outbreaks on the sun as a necessary consequence of the interaction of the solar atmosphere and the layers adjoining the photosphere, we shall now proceed to trace the distribution of these eruptions on the solar surface.

If the sun did not rotate on its axis, and if it were uninfluenced by bodies outside itself, the probability of the occurrence of an eruption would be the same for every point of its surface. On a rotating globe, however, the conditions are altogether different. Owing to the greater centrifugal force at the equator, the reflecting atmosphere in this

region must be somewhat deeper, and the total number of reflecting particles correspondingly larger, than in higher latitudes. This circumstance will *a priori* tend to enhance to some extent the conditions favourable to the occurrence of outbreaks in the equatorial regions. The centrifugal force on the sun is no doubt small, but the *intensity* of this force has no important bearing on our question. However small may be the advantage possessed by the equatorial regions in this respect, it is quite sufficient for our purpose to know that there is such an advantage at all.

For, as an immediate consequence of an upheaval at any point, there will obviously be an indraught of gases from every side towards the centre of rarefaction. This is true not only of the photospheric layers, but also of the overlying atmosphere, which is likewise penetrated by the eruption. Hence, if equatorial outbreaks show a preponderance, however slight, in frequency and intensity, they must induce an inflow of reflecting atmospheric matter from higher latitudes towards the equatorial regions.

Since, then, by this influx of absorptive matter, the amount of heat-reflection towards the photosphere must steadily increase at the equator, the conditions requisite for subsequent outbreaks will become more and more favourable there, whilst in higher latitudes the very opposite will be the case; and however insignificant the excess of equatorial disturbances may be at first, it is bound in course of time to become gradually more and more prominent.

Explanation of Janssen's "réseau photospherique."

34. As has been stated, one effect of an eruption must be that the photospheric matter is drawn with great vehemence towards the centre of the cyclonic disturbance. This explains a very remarkable phenomenon first recorded by Janssen,* for which, so far as we are aware, no explanation has yet been offered.

The phenomenon in question is the so-called "réseau photospherique." In the annexed figure (4a), suppose an eruption to originate and to attain its maximum intensity at the centre of each circle, while the circumference bounds its sphere of effective activity. All the points not included in one or other of the circles must then be supposed to remain unaffected by the eruptions in question. If several of these happen to take place in such close proximity to one another that their fields of action overlap, we obtain the formation represented in Fig. 4a.

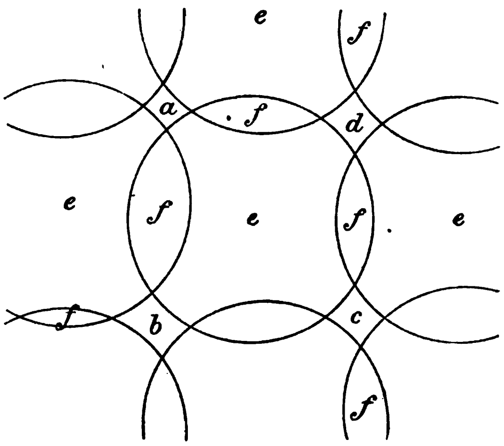


Fig. 4a.

All the points within the small spherical quadrilaterals a, b, c, d remain undisturbed by any of the outbreaks, and the solar surface in these places shows the appearance of normally developed granulation, in which the bright, well-defined granules contrast sharply with the dark interstices. On the other hand, the nearer we approach the centres (e, e) of eruption, the more diffuse and indistinct does the granular structure become.

* "Annales de l'observatoire d'astronomie physique de Paris," Tome premier.

But especially peculiar is the appearance of the surface within the lunes *f*, *f*, *f*. For since these areas fall at one and the same time within the spheres of action of two neighbouring eruptions, a secondary depression must evidently occur near *f*. Hence the currents starting from *a*, *b*, *c*, and tending towards the centres of eruption, will to some extent be first directed towards the seat of this secondary depression, thus presenting the appearance of bright and dark trails pointing in the direction of the major axes of these lunes.

We have tried to illustrate roughly this explanation of the phenomenon in Fig. 4b, the close resemblance of which to M. Janssen's photographs, and particularly to that of Plate IV. of the "Annales," cannot fail to be recognised.

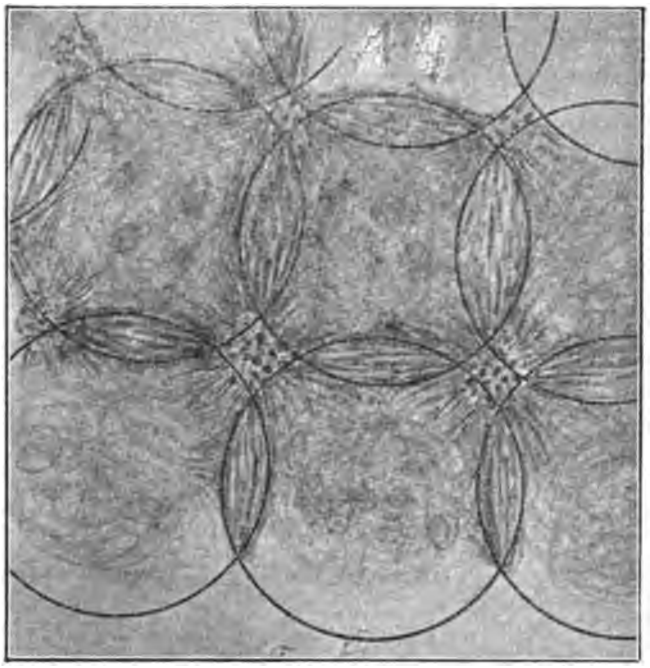


Fig. 4b.

Existence of surface currents with meridional direction flowing towards the equator as well as towards the poles.

35. Let us now consider more closely the conditions existing at the moment when the equatorial outbreaks have clearly begun to outnumber those in higher latitudes. Whenever an eruption occurs, the ejected masses must be supposed to fall back towards the photosphere. But in consequence of the sun's rotation they will not strike the photosphere at the same point from which they were thrown up, but must impinge on a place somewhat to the east of the starting point, as viewed from the earth. In the diagram 5, let the plane of the paper represent a portion of the solar surface, and suppose that the matter ejected from an eruptive district at C′ on the equator arrives once more on the photosphere at C, lying farther east. It is practically certain that besides C′, there will be other districts situated along the equator which are simultaneously in a condition favourable to the formation of eruptions. Let C″ be one of these,

the ejected matter from which reaches the photosphere again at a point still farther east.

When the material ejected from C′ impinges on the photosphere at C, it will spread out in all directions, forming a net of currents emanating from C. The diagram shows how these currents are influenced by the general inrush of matter towards the partial vacua produced at C′ and C″. Obviously the currents arising from impact at C will tend to be deflected *towards* the cyclonic centres, and the more so the smaller the inclination of the current to the equator. Thus those currents setting out from C in the line of the meridian will be least affected by the lateral suction caused by the eruptions at C′ and C″, and will deviate but little from their original course carrying them from the equator towards the poles, that is to say, away from the sphere of influence of cyclonic disturbances altogether. Now, owing to the conflux of reflecting matter towards C′ and C″, the conditions necessary for outbreaks at these localities will

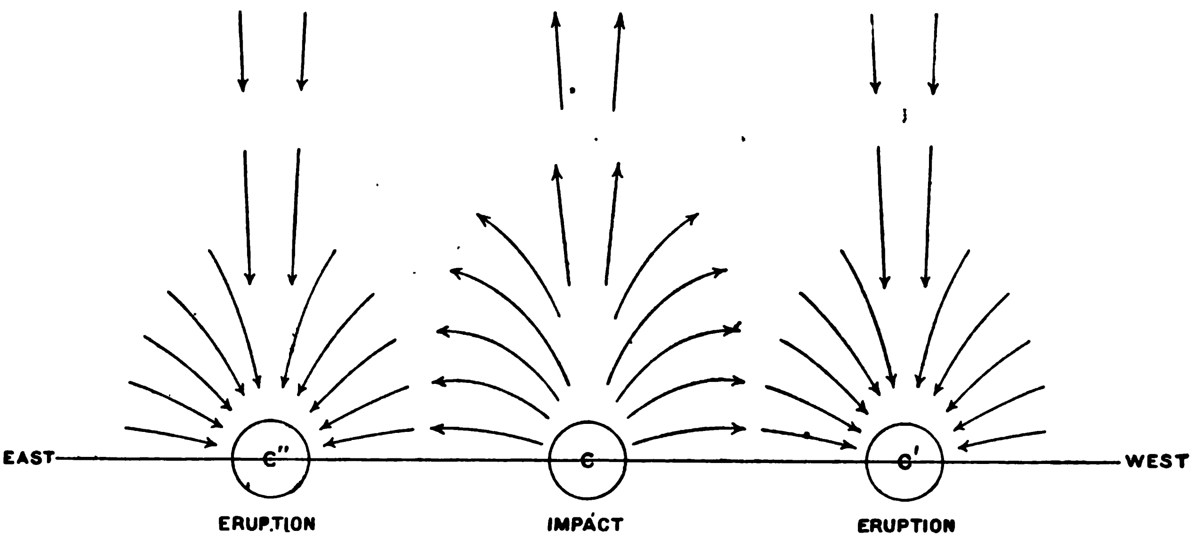


Fig. 5.

gradually become more pronounced, and eruptions will occur more frequently. Consequently, impacts at C and other similarly situated places grow more intense, and the meridional currents setting out therefrom will therefore gain strength continuously. Hence it appears that the impacts of matter on the photosphere caused by neighbouring eruptions must eventually give rise to currents flowing in a decidedly meridional direction from the place of impact towards the poles.

Let us next consider the opposite case of an eruption flanked by two impacts. When the outbreak takes place, for example at C′, there must be an inrush of matter from every side towards the vacuum produced there. As regards the directions east and west of C′, this inrushing matter is readily supplied by the impact matter moving from C towards C′. But in the case of the directions north and south of C′ there is no such supply of impinged matter available to fill up the gap; consequently the store of matter in the photosphere *beyond* the equatorial belt of eruptions must

needs be laid under contribution to supply the necessary inrushing matter. Thus the vacuum is filled by a rush of impact matter from the east and west, and by a rush of matter belonging to the outside photosphere from the north and south. The presence of this impact matter lying east and west of C′ obviates the need of drawing, to any great extent, for the indraught supply upon the part of the photosphere lying beyond the belt of eruptions in the directions from which the impact matter is already flowing. Hence this phenomenon also necessitates the formation of currents flowing in a decidedly meridional direction, though in this case their course is from the poles towards the centre of disturbance at the equator.

It is to these two systems of currents that we invite the attention of our readers. For, taking into consideration the stupendous forces exhibited in the solar eruptions, as well as the fact that the conflux of reflecting matter supplies a district of outbreaks, once established, with the conditions favourable for frequent repetitions of its tremendous convulsions, we must consider these currents as most important factors in the dynamical processes going on at the solar surface.

Comparison between the velocities of the pole-equator and the equator-pole currents.

36. Here the question arises as to whether the two currents are of equal velocity, or whether they must not rather be assumed to differ in this respect. As regards the absolute amount of these velocities we can of course know nothing, seeing that their determining factors form a system of the utmost complexity.

But, in accordance with the laws of ordinary impact, it seems clear that the ejected matter, when eventually coming into contact with the photosphere, must lose a considerable amount of mechanical energy. The assumption thus seems warranted that the velocity with which this matter afterwards spreads out from the place of impact along the solar surface will be less than the velocity with which the masses drawn towards the vacua approach the centres of outbreak. This conclusion, as we shall find, is of great importance as regards the peculiar phenomena of the sun's axial rotation deduced from observation.

Deviation of the currents from the meridional direction owing to the sun's rotation.

37. When the fact of this rotation is taken into account, our conclusions with regard to the directions of the two currents have to be somewhat modified. Neither of them can move exactly along a meridian such as A B (Fig. 6), but each will deviate from that course in a manner analogous to our trades and anti-trades. The pole-equator current, as seen from the earth, will appear as a north-west current in the northern and a south-west current in the southern solar hemisphere; while the equator-pole current on the northern hemisphere will move from south-east to north-west and on the southern hemisphere from north-east to south-west.

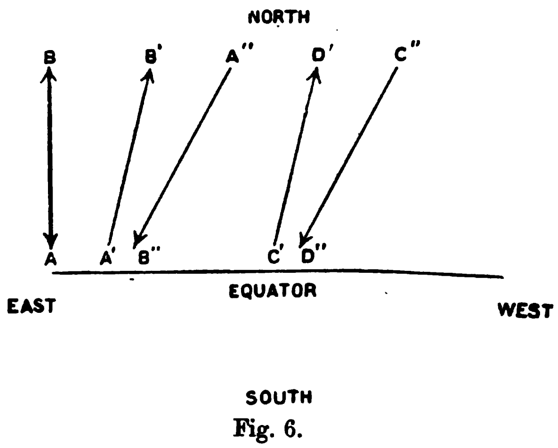


Fig. 6.

If the currents were of equal strength their directions would be parallel; but since the quicker current undergoes more pronounced deviation owing to the greater change

of latitude of its elements in a unit of time, the pole-equator current must evidently assume a direction represented by A″ B″, and the equator-pole current a direction indicated by A′ B′. Hence the two currents set up by one and the same district of eruptions cannot meet under ordinary circumstances.

Conditions necessary for the collision of two oppositely directed currents. A solar spot the result of such a collision.

38. Suppose, however, that there comes into play another outbreak-locality immediately to the west of the one whose resulting currents we have just considered. This is indeed a very probable occurrence, since, as has already been mentioned, there is sure to be more than one district in eruption at the same time on the solar equator. Granting this assumption, a conflict between the two currents A″ B″ and C′ D′ will in course of time become inevitable, provided the eruptive power in the two districts is maintained long enough. From our previous investigation we know that such a condition is likely to be satisfied at the beginning of a cycle, when the heat reflected to the photosphere grows more and more intense every moment, and the eruptive forces are continually on the increase. It is, then, to the encounter of two such currents that the formation of a *solar spot* is due, which, so far in accordance with Faye's theory, has to be considered as the result of a vortex motion generated by the concourse of two oppositely directed currents. The question of the physical nature of sunspots from this point of view will be investigated in a later section of this paper. For the present moment, however, we are more concerned with the explanation of their peculiar heliographic distribution, and with the possibility of doing so by means of the theory just given as to their origin.

Reason why the first spots must appear in high latitudes.

39. We have seen that when an eruption takes place, there is a conflux not only of photospheric matter but also of parts of the reflecting atmosphere towards the centre of the outbreak. By this removal of the superimposed reflecting material from the districts adjacent to the scene of eruption, the potency of these localities to produce similar outbreaks is greatly reduced. Thus, at a time when the reflecting power of the atmosphere is feeble, viz., at the beginning of a cycle, the establishment of *one* district of cyclonic disturbance may be assumed to counteract the formation of others in its near vicinity. Indeed if the reflecting power of the atmosphere over such a neighbouring district only slightly exceeds that which is required for the maintenance of thermal equilibrium, then the eruptions close at hand may not only carry off all this surplus, but may also, for some time after, prevent any increase in the absorbing power of the atmosphere in the localities neighbouring the outbreak. As a consequence of such circumstances a state of unstable equilibrium cannot well be produced in these localities.

Hence, at the beginning of a cycle, the conditions favourable to the formation of another district of cyclonic disturbance can only be found *at a considerable distance* from the one first established. Under all circumstances the two districts must be so far apart that the reflective power of the atmosphere overlying the second suffers little or no diminution from the outbreak in the first, and thus increases progressively in accordance with the law previously laid down.

Let us now turn to account these conclusions, viz., that two eruptive districts are requisite for the generation of a spot, and that the distance between them will be

greatest at the beginning of a cycle. Obviously the greater is this lateral distance between the two equatorial centres of eruption, the farther removed from the equator will be the point of collision of the two approximately meridional currents A″ B″ and C′ D′. But since it is this collision which leads to the formation of a spot, it follows that the first spots of a new cycle must make their appearance in *high* latitudes. Moreover the fact that these early indications of renascent spot-life are generally very slight as compared with the intensity exhibited in its subsequent development, finds a ready explanation in the relative feebleness of the first eruptions, combined with the great distance traversed by the two currents before they come into conflict. With the increasing reflective power of the atmosphere the lateral distance between the districts will gradually be diminished, with the result that the points of collision, and therefore the position of the spots, will more and more approach the equator.

The vicinity of a spot must become the seat of eruptive disturbance. "Primary" and "Secondary" districts of outbreaks.

40. It must be confessed, however, that the conclusions drawn from the foregoing considerations regarding the changes in the heliographic distribution of spots will not altogether account for the phenomena actually observed on the solar surface unless we modify them in accordance with a further very important circumstance. Observation shows clearly that the neighbourhood of a sunspot is always the seat of violent eruptions. Indeed if it be true that a spot is a vortex, its immediate action must be to draw the surrounding atmosphere towards its centre. Thus the reflecting matter must be denser over the vortex than over the spotless parts of the solar surface, whence the conditions for the formation of fresh eruptions must be greatly enhanced in the neighbourhood of a spot. As soon, therefore, as spots appear they give rise to what may be called "*secondary*" districts of eruption, in contradistinction to the "*primary*" districts on and near the equator.

Influence of the secondary districts of eruptions on the formation of spots.

41. Such a secondary district of eruption will in its turn originate currents towards the poles and towards the equator, and through their collision will occasion the formation of new spots. Though the earlier spots on account of their relative feebleness may not give rise to currents of great intensity, yet as time passes and the development becomes stronger, their influence on the formation of subsequent spots will become more and more appreciable, entailing at the same time an important modification of the heliographic distribution of spots.

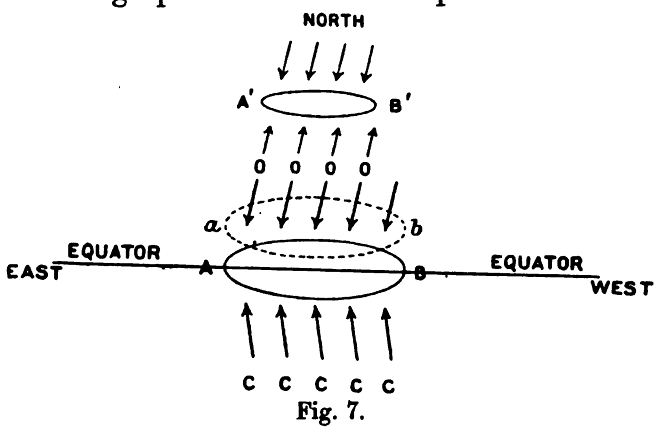


Fig. 7.

One remarkable effect of this secondary formation of eruptions will be at once apparent. Let A B represent a primary district of eruptive disturbance at the equator, A′ B′ a secondary district in a high northern latitude caused by the action of spots within A′ B′ (Fig. 7). The arrows indicate the direction of the currents flowing towards these districts. Since the directions of the

currents between A B and A′ B′ are mutually opposed to one another, their velocity must become weakened. Thus the force of the currents passing from 0, 0, etc., to A B is less than that of the currents coming towards A B from C in the southern hemisphere. Hence A B, that is the locality where atmospheric reflection is greatest, must gradually move towards A′ B′, or from the equator towards the north.* So far, it is true, no reason has been brought forward to explain why the formation of spots should be *different* on the two hemispheres of the sun. It might therefore be objected that the chances for spots and consequently for the formation of secondary districts of cyclonic disturbances on a particular meridian should be the same for both hemispheres. If this were so, the equatorial eruptions would be subjected to the equal and simultaneous influences of a northern and a southern current, and a change in the position of the equatorial district would of course be impossible.

We are, however, justified in assuming that such a coincidence can rarely happen. The phenomena north and south of the equator will not, as a rule, appear at the same time; and if, by any chance, they should occur simultaneously, they will not at any rate be of the same dynamical power.

Let us suppose, then, that one has, at the beginning, some small advantage over the other, either as regards time or intensity. As the development of each proceeds, this advantage will become more and more pronounced, and, in the end, must entail a decided preponderance of that phenomenon which made its appearance earlier or with greater intensity.

To illustrate this let us take the spot-display in the northern hemisphere at A′ B′ as the earlier or stronger of the two, through whose influence the equatorial primary seat of disturbance at A B has been removed to *a b* (Fig. 8). Let E C be a pole-equator current rushing towards the vacua at *a b*, and D E a neighbouring equator-pole current, E being their point of collision. Then on the southern side of *a b*, E′ C will represent the pole-equator current, D E′ the equator-pole current, E′ being their point of encounter. Now, when E′ C′ crosses the equator it changes from a S.-W. to a S.-E. current, and similarly D E′ becomes a N.-E. current south of the equator. If C′ D′ be points on the southern hemisphere corresponding in latitude to C D, then the distance C′ D′ is equal to the distance C D. Hence, when the equator-pole current D E′ has reached D′ it is still no nearer its neigbouring pole-equator current E′ C than it was at the beginning of its course, and, since the curves E C C′ E′ and E D D′ E′ must be symmetrical with reference to the equator, the point E′, where these currents collide, must be, so to speak, the image of E in A B.

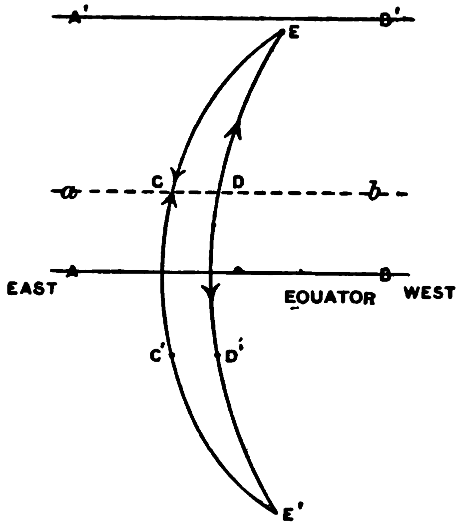


Fig. 8.

* This phenomenon is altogether analagous to the seasonal displacement of the belt of calms on the earth.

The current D E′ has therefore a greater distance to traverse before it encounters E′ C than has D E before it encounters E C: consequently D E′ and E′ C cannot meet so soon; nor will their collision be so forcible as that at E in the northern hemisphere, seeing that D E′ must lose more of its original intensity by the prolonged resistance of the photospheric medium through which it has to force its way.

Thus the formation of new spots will take place earlier and more rapidly at E than at E′. The same reasoning applies to the secondary districts of eruptions formed at these two places. Those at E being stronger will weaken the current E C, so that the current E′ C will tend to push the line *a b* still farther north. But with every further change of this line in a northerly direction the conditions conducive to spot-production on the southern hemisphere become less and less favourable; so that, in course of time, there will be a predominance of spots and secondary eruptions on the northern hemisphere on this particular meridian.

The simultaneous appearance of spots in both hemispheres must be scarce. Explanation of the temporary preponderance of one hemisphere over the other as regards the development of spots.

42. There are two interesting results to be derived from this discussion:—

(*i.*) When, on a particular meridian, spots have developed strongly in one hemisphere, there can only be a weak development of spots and eruptions on the other. As a matter of fact the spot maps of Carrington and Spörer show that this is almost invariably the case.

(*ii.*) We may now understand why the average number of spots is not always the same on both sides of the equator—a phenomenon to which Spörer has repeatedly called attention, and which is placed beyond all doubt by observation.

From the foregoing considerations it appears that there must always be a tendency for the reflecting atmosphere to move somewhat more rapidly and in greater force towards that hemisphere on which the first spots of a cycle appear.

However slight and insignificant this tendency may be at the outset, it must steadily increase, since with every new addition of atmosphere the conditions requisite for further eruptions become more and more favourable. Thus the development of spots and eruptions in this hemisphere will eventually preponderate to such an extent as to maintain its superiority for a considerable length of time.

Throughout the whole argument it is of the utmost importance to remember this remarkable principle—that every spot, by drawing towards its centre the reflecting matter of the atmosphere, increases the state of unstable equilibrium in its vicinity, and consequently enhances the fertility of the surrounding district in the production of eruptions.

Explanation of the clustering of spots along two principal meridians 180° apart.

43. This principle also explains a curious phenomenon recently made known by the observations of Prof. Wolfer,* viz.—that sunspots evince a remarkable tendency to cluster along two meridians about 180° apart.

Owing to the conflux of reflecting atmospheric matter towards it, that meridian on which the first spots appear must from the outset have a distinct predominance over all others in regard to the development of eruptions and spots.

* Publicationen der Sternwarte des eidgenöss. Polytechnicums zu Zürich, Band I., 1897.

The districts in which afterwards eruptions occur will be impeded in their explosive development by the removal of the atmosphere drawn towards the first spot-meridian. But obviously they will be the less so, the farther they are removed from the initial spot-meridian. It is therefore to be expected that on the meridian diametrically opposite to the primary district there will exist the most favourable conditions for another great centre of solar activity.

Changes in the heliographic distribution of spots during a cycle.

44. We may now proceed to trace out the changes in the heliographic distribution of spots during a whole sunspot period, as they appear to follow from the theoretical views here proposed.

The comparatively sparse distribution in longitude of the first equatorial eruptions, along with their imperfect development, will, at the early stage of spot-development, give rise to but a few feeble collisions at a great distance from the equator. But with the steadily increasing frequency and intensity of these eruptions, their longitudinal distance from one another becomes lessened, and the collisions of the currents will occur oftener and, at the same time, nearer the equator. This, however, does not involve the complete disappearance of the spots in high latitudes; for the secondary eruptive districts originated in the neighbourhood of the first spots will, by generating currents in opposite meridional directions, give rise to collisions in the direction of the pole as well as of the equator. Thus, while the chief zone of spots and secondary eruptions moves in the direction of the equator, a possibility still remains for the formation and perpetuation of spots in high latitudes—at least, as long as the general intensity of eruptive power is on the increase. Now, while the maximum zone of spots and secondary disturbances gradually approaches the equator, the zone of primary disturbance travels, as has been shown, in the opposite direction. Hence, instead of one equatorial belt of eruptions such as we observed at the beginning of the cycle, there will appear in course of time two ranges of cyclonic disturbance on each side of the equator, tending to approach one another.

The conditions most favourable to collisions must evidently occur in those localities where oppositely directed currents are present in greatest abundance. Prominent in this respect are the areas between the primary and secondary districts, and it is here that the chief development must accordingly be looked for.

In the annexed diagram (Fig. 9) let *a b* indicate the zone of primary eruptions and *a′ b′* that of spots and secondary eruptions at a given moment: then the space between these two lines will be the principal area of the conflicting currents, and, as a consequence, the district where new spots will most frequently appear. This increasing development of spots on the equatorial side of *a′ b′* must necessarily move the line of maximum spot-formation in the direction of the equator,

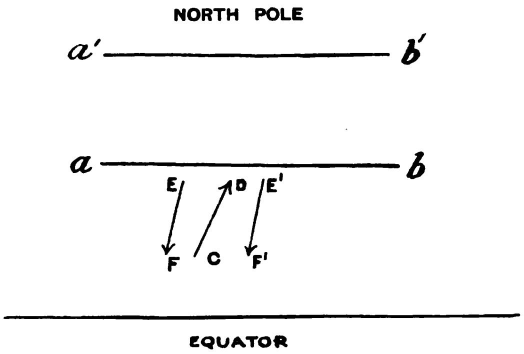


Fig. 9.

whereas the increase of secondary eruptions will cause the line *a b* to move farther from the equator. The gradual approach of the two zones of activity will continue until these zones coalesce, which may be expected to happen about the time of the spot-maximum.

The existence of spots between *a′ b′* and the pole depends entirely on the activity of the secondary eruptions grouped along *a′ b′*. What might be supposed is that since this line is moving towards the equator the poleward limit of spot phenomena ought to change in the same direction. It must be remembered, however, that during the earlier part of a cycle the intensity of the outbreaks along *a′ b′* is steadily on the increase, so that their sphere of influence is widening—whence it follows that the poleward limit of spots need not move towards the equator as quickly as *a′ b′* does, but may even remain stationary as regards latitude right up to the time of maximum.

It remains to consider the conditions of spot-formation in the zone between *a b* and the equator. Let us suppose two currents C D and E F (Fig. 9), of which, by our previous reasoning, C D has the greater velocity, and therefore suffers the larger deviation from the meridian. These currents may, indeed, come into collision on or near the equator, and produce a spot there. But, seeing that the meridional deviation near the equator cannot but be comparatively small, the collision could only take place if the currents originate very close to one another—a condition only fulfilled when there is a great display of eruptions along *a b*. We may consequently conclude that spots near the equator are not likely to originate in the first stages of a cycle, but will probably appear later, about the time of maximum spot-development.

Theoretical conclusions as to the heliographic distribution of spots at the time of maximum.

45. As a general result of the preceding enquiry, it follows that at this time of maximum the spot-density will be greatest at a certain distance from the equator, somewhere between *a b* and *a′ b′* (Fig. 9), and that from this zone the frequency of spots will decrease in both directions—a precisely similar distribution taking place in both hemispheres at the same time.

All these theoretical conclusions find ample verification in the results derived from observation, and it will be well known to all who have taken part in such research that the observed changes in the heliographic distribution of sunspots up to the time of maximum regularly occur in the precise manner outlined above.

Changes in the heliographic distribution of spots after the maximum.

46. Let us now briefly consider the further development after the maximum has been passed. In accordance with the first part of our theory, the quantity of reflected heat is now decreasing, and therefore the spot phenomenon must decrease correspondingly.

Decisive influence of centrifugal force.

The question then is—at what parts of the two spot-zones will a reduction in the number of spots first become noticeable? On general grounds it may be expected that such a falling off may be expected to take place at the extreme verge of each zone—either on the polar limits, or on the equatorial boundaries. On closer examination it will appear that the spot phenomenon must begin to die away on those edges of the two belts lying towards the poles of the sun; whilst the life of the cycle will be most prolonged on the edges next the equator.

The reason is that the elements of the photosphere and of the overlying atmosphere are subjected to a greater centrifugal force on the equator than at any other part of the solar surface, a condition which, as we have seen, facilitates the generation of cyclonic outbreaks. Thus, the currents on the equatorial side of a spot-zone will be stronger than those on the polar side, and the outcome of their collisions will be more considerable.

Owing to this decisive influence of the solar rotation, the eruptions will tend to become more intense towards the equator, and less so towards the poles. Hence, on the one hand, a gradual decay of spots will now be observed in high latitudes, and, on the other, a steady approach towards the equator on the part of the existing maximum-ring of eruptions and spots.

The last visible signs of spot-activity must therefore appear in the region where the centrifugal force is most effective, *i.e.*, in the vicinity of the equator.

Of that great zone of solar activity—extending, at the time of maximum, from the equator almost to the parallel of 40°—there remains at the minimum nothing but a thin ring near the equator, itself doomed to ultimate extinction through the gradual ebbing away of the currents to which it owes its vitality.

Then, with the disappearance of the last spot, would come the time when the solar surface has once more arrived at a state of thermal equilibrium. This state, however, as we have seen, is not attained by the whole surface at one and the same time, a new cycle of spots having already begun in high latitudes before the last traces of the old phenomenon have completely died away at the equator.

SECTION IV.

The Law of the Sun's Axial Rotation, and the Changes of its Apparent Velocity during a Sunspot Cycle.

47. From their long series of sunspot observations, Carrington and Spörer drew certain empirical conclusions regarding the law of rotation of the solar surface. Within recent times further evidence on this point has been amassed through Prof. Dunér's spectroscopic observations on the rotation of the photosphere, and through M. Stratonoff's photographic researches regarding the period of rotation of solar faculæ. **State of present knowledge regarding this law.**

The chief results arrived at by these investigators may be thus summarised :—

Carrington found that the whole of the sun's surface did not rotate uniformly, but that spots near the equator exhibited a shorter period of rotation than did those in higher latitudes. This result he eventually expressed by one or other of the following formulæ for the daily angular rotation of the solar surface in different latitudes :

$$865' \mp 165' \text{ Sin } {}^7\!/\!_4\, \beta$$
$$865' \mp 156' \text{ Sin } {}^7\!/\!_4\, (\beta - 1^\circ),$$

where β is the heliographic latitude.

Spörer fully confirmed Carrington's discovery, proposing, however, to substitute the simpler formula

$$\xi = x + y \operatorname{Cos} \beta,$$

which, he found, most closely represented his own observations when

$$\xi = 512'{\cdot}9 + 347'{\cdot}9 \operatorname{Cos} \beta.$$

By spectroscopic observations of the velocity in the line of sight of the metallic vapours near the photosphere on both limbs of the sun, Prof. Dunér established the fact that the diurnal angular rotation of the sun's general surface (photosphere) is somewhat smaller in all latitudes than that of the spots—the difference at the equator being 12′.

M. Stratonoff, on the other hand, by measurements of photographs of faculæ, found that the strata represented by these formations rotate more quickly than even the spots do; with this peculiarity, that as the latitude increases the velocity differs from the equatorial velocity in a manner which shows systematic deviations from the general formula used by Spörer.

Rotational anomalies due to action of surface currents.

48. The question therefore arises as to how far the theory here propounded is capable of explaining satisfactorily the peculiarities in the movements on the solar surface, which must be taken as the cause of these singular rotational phenomena.

In the light of the preceding discussion it appears evident that the apparent anomalies in the sun's rotation are mainly due to the system of currents, which in their circulation exchange surface matter between different latitudes. Now, although the complexity of these currents is such that an exact scientific analysis of their action and effects seems at present altogether impracticable, it is nevertheless possible to obtain some idea of the manner and degree in which they must affect the rotation in different latitudes.

Preponderating influence of currents flowing towards the equator.

Reasons have already been given for the fact that the intensity of the currents flowing towards an eruptive district is greater than that of those moving in an opposite direction. Hence, since the greatest and most enduring districts of eruption are to be found at a more or less close proximity to the equator, the rotation of the photosphere must be chiefly governed by the pole-equator currents.

By constantly transferring surface matter from higher to lower latitudes, these currents will to a certain extent act as a retarding force on the rotational velocity of the districts through which they pass.

Such an effect finds a general expression in the formula

$$\xi = \xi_p + a \operatorname{Cos} \beta + b \operatorname{Sin} \beta + c \operatorname{Cos} 2\beta + d \operatorname{Sin} 2\beta + \ldots\ldots,$$

where ξ represents the angular velocity of rotation for the latitude β, and ξ_p, a, b, etc., are constants.

Although theory is unable to pronounce definitely as to the values of the different constants in this expression, their evaluation can nevertheless be effected empirically from observations. Now, as has been stated, Spörer found that the simple formula

$$\xi = \xi_p + a \operatorname{Cos} \beta$$

perfectly represents his observations, and therefore we do not hesitate to accept this expression as at least a very approximate representation of the actual phenomena, and to make it the basis of a special inquiry into the changes which the solar rotation undergoes during a sunspot period.

Variation of the angle a in Spörer's formula during the spot-cycle.

49. The quantity a in Spörer's formula must be proportional to the intensity of the meridional currents. If a were zero we should have the case of a constant angular rotation over the whole solar surface, thereby indicating the absence of any transference of material from one latitude to another.

The stronger the currents the greater does a become. But the frequency and strength of these currents depends on the number and intensity of the eruptions: whence we may conclude that the quantity a will vary with the intensity of eruptions and spots, and will attain its maxima and minima simultaneously with them. The data by which this conclusion can be tested are contained in the various publications of sunspot observations made by Spörer from 1861 to 1893. In order to make the test as rigorous as possible, only such spots have been selected as were observed in at least two solar rotations. This material is here divided into three groups; the first comprising all observations made during a time of copious displays, the second those obtained at times of medium activity, and the third those made when the spot phenomenon was at its feeblest. As a further step, what may be termed the mean number of spots for each group has been deduced in the following manner.

Suppose the group of observations contains a_1 spots observed during the year y_1, a_2 during the year y_2, etc., the relative spot numbers of the several years taken from Spörer's observations being denoted by n_1, n_2, . . . Then the mean number of spots for the group is derived from the relation

$$n = \frac{a_1 n_1 + a_2 n_2 + \ldots\ldots}{a_1 + a_2 + \ldots\ldots}.$$

From the single diurnal angles of rotation in each group, arithmetical means have been formed, each of which comprises about 5 degrees of latitude. By substituting the numerical values thus obtained for ξ and β in Spörer's formula

$$\xi = \xi_0 + a \operatorname{Cos} \beta$$

the probable value of ξ_0 and a have been determined by the method of least squares.

The following Tables show the results of this computation:—

MAXIMUM-GROUP.

$n = 977$	β	ξ Obsd.	ξ Compd.	O − C
	28°·20	13°·690	13°·685	+ 0°·005
	22°·43	13°·923	13°·938	− 0°·015
	17°·38	14°·132	14°·115	+ 0°·017
	12°·85	14°·232	14°·237	− 0°·005
	8°·53	14°·308	14°·320	− 0°·012
	2°·84	14°·388	14°·377	+ 0°·011

$$\xi = 8°{\cdot}511 + 5°{\cdot}874\ (\pm\ 0°{\cdot}100) \operatorname{Cos} \beta.$$

MEDIUM-GROUP.

$n = 703$

β	ξ Obsd.	ξ Compd.	O − C
27°·65	13°·752	13°·718	+ 0°·034
21°·66	13°·930	13°·952	− 0°·022
17°·50	14°·042	14°·082	− 0°·040
13°·10	14°·159	14°·191	− 0°·032
8°·42	14°·293	14°·273	+ 0°·020
4°·24	14°·357	14°·317	+ 0°·040

$$\xi = 8°{\cdot}958 + 5°{\cdot}373\ (\pm\ 0.280)\ \mathrm{Cos}\ \beta.$$

MINIMUM-GROUP.

$n = 250$

β	ξ Obsd.	ξ Compd.	O − C
25°·71	13°·813	13°·821	− 0°·008
17°·55	14°·077	14°·069	+ 0°·008
12°·11	14°·220	14°·184	+ 0°·036
8°·63	14°·206	14°·236	− 0°·030
4°·92	14°·266	14°·271	− 0°·005

$$\xi = 9°{\cdot}561 + 4°{\cdot}728\ (\pm\ 0{\cdot}240)\ \mathrm{Cos}\ \beta.$$

Spörer's observations confirm change of a during a cycle. Further tests.

50. These computations show most unmistakably the change of value which a undergoes during the course of a sunspot period, and confirm the conclusion arrived at in our theoretical discussion, viz., that this value is greatest for the maximum group and smallest for the minimum group.

To make quite sure, however, that this result is not accidental, we have extended the computations over two other sets of data, sufficiently reliable to serve for our purpose. These are obtained from Carrington's observations,* embracing the period immediately previous to that taken up in Spörer's researches, and from a very valuable series of observations made at Toulouse† during the five years, 1874–78. Carrington's observations give :—

$$n = 987, \qquad \xi = 8°{\cdot}485 + 5°{\cdot}907\ \mathrm{Cos}\ \beta,$$

while from those made at Toulouse there results :—

$$n = 335, \qquad \xi = 9°{\cdot}474 + 4°{\cdot}829\ \mathrm{Cos}\ \beta,$$

which values entirely corroborate those previously obtained.

By arranging all the different values of a according to the mean spot-number we get

n	a	a_0	$a - a_0$
987	5°·907	5·916	− 0·009
977	5°·903	5·895	+ 0·008
703	5°·373	5·372	+ 0·001
335	4°·829	4·829	0·000
250	4°·728	4·727	+ 0·001

* Observation of the Spots on the Sun from November 9th, 1853, to March 24th, 1861, made at Redhill.

† Annales de l'Observatoire de Toulouse, tome premier.

The figures of the columns headed a_0 are derived from the empirical formula

$$a_0 = 4^\circ{\cdot}478 + 0^\circ{\cdot}000839\, n + 0^\circ{\cdot}000000626\, n^2.$$

Since the agreement between a and a_0 is practically perfect, we may assume that this numerical expression represents a very close approximation to the actual facts.

Explanation of fact that the equatorial angle of rotation is greater at times of spot-maxima.

51. But there is still another result to be derived from these observations which also proves the correctness of the conclusion that the currents flowing towards the eruptive districts are the cause of the peculiar law of solar rotation.

We know that at a time of maximum the densest region of eruptions is about 15° distant from the equator, and that this zone gradually approaches the equator as the solar activity attains its minimum. If in the annexed diagram (Fig. 10), the parallel $\epsilon\epsilon'$ represent the region of maximum eruption, the spherical segment of the photosphere between $\epsilon\epsilon'$ and the pole will be influenced by pole-equator currents, and consequently there will be a decrease of angular rotation with every increase of latitude from $\epsilon\epsilon'$ towards the pole.

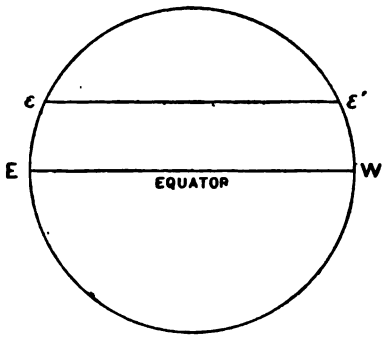


Fig 10.

But between $\epsilon\epsilon'$ and the equator the predominant currents will flow in the opposite direction, *i.e.* from the equator towards $\epsilon\epsilon'$, and thus, by carrying matter from lower to higher latitudes, will tend to accelerate the angular rotation in this particular area.

On the other hand, at the time of minimum the parallel $\epsilon\epsilon'$ coincides with the equator, and hence pole-equator currents will then predominate over the whole surface. A spot making its appearance near the equator ought therefore to exhibit a greater rotational velocity at times of maximum than at times of minimum—a phenomenon which is clearly shown by the observations.

If in Spörer's formula we equate β to zero, we obtain the following values for ξa, the diurnal angular rotation at the equator:

n	ξa
987	14°·392
977	14°·385
703	14°·331
335	14°·303
250	14°·289

From these figures it is evident that the diurnal angular velocity of a point on the sun's equator is fully one tenth of a degree less at a time of minimum than at that of maximum. This corresponds to about 750 miles, or 1200 kilometres, per day.

The fact that the apparent solar rotation is influenced to such an extent by the action of surface currents has always been a great obstacle to the determination of the true period of solar rotation. This difficulty, however, may now be almost entirely overcome by the following simple consideration.

Obviously the sun's true axial rotation will be shown at places where the retarding influence of the one set of currents is exactly counterbalanced by the accelerating effect

of the other. This condition will be approximately satisfied at all points along $\epsilon\epsilon'$ (Fig. 10). Whatever the distance of this line of maximum activity from the equator, spots situated on or near it must always show the same angular velocity of rotation.

Thus, if we knew, for each of the above formulæ for ξ, the heliographic latitude of $\epsilon\epsilon'$ in that particular instance, we should, by substituting this value for β, obtain the same angular rotation from all these different formulæ. Now, from Spörer's observations (Publ. d. Astron. Gesellschaft, xiii., p. 138; and Publ. des Astrophysik. Observ. zu Potsdam, x., p. 142) the latitude of $\epsilon\epsilon'$ at different stages of spot activity can be readily determined. Some of these are given in the subjoined Table :—

Relative Spot Number (Spörer).	Latitude of Greatest Activity, $\epsilon\epsilon'$.
1197	15°·6
784	12°·2
568	11°·1
350	10°·0
148	9°·0
34	7°·5

By means of these values, and the preceding formulæ for ξ, we obtain the following results :—

Relative Spot Number.	Latitude of Parallel of Greatest Activity.	Diurnal Angular Rotation of Spots in this Parallel.
987	13°·7	14°·224
977	13°·6	14°·220
703	11°·7	14°·220
335	9°·9	14°·231
250	9°·5	14°·224
	Mean =	14°·224

The constancy of the angular rotation as given in the last of these columns is certainly most striking, when compared with the systematic differences previously shown to exist in the values for the equatorial rotation ξ_a. Such a result clearly lends additional support to the contention that the apparent anomalies in the law of solar rotation must be attributed to currents flowing in a quasi-meridional direction towards the temporary seat of maximum development of eruptions and spots.

Change of rotation during a cycle already suspected by Spörer.

52. That the possibility of a connection between the quantity a and the spot period had already occurred to Spörer may be inferred from some of his remarks, though it does not seem that he ever attempted to pursue the subject farther. It may suffice to refer to the following passage in "Publ. der Astr. Gesellschaft," xiii., p. 153:—

"Der Nachweis dass innerhalb der elfjährigen Periode bei den Jahrescurven der für verschiedene Breiten aufgestellten Mittelwerte der Rotationswinkel charakteristische Verschiedenheiten (vor und nach dem Minimum) auftreten, schliesst vollständig den Gedanken aus, dass an der Sonnenoberfläche eigentümliche Rotationsverhältnisse nach feststehenden Parallelzonen stattfinden könnten, und werden wir darauf hingewiesen, die vermittelst der Flecke gefundenen Rotationsverhältnisse solchen Strömungen zuzuschreiben, welche mindestens ebenso veränderlich sind, wie die Häufigkeit der Flecke und deren Verteilung nach der heliographischen Breite."

Cause of the peculiar displacement of spots in latitude.

53. From the hypothesis that the currents flowing towards eruptions exceed in velocity those moving in the opposite direction, an important conclusion may be derived. Such a difference of velocity must necessarily give rise to a temporary accumulation of photospheric matter near the places of eruptive disturbance. Gravitation, on the other hand, will naturally tend to remove this inequality of level. Thus, if a spot were to appear in a district where this extra matter had accumulated, the motion of the photosphere, as it yielded to gravitational pressure, would make itself apparent in a change of position on the part of the spot.

Since the region of greatest spot frequency is also the locality of highest pressure, the accumulated matter is gradually pushed away from this region towards the equator and towards the poles. In the accompanying diagram (Fig. 11.) let O be such a locality of maximum pressure. Then a spot at F will be carried along $O\,E$, to the lower level at E; while a spot at F' makes its way along $O\,E'$, towards E'.

The two spots are therefore moved in opposite directions from O. Hence, spots situated *between* the maximum zone of eruptions and the equator will be driven *towards* the equator, whereas spots lying between this zone and the pole will be impelled in the poleward direction. A spot situated in such a position as at O will, of course, exhibit no change of position whatever.

Now, at the time of minimum, as we have seen, the region O forms a ring round the sun on or near the equator, and with increasing spot-development divides into two rings drifting in opposite directions away from the equator. These rings attain their greatest latitude at the moment of maximum, after which they again approach the equator until the next minimum. We should therefore expect to find a corresponding change in the latitudinal motions of the spots.

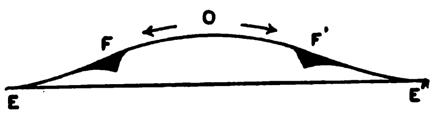


Fig. 11.

Theoretical conclusions confirmed by observations.

54. To test this point we have taken the difference between the observed latitudes at the end and at the beginning of the observations of each individual spot as given by Spörer. This amount of change in latitude, divided by the number of days during which the spot was under observation, gives the daily change of latitude for that particular spot.

As a rule, only such spots have been included as were under observation for at least a week.

The values thus obtained are arranged in three groups corresponding to maximum, medium, and minimum frequency of spots, and from the single values in each group arithmetical means have been taken, each extending over about 5 degrees of latitude.

In this way values have been secured for the average latitudinal change of position during 10 days. The results are given in the subjoined Table, where the positive sign indicates a movement towards the pole, the negative sign a drift towards the equator, while the asterisks denote those regions where there is no observed latitudinal motion.

	0° to 5°	5° to 10°	10° to 15°	15° to 20°	20° to 25°	25° to 30°
Max.	− 0°·90	− 0·26	− 0·10	* + 0·18	+ 0·30	+ 0·53
Med.	− 0°·20	− 0·18	* + 0·07	+ 0·13	+ 0·23	+ 0·34
Min.	− 0°·08	* + 0·01	+ 0·07	+ 0·26	+ 0·43	

Bearing in mind that the highest pressure must occur where the motion of the spots changes sign, we see from the above Table that, at the time of maximum, this locality is actually, as we inferred, at its greatest latitude, and that as the time of minimum comes on it gradually appproaches the equator.

Reason why the rotation of a spot is quicker than that of the photosphere.

55. The difference between the rotational velocity of the photosphere and that of the spots admits of the following explanation.

According to our conclusions, spots are vortices in which photospheric and atmospheric matter is whirled from a higher to a lower level, *i.e.*, nearer to the sun's centre. This matter, having a greater linear velocity of rotation than that of the layers to which it is transferred, must tend to increase the angular velocity of these lower layers.

Thus, the apex of the spot-vortex has a greater angular rotation than the visible photosphere, and, since the apex is the principal seat of the gyrating force, it must consequently quicken the rotation of the visible spot as compared with the surrounding photosphere.

Experimental Illustration.

56. This phenomenon may be illustrated by the following experiment. Take a vessel, as in Fig. 12, *a*, the bottom of which is fitted with a pipe *E F*, having a tap at *E*, which can be moved along *C D*. Fill the vessel with water, and open the tap. There will at once be a flow of water towards *F*, somewhat resembling the gyratory motion of the photosphere in the spot. If now the pipe be moved towards *D* the gyration will at first assume a shape like *G H F* (Fig. 12, *b*), but very soon it will follow the motion of the apex *F*, and finally settle as *G′H′F*, thus indicating that the motion of the whole vortex is governed chiefly by that of its apex. As long as *F* is moving the axis of the vortex naturally remains at a certain inclination to the vertical.

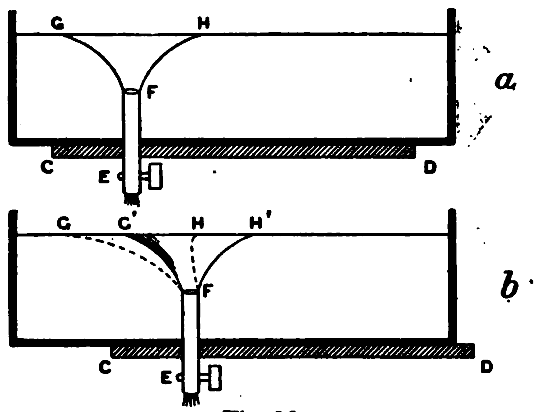


Fig. 12.

This is precisely the state of matters we should expect to find exemplified on the solar surface, where the apex of a vortex is constantly moving in a forward direction. The tendency of the gyration to keep the axis of the cone coincident with the radius of the sun is interfered with by the resistance of the photospheric matter, so that the axis will not attain its normal position, but will be kept at a certain angle to the radius. The apex of the vortex will therefore somewhat *precede* the centre of the visible spot.

Explanation of the peculiar shape of spots whose activity is declining.

57. These considerations explain a phenomenon first pointed out by Secchi, viz. —that many spots do not present the usual typical circular formation, but partake rather of the shape of a deformed ellipse. He gives, on p. 91 of his work "Le

Soleil," an illustration which markedly brings out this characteristic, and gives the spot all the appearance of forcing its way forward in the direction of rotation through what looks like a resisting medium—that is to say, the spot shows evidence by its formation of its advancing more rapidly than the photosphere in its apparent rotation round the sun's axis.

The photosphere acting against the preceding side as a resisting force compresses the vortex, whilst on the following side it tends to draw the parts away from the centre, thus producing an elongated formation, as shown in the annexed sketch (fig. 13).

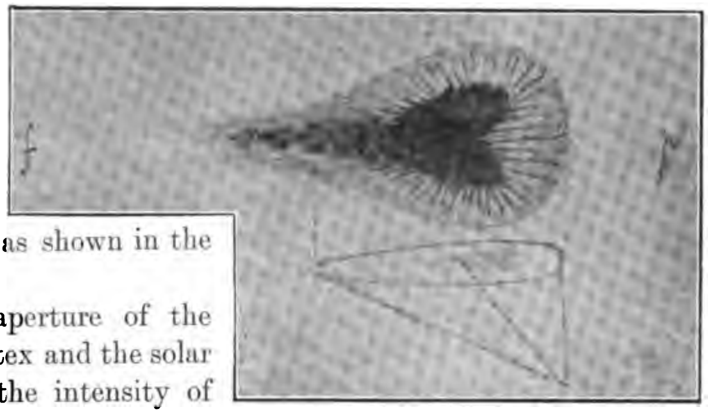


Fig. 13.

It seems evident that the aperture of the angle between the axis of the vortex and the solar radius must chiefly depend on the intensity of the vorticose motion. The stronger the gyration the more readily will the entire spot follow its apex, since it will more easily overcome the resistance offered by the slower-moving photosphere. Hence the peculiar elongated spot-formation which we have been discussing will naturally be characteristic of spots whose gyratory force is already on the decline. As this force decreases, the angle between the axis of the vortex and the solar radius increases, and the visible part of the spot becomes less able to plough its way through the photosphere, which, therefore, on the preceding side will more and more encroach upon the spot, thus steadily diminishing and distorting it. Eventually there will come a moment when the photosphere, like a mighty lava current, will overspread the whole surface of the visible spot, completely blotting it out. This event, indicating to our perception the apparent end of the spot's existence, may, however, take place at a time when the gyratory action of the *apex* has not yet wholly expired. The motion may still exist at a lower level, although it has become too weak to affect the visible photosphere.

Occasional reappearance of a spot.

58. Now, suppose that at this stage the same currents which had previously called the spot into being are, by some means or other, reinforced so much as to increase the gyratory motion at the apex. Then the new spot thus caused will break through the photosphere at a place in advance of that at which the old spot disappeared. For, on account of the increased intensity of gyration, the angle between the new spot-axis and the solar radius must be smaller than was the corresponding angle in the last stage of the old phenomenon. (*See* Fig. 14). Seen from the earth, then, the centre of

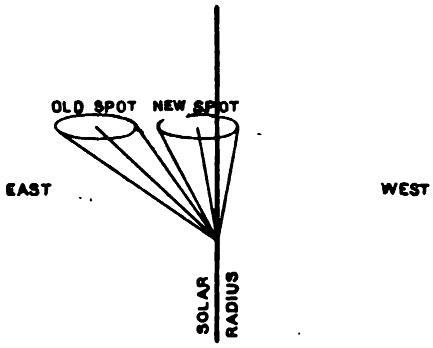


Fig. 14.

Revival of gyratory forces usually associated with a sudden increase of rotation.

the new spot must lie to the west of the old one—a result in perfect agreement with observation, as must be well known to every observer of the sun's surface.

59. Such a revival of currents may, of course, occur at a moment when the spot has not yet completely vanished. We should then expect a sudden displacement of the spot towards the preceding side, and this phenomenon has also frequently been observed. In fact, any change in the appearance of a spot or group of spots indicating a revival of the generating forces, is always accompanied by a change of the spot's position in a forward direction.

In a group of spots those preceding are always more strongly developed.

60. The same considerations may be applied to the case of a group of solar spots. The immediate consequence of a collision of currents will not be, as a rule, the formation of a single strongly-developed gyration, but rather that of a group of small vortices distributed along the whole line of collision. Now, the part of the solar surface covered by these vortices will tend to outrun the neighbouring photosphere in a westerly direction. The small spots on the preceding side will therefore be crowded together, while those on the following side will have a tendency to separate. Hence, since large spots are usually formed by the coalescence of a number of small ones, it is easily seen why a large spot is more readily originated on the western than on the eastern side.

Rotation of the Faculæ.

61. The facts thus detailed may suffice to explain why sunspots yield a shorter period of rotation than the photosphere. Let us now turn our attention to the law of rotation found by Stratonoff* to be characteristic of the faculæ. An explanation of this law requires in the first place a clear conception as to the nature and origin of the facular phenomena. On this point we cannot do better than accept Father Fenyi's view as expounded in his essay "Ueber einen neuen Gesichtspunkt und neue Erklärung der Erscheinungen auf der Sonne," A.N., No. 3355:—

Fenyi's theory adopted.

"Die von der Sonne mit enormer Geschwindigkeit in den leeren Raum geschleuderten Hydrogenium-Massen† werden natürlich unter dem Einflusse der Gravitation allein stehend in geraden oder krummen Bahnen zur Sonne wieder zurückkehren. Sie werden mit derselben Geschwindigkeit auf der Oberfläche derselben ankommen, mit welcher sie ausgegangen sind, und müssen demnach Meteoren gleich in die Sonnenatmosphäre einschlagen. Schon in den höchsten Schlichten der Atmosphäre werden sie daher, je nach dem Maasse der eintretenden Hemmung, enorme Wärmegrade hervorrufen, welche die Wärme der übrigen Oberfläche weit übersteigen. Dass die so hoch erhitzten Stellen der Oberfläche auch heller leuchten müssen, kann wohl nicht bezweifelt werden. Solche hellere Stellen der Oberfläche sind bekanntlich die Sonnenfackeln. Die Verhältnisse ihrer Beobachtung stimmen recht gut mit der Annahme überein, dass sie eben nichts anderes sind als jene Stellen, wo die Ströme der die Sonne umgebenden Gasmeteore auf dieselbe niederstürzen. Die Ströme werden zwar in ihrer Bahn in grossen Höhen sich etwas ausbreiten, allein in der Nähe der Oberfläche mit den dort häufigen Gegenströmen zusammentreffen und in

* Rotation du Soleil déterminée par des facules. A.N., Vol. 140, page 113.

† Instead of this expression, we should prefer to use the term "photospheric masses."

Bahnen gelenkt werden, welche jene langgezogenen Formen erzeugen können, die den Fackeln eigentümlich sind. Dass sie die Fleckenzone nicht weit überschreiten und um die Fleckengruppen dichter stehen, erklärt sich daraus, dass dort die Eruptionen auftreten, deren Folgen sie eben sind."

This explanation of the faculæ is in perfect harmony with what we have found to be the features characteristic of the descending residue of erupted matter:—the greater luminosity of the faculæ due to force of impact; their elevation above the level of the photosphere; their habit of clustering over extensive meridional areas ("Meridian-streifen" according to Spörer)—all these peculiarities find at once a ready explanation.

Reason why faculæ must have a *greater* angular rotation than the photosphere.

62. During their ascent to the higher layers of the atmosphere, the masses thrown out by eruptions may receive an additional rotatory impulse from the greater linear velocity of these higher layers. Thus, on falling back to the photosphere their angular velocity is slightly greater than it was before their ejection. Moreover, it must be borne in mind that, on the greater part of the solar surface, the faculæ represent currents flowing in the direction of the poles.

Thus, by carrying matter from lower to higher latitudes, as well as from higher to lower levels, the facular masses must possess a general tendency to greater rotational velocity than the photosphere.

At the level of the photosphere, it is true, this effect is more than counterbalanced by the retarding influence of the stronger pole-equator currents. But as regards the higher crests of the facular currents—which, indeed, are the only parts exposed to observation—the neutralising effect of the pole-equator currents is certainly far less marked, and these crests, therefore, should show a quicker rotation than the photosphere.

Explanation of a peculiarity in M. Stratonoff's curve.

63. These considerations sufficiently explain the principal deviation of M. Stratonoff's results from those obtained by Professor Dunér. But in this connection M. Stratonoff's curve shows a peculiarity which calls for attention. We allude to the appearance of a clearly marked secondary maximum in about 25° latitude (*see* Fig. 15). To explain this anomaly—of which, by the way, slight traces are also to be found in Spörer's spot curve

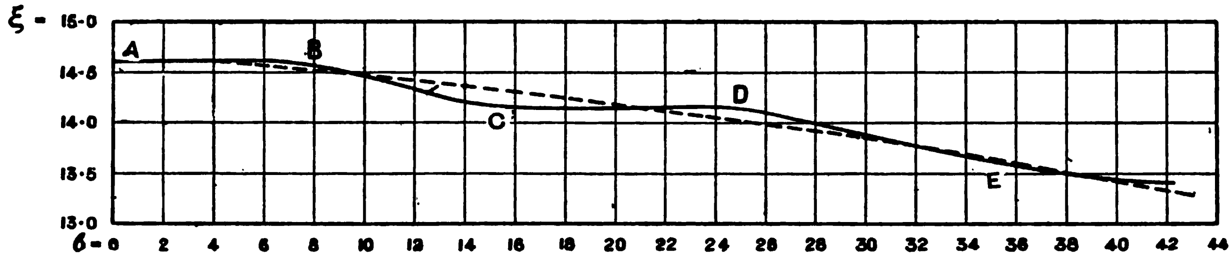


Fig. 15.

—it should be pointed out that the observations discussed by M. Stratonoff extend over the years 1891–94, *i.e.*, on the ascending branch of a spot curve. According to our theory a zone of primary eruptions was at that time situated near the equator (at *B*, 8° lat.), and a weaker zone of secondary eruptions in a higher latitude (at *D*, 25° lat.—Fig. 15).

From the fact that the rotation is principally modified by currents flowing towards the eruptive zones, it clearly appears that between AB and CD, in Fig. 15, the photosphere is chiefly influenced by equator-pole currents, while between BC and DE the pole-equator currents prevail. The more rapid decrease of angular rotation in the latter district, as compared with the former, thus readily explains itself. Instead of the normal curve (the dotted line in the figure), we have the full drawn-out curve which, by the secondary elevations at B and D, indicates the existence of the two zones of eruptions—the one near the equator and the other in about 25° of latitude.

Our theory, however, requires further that the two districts of eruptions shall approach each other, and shall finally coincide at the time of spot-maximum. The latitudinal distance between the elevations B and D must therefore gradually decrease as the maximum is approached, and must eventually vanish altogether when that time is reached. Stratonoff's researches confirm this theoretical result to this extent, that in 1893, the year of greatest spot activity, the double maximum character of the curve, so strongly marked in the preceding years, had indeed completely disappeared.

SECTION V.*

THE MOTIONS OF THE UPPER SOLAR ATMOSPHERE, AND THE QUIESCENT PROMINENCES.

Increase of depth of pole-equator currents near the poles.

64. The rotation in higher latitudes is modified chiefly by currents drawn from the pole towards the equator. These, it is evident, must become deeper as the poles are approached on account of the decrease of area of the solar surface.

On emerging from the polar regions, the currents, by spreading out, will bring to the surface materials from greater depths. This will give rise to a greater luminosity of the solar surface in these regions, the result being a circum-polar ring of granulations brighter than the average, almost resembling small faculæ. The position of this ring as well as its intensity will depend on the vigour of the eruptive action in lower latitudes.

Secchi's polar ring of faculæ and prominences.

65. The existence of such a ring round the poles at times of great solar activity seems to be placed beyond doubt by Secchi's observations. In the German translation of his book by Schellen, we find the following remarks :—

"Wenn man die Granulation der Sonne sorgfältig untersucht, so findet man in der Nähe der Pole eine ziemlich scharf gezogene Grenzlinie, die sich durch einen Gürtel von kleinen Fackeln zu erkennen gibt; leztere sind zwar nicht so hell wie die der Königszonen, aber sie sind doch deutlich zu sehen. Nachdem wir diese Grenzlinie der Granulation eine lange Zeit hindurch sorgfältig untersucht haben, hat sich Folgendes ergeben :—

a. Die Grenzen der Granulation sind sehr constant;

* *Note.*—In this section we abandon the restriction placed, at the outset of our investigation, upon the meaning to be assigned to the term "atmosphere." As used in the sequel, the word must be understood to refer not merely, as hitherto, to the comparatively thin stratum constituting the *reflecting* atmosphere, but to the atmosphere *in its entirety*, *i.e.* to the *whole* of the sun's gaseous appendage outside the photosphere.

That this point should be clearly comprehended is rendered absolutely necessary by the very unfortunate ambiguity attaching to the meaning of the term "atmosphere" as used by different writers.

b. wo diese Gürtel von Fackeln den Sonnenrand treffen, findet man häufig Protuberanzen;

c. das secundäre Maximum der Protuberanzen (70° bis 80°) fällt mit diesen Grenzen der Granulation vollkommen zusammen;

d. die von diesen Gürteln um die Pole herum abgegrenzten Kugelschalen sind excentrisch zu den Polen und ihre Lage ist wesentlich durch die jeweilige Wirkung der Sonnenkräfte bedingt."

Differences between eruptions and quiet prominences.

66. This quotation raises another question requiring further consideration. In our previous discussions we have carefully avoided the use of the ambiguous term "protuberances," preferring rather to adopt the expression "eruptions" for what are generally called "metallic protuberances." In doing so we desired to emphasise the tremendous dynamical energy of these phenomena, the marvellous velocity of their ascent, and the important part they thus play in the mechanism of the sun.

The "protuberances" mentioned in the above extract are, however, of quite another character. Indeed, they do not belong to the class of eruptive phenomena at all. The name "quiescent protuberances," by which they are usually known, indicates the small share they are believed to have in the dynamical events occurring on the solar surface.

The essential arguments of the preceding theory are therefore but little affected by their existence. Nevertheless, we take this opportunity of stating, in a word, our opinion regarding the nature of these remarkable objects.

Opinion as to nature of quiet prominences.

67. On the whole, these quiescent prominences seem to be "the debris and relics of eruptions, consisting of gases which have been ejected from beneath the solar surface, and then abandoned to the action of the currents permeating the sun's upper atmosphere." (Young, p. 221).

Prof. Young himself seems rather inclined to reject this hypothesis. He asserts that "near the poles of the sun distinctively *eruptive* prominences never appear, and there is no evidence of aërial currents which would transport to these regions matter ejected nearer the sun's equator." Observational evidence, however, disproves the latter part of this statement. Both Secchi and Spörer are strongly convinced from their observations, that in higher layers of the atmosphere currents are found to prevail flowing from the equator towards the poles.

Since the photosphere and the adjacent lowest layers of the superincumbent atmosphere show a predominant motion towards the equator, a rarification of atmosphere at the poles must ensue, thereby causing a tendency on the part of the higher atmospheric layers to fill up the polar vacua.* Owing to the gradual decrease

* As proving the assertion that in the higher layers of the solar atmosphere the prevailing currents have an opposite direction to those of the lower layers, we quote the following passage from Secchi (German Transl., p. 437):—

"Wenn die in die Höhe steigenden Säulen nach einer Seite abbiegen, so gewinnen sie das Aussehen der aus unseren Kaminen bei einer bestimmten Windrichtung aufsteigenden und abgelenkten Rauchwolken; es kommt nicht selten vor, dass der untere Stamm nach der einen Seite gerichtet ist, während der obere Theil nach der entgegengesetzten Seite abbiegt, ein Beweis, dass in der Sonnenatmosphäre, wie auf der Erde, in verschiedenen Höhen Strömungen von verschiedenen Richtungen vorkommen."

Later on (p. 612) the same author says:—" Überall da, wo die Thätigkeit der Flecke und die aus dem Innern stammenden Eruptionen nicht hindern, sind die Büschel und die wolkigen Säulen der Protuberanzen nach den *betreffenden Polen hin* gewendet."

in superficial area as the pole is approached, these upper currents will descend and deepen, and their descent will take place in the same latitude as that in which, for similar reasons, the surface layers ascend and broaden out.

The lighter constituents of the eruptions, especially hydrogen, may naturally be expected to be swept away by these upper currents more easily than the heavy metallic vapours, just as in the case of a rifle discharged into moving air, the powder-smoke is more strongly influenced by the atmospheric currents than the bullet. The existence, therefore, of quiescent prominences near the poles, with a maximum of frequency near the bright circum-polar ring discovered by Secchi, finds a sufficient explanation from this theory.*

* The above remarks regarding the great movements in the solar atmosphere may readily be applied to the atmospheric currents in the vicinity of a district of spots. We have repeatedly had occasion to refer to a tendency on the part of the lower layers of the atmosphere to flow towards such districts. This being so, the upper layers must tend to move in the opposite direction. Bearing in mind that, owing to the greater density of the reflecting atmospheric matter near a spot, eruptions must become frequent there, and that the lighter constituents of these eruptions are carried away by the atmospheric currents, the shape of the so-called hydrogen prominences near the spot district may be pictured as follows:—In the lower layers a prominence will appear to be directed towards the spot, while in the upper strata it will drift from the spot. At a greater distance from the spot, the upper currents will gradually descend, thereby giving rise to an accumulation of clouds of hydrogen. The occurrence of phenomena such as here described is clearly proved by Spörer's observations. Moreover, whenever a spot-group shows a considerable extension in longitude, we should expect to find, in accordance with this theory, parallel chains of hydrogen-clouds lying to the north and south of the group. On this point we again quote Spörer (Publ. d. Astr. Gesc. xiii., ii., p. 132):—

"Spätere Untersuchungen haben ergeben, wie langgestreckte Fleckenketten nördlich und südlich (in einigem Abstande) durch lange Reihen hoher Protuberanzen, Gebirgszügen vergleichbar, eingefasst waren, so dass sich de Fleckenketten gleichsam in einem weiten Thale befanden."

An excellent corroboration of these observations is also contained in the plates appended to a recent publication of the Zurich Observatory by Prof. Wolfer, already mentioned before.

The striking similarity of these prominences to the clouds in our own atmosphere is strongly insisted on by all observers.

There have been frequent instances where no visible connection between the protuberance and the underlying chromosphere could be traced; and some prominences have even been recorded as having originated without any direct connection with the solar body—much in the same way as the invisible moisture in our atmosphere is sometimes suddenly converted into a visible cloud by a critical change of temperature.

Indeed, if we consider the great and manifold changes to which the photospheric masses and the reflecting portions of the overlying atmosphere are constantly subjected by the numerous photospheric currents, we can scarcely escape the conviction that a local increase of radiation into the higher parts of the atmosphere must occasionally happen through the sudden displacement of absorbing matter in the lower strata. If, over such an area, there should chance to be an accumulation of hydrogen cooled below the point of luminosity, the sudden accession of heat due to the sharp increase of radiation from beneath would, under specially favourable conditions, be sufficient to eventually restore the hydrogen to its former state of incandescence.

SECTION VI.

THE VORTICOSE NATURE OF SUNSPOTS.

68. In an earlier section of this inquiry we arrived at the conclusion that solar spots must be regarded as vortices in the photosphere, caused by the collision of two oppositely directed currents. This result, so far, is identical with that derived from Faye's spot-theory.

Comparison of the new spot-theory with that of M. Faye.

It need scarcely be mentioned, however, that the causes to which we have been led to attribute the generation of these currents differ widely from the hypotheses of that eminent astronomer. This will be at once apparent from the following very lucid exposition of Faye's theory given by Professor Young:—

"Faye supposes the sun's peculiar law of rotation to be due to the fact that the ascending masses of vapours (which form the photosphere by their condensation) start from a stratum whose depth below the visible surface regularly diminishes from the equator towards the poles. Hence result currents parallel to the equator, and the consequence is that, generally speaking, neighbouring portions of the photosphere have a relative drift. At the equator and at the poles this drift vanishes, but is most considerable in the middle latitudes. Now, it is Faye's theory that, in consequence of this relative drift, eddies are formed . . ., these eddies become cyclones or whirls precisely analogous to those seen in water where a rapid current is obstructed by an obstacle."

69. The chief objection urged against this theory is the feebleness of the drift which, in Faye's view, is the principal factor in spot-generation. The theory here propounded, on the other hand, is free from this objection; for, judging from their effects on the rotational phenomena, the currents which we suppose to be the primary cause of spot-development must be extremely powerful, and thus quite capable of producing vast gyrations at their place of encounter. But even when this objection is disposed of, we have yet to see whether the vortex theory really accounts for all the complicated phenomena observed in solar spots.

Chief objection to Faye's theory.

70. The striking characteristic of the relative darkness of spots is easily explained. For since the gyratory motion of such a vortex tends to draw down with considerable force the absorbing and reflecting elements of the neighbouring atmosphere, and to compress them more and more strongly the nearer they are brought to the apex, an increase of both light- and heat-absorption in the funnel of the spot will naturally ensue.

Explanation of the darkness of a spot. The matter filling the cavity identical with the ordinary solar atmosphere.

Inasmuch as the atmosphere consists of gases and metallic vapours as well as of pulverulent solid particles, their increased power of absorption will be indicated respectively by a broadening of the lines peculiar to these gases and vapours, and by an increase of general absorption, especially in the blue end of the spectrum. In fact,

the matter collected in the cone of a sunspot is, in chemical composition, absolutely identical with the constituents of the general solar atmosphere, but differs in point of temperature and density.

Possibility of a theoretical comparison between the radiation of a spot and that of the photosphere.

71. As regards these two differentiating properties, at least approximate mathematical deductions may be made from the principles of vortex motion; and it would therefore appear to some extent practicable to institute a comparison between the amount of energy radiated into space from any point of the photosphere within a spot, and that emitted by an average point on the undisturbed photospheric surface.

Such a comparison, as it happens, has actually been made already from direct observation; so that we are thus afforded a means of testing the sufficiency of the results obtained from our proposed theoretical inquiry.

Mr. Maunder's objection to the absorption theory.

72. In the reports of the Brit. Astron. Assoc., Vol. vii., No. 3, Mr. Maunder publishes a paper in which he attacks the assumption that solar spots are phenomena of increased absorption. He strongly urges that "if the main cause of the darkness of a spot be increased absorption, then, whether the spot be a depression or an elevation, the effect of foreshortening will be to greatly increase the amount of that absorption." The consequence, therefore, in his view, should be an increase in the darkness of the spot when approaching the solar limb; but as this conclusion is by no means borne out by observation, we should have to ascribe the darkness of a spot to a decrease of radiation rather than to an increased absorption.

At first sight, no doubt, this objection appears very plausible, especially as it seems to be corroborated by the observations of Langley, Frost, and Wilson regarding the thermal radiation of spots. These observations, to which we shall have occasion to revert later on, prove clearly the absence of an appreciable decrease of heat-radiation from the spot when approaching the limb; and this fact, when considered from the point of view of the absorption theory, certainly indicates that a ray proceeding from the photosphere within a spot always traverses the same quantity of absorbing material whether the spot be in the middle of the solar disc or in the immediate vicinity of the limb.

Vertical changes of density in a vortex.

73. Nevertheless we are unable to accept the general application of Mr. Maunder's objection, on the ground that he has underrated the important part which, under certain circumstances, is played by the distribution of density in the absorbing medium.

A vortex generated in a homogeneous fluid assumes approximately the form of a cone, the axis of which, in the case under discussion, may be supposed to be directed towards the sun's centre. The absorbing matter filling the vortex may also be assumed to have a conical shape, and it is when the density of this matter increases towards the apex that Mr. Maunder's argument fails to hold good.

In Fig. 16 let A C B represent a central section through the cone perpendicular to the photosphere; O, the centre of the spot as seen from the earth, being the point towards which the measuring apparatus is directed.

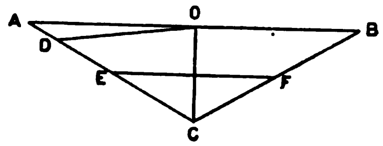


Fig. 16.

If the spot is in the middle of the solar disc, then the apparatus receives the rays starting from the point C of the photosphere, and passing through the cone along C O; but when the spot is near the limb, the rays falling on the apparatus will start from D and pass along D O. Now, although D O may be longer than C O, the conditions of density may nevertheless be such as to affect the shorter ray C O more strongly than D O. If, for example, the density of the matter increases from O towards C, so that the density along E F is greater than along A B, the ray D O will traverse on its whole path a relatively rare medium, while C O, which passes through the denser layers near C, may on this account experience just as much or even more absorption.

Now, in a vortex the conditions for such a distribution of density are certain to exist.

Let A B and E F (Fig. 17) be the diameters of two sections at right angles to the axis of the cone, both of equal thickness. The law of continuity requires that in a unit of time equal masses pass through each of these sections.

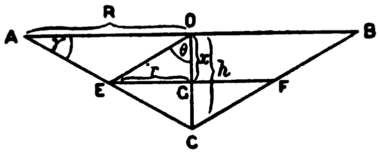


Fig. 17.

Hence it follows that the respective densities are inversely proportional to the areas of the sections. If the distribution of absorbing matter were supposed to be uniform at every point of the same horizontal section—the density being thus exactly the same at A, at O, or at any other point in the section whose diameter is A B—then it would be easy to determine the whole amount of absorbing matter encountered by any ray on its way through the cone towards O. For if, on this assumption, we denote by ρ_0 the quantity of absorbing elements contained in a given unit of volume of the section A B, and by ρ the quantity in the same unit in a section at a distance O G from O, then, putting O G $= x$, A O $=$ R, E G $= r$, and C O $= h$, we obtain the following relation :—

$$\rho : \rho_0 :: R^2 : r^2 :: h^2 : (h-x)^2$$
$$\therefore \rho = \rho_0 \frac{h^2}{(h-x)^2}$$

For the unit of volume let us take a cylinder with the radius $d\,r$ and the height $d\,h$. Let us now consider a cylindrical pencil of rays with the radius $d\,r$ starting from C in the direction of O. To find the whole quantity of absorbing elements encountered by such a cylinder from G to O, we have to integrate the above expression from o to x:—

$$\int_o^x \rho\,d\,x = \rho_0\,h^2 \int_o^x \frac{d\,x}{(h-x)^2}$$

If now we take a cylinder of rays with the same radius passing in the direction E O, then, denoting the angle E O G by θ, we obtain for the whole quantity of absorbing elements along E O:—

$$\rho_0 h^2 \operatorname{Sec} \theta \int_0^x \frac{d\,x}{(h - x)^2}$$

$$i.e. \qquad \rho_0 \frac{h\,x}{h - x} \operatorname{Sec} \theta \qquad (1).$$

Thus, provided that ρ_0, h, and the angle of the cone, γ, are known quantities, the whole sum of absorbing elements could be computed for any cylinder of rays starting from any point on the inner surface of the spot, and travelling towards O.

But, apart from the inadmissibility of the assumption as to the homogeneity of density throughout each horizontal layer—of which more hereafter—there are two other reasons which forbid the acceptance of (1) in its present form.

The formula not applicable near the apex of the cone.

74. First, the hypothesis that A C and B C are straight lines involves the consequence that the density of the absorbing matter at C shall be infinitely great—a conclusion by no means in agreement with fact. Indeed, what has really to be taken as the true form of the vortex is not so much a cone as a body of rotation, the axial section of which is represented in Fig. 18, where the arrows indicate the directions of the motions in the photospheric matter (*cf.* Faye, "Explication des taches solaires;" Comptes Rendus 76, 1873, p. 391).

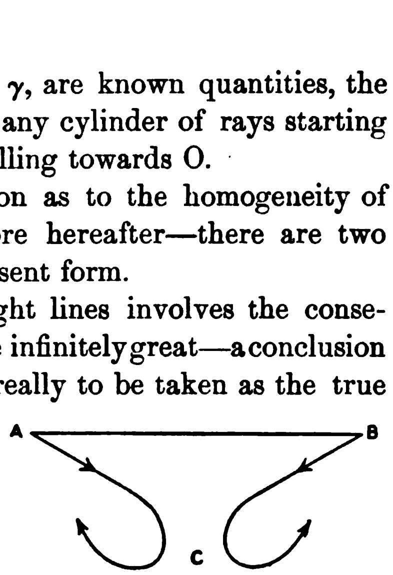


Fig. 18.

The expression (1) is therefore not applicable to very small values of r and θ.

Influence of heat generated by compression on the absorptive power of the solid atmospheric particles in the spot.

75. The second reason which makes it impossible for (1) to correspond with actual fact is of even greater importance, inasmuch as it applies to all the horizontal layers within the spot, and not only to those near C.

The principal absorption of light and heat, as has been repeatedly stated, is caused by dust-like solid particles suspended in the gases and vapours of the atmosphere. The compression of these gases within the vortex is bound to generate a certain amount of heat, sufficient to appreciably raise the temperature of the solid elements and thereby reduce their power of absorption, or, what comes to the same thing, to increase their power of radiation. Thus a particle of given dimensions at the level A B has, by being cooler, a greater absorptive power than a similar equal particle at C, and to neglect this temperature influence in our computations would necessarily lead to an erroneous result.

Elimination of the heat-influence by altering the assumptions as to density.

76. Let us then consider two units of volume at different levels inside the vortex; let ρ_0 be the number of particles all of equal dimensions in the one, and ρ_1 the number in the other—ρ_1 being $> \rho_0$. If the temperature in both units were the same, and if α represent the quantity of radiation absorbed in a single particle, then the whole amount of heat absorbed by the first unit would be $\alpha\,\rho_0$ and by the second $\alpha\,\rho_1$.

But if the temperature of the first unit be lower than that of the second, the quantities of heat absorbed by them would be denoted by $\alpha_0\,\rho_0$ and $\alpha_1\,\rho_1$ respectively, where $\rho\alpha_0 > \alpha_1$. If now we take $\alpha_1\,\rho_1 = \alpha_0\,\rho'_1$, this equation would indicate that instead of the actual number of particles ρ, of the absorptive power α_1, a fictitious number of particles $'\rho_1$ have been substituted of the same absorptive power as those contained in the

first unit. The total effect of absorption is absolutely the same in both cases; so that by assuming a different density the influence exerted by increased temperature on the absorptive power of the affected elements may always be accounted for.

By an appropriate modification of the law expressing the change of density, we may therefore, in any case, justify the assumption that the absorptive power of a particle remains unchanged on its whole way through the vortex. It remains, then, to ascertain what modification of the above law, regarding the changes of density in a spot vortex, is required in order to comply with this assumption.

Let us suppose for a moment the extreme case in which by moving the absorbing matter from a horizontal layer, A B, to another, A′ B′ (Fig. 19), the heat generated by compression is just sufficient to counterbalance the increase of absorptive power caused by increased density. If we take a unit of volume in each of the two layers, this supposition evidently means that while the number of elements contained in such a unit is greater in A′ B′ than in A B, at the same time the temperature in the former will be greater than in the latter. Thus, while the increase of density would increase the absorptive power of the unit in A′ B′, the simultaneous rise of temperature in its constituents would have quite the contrary effect—sufficiently so in this case to neutralise the other. Thus a ray traversing A B suffers the same amount of heat absorption as when crossing A′ B′ under the same angle. Furthermore, the generated heat being a direct effect of compression, a complete compensation of the one effect by the other must take place throughout the whole cone; hence in such a case any two layers of the cone of equal thickness will have the same absorptive power whatever be their level.

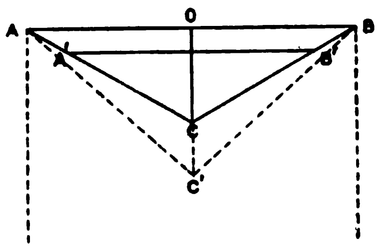


Fig. 19.

Now, exactly the same distribution of thermal opacity as has been found to exist in this cone must also exist in a cylinder on the same base A B, and of uniform density throughout. Since, then, in the case of this cylinder, the point C becomes infinitely removed, the relation between the whole absorbing effects of the layers A B, A′ B′, will be obtained by substituting ∞ for h in the expression (1). This leads to the equation $\rho' = \rho_0$, which likewise results from our present hypothesis. The actual conditions in a spot will not, of course, correspond to this extreme case any more than they will to that in which the absorptive power is not influenced at all by the heat resulting from compression of the particles.

The reality, in fact, will be found somewhere between the two extremes: thus the supposed cylinder must be replaced by a cone, the length of whose axis is $> h < \infty$. The number of absorbing elements per unit of volume in any horizontal section of this new cone is determined by the relation

$$\rho' = \rho_0 \frac{h'^2}{(h' - x)^2}\,;\; h < h' < \infty\,.$$

So that the absorptive power of any unit of volume in a section at a distance x from A B is obtained from

$$\alpha_0 \rho' = \alpha_0 \rho_0 \frac{h'^2}{(h' - x)^2} \qquad (2).$$

Formula representing the quantity of absorbing particles encountered by a cylinder of rays passing in any direction through the spot.

77. Let us now consider a cylinder of rays, with the radius dr, starting from the photosphere at E and passing through the vortex in the direction of O. We require to find the quantity of elements encountered by these rays on their whole journey from E to O (Fig. 20).

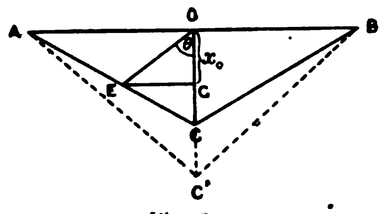


Fig. 20.

For this purpose we have to integrate the right hand side of (2) according to x from o to x_0, and to multiply the result by sec θ. We thus obtain

$$\int_o^{x_0} \rho' \, dx = \rho_0 \, h'^2 \, Sec \, \theta \int_o^{x_0} \frac{dx}{(h' - x)^2} = \rho_0 \, Sec \, \theta \, \frac{h' \, x_0}{h' - x_0} \qquad (3).$$

Horizontal changes of density in the vortex.

78. In the foregoing deductions we have taken into account only the vertical changes of density in the vortex, assuming that in each horizontal layer A B or E F the density is the same for every point along A B or E F. Such a supposition must, however, be far from the actual fact.

Indeed, it is quite evident that in none of these horizontal layers the density is uniform, but that it must increase towards the centre of each. The next step, then, is to find an, at least, approximate mathematical expression for this horizontal change of density.

Relation between velocity and pressure of wind.

79. According to a fundamental law of anemometry, the relation between the pressure of the wind, p, and its velocity, u, is expressed by the relation

$$p = p_0 (1 - e u^2)^* \qquad (4),$$

where p_0 indicates the wind pressure for $u = o$, *i.e.*, for the case where the whole velocity is annihilated.

* An analytical deduction of this equation is found in Ferrel's treatise "On Cyclones, Waterspouts, and Tornadoes;" U.S. Coast and Geodetic Survey, App. No. 10, Report of 1878, p. 74, under (j)

His equation has the somewhat different form

$$a\left(1 - \frac{P}{P_0}\right) = u^2 - u_0^2$$

where P_0 and u_0 indicate the *initial* pressure and velocity of the wind. If, however, we introduce p_0 = the pressure at the moment when $u = 0$, we can write this equation in the form

$$P = p_0 \left(1 - \frac{P_0}{a \, p_0} u^2\right)$$

which agrees with that given above.

Thus, if the changes in the horizontal component of velocity with which, in our vortex, the atmospheric matter is impelled from A to O or from E to G were known, the horizontal changes of density could be obtained by means of equation (4). Generally speaking, the horizontal component (u) of the wind-velocity in the vortex will be a certain function of r, the distance from the axis, so that

$$u = \phi(r)$$

$$\phi(r) = a + br + cr^2 + \ldots\ldots\ldots + \frac{b^1}{r} + \frac{c^1}{r^2} + \ldots\ldots\ldots$$

Now, one fact at least is certain—namely, that the *horizontal* velocity in the centre O or G must be zero. For any particle arriving at O must encounter another particle coming from the opposite direction with the same velocity, and the two will mutually destroy one another's horizontal motion. Hence, in the above expression for $\phi(r)$, the coefficients of all the powers of r smaller than the first must disappear, thus yielding the simpler formula

$$u = \phi(r) = br + cr^2 + \ldots\ldots\ldots \qquad (5).$$

Expression for the horizontal change of pressure in the vortex. Final formula for the whole quantity of absorbing elements in any direction.

80. The simplest assumption would of course be

$$u = b\,r.$$

In this case, which shall here be treated as a first approximation,

$$\int_o^{x_0} \rho'\,dx = \rho_0\,h'^2\,Sec\,\theta \int_o^{x_0} \frac{1 - \epsilon\,x^2\,tg^2\,\theta}{(h' - x)^2}\,dx \qquad (6).$$

But

$$\int_o^{x_0} \frac{x^2\,dx}{(h' - x)^2} = \frac{h'\,x_0}{h' - x_0} + 2\,h'\,ln\,\frac{h' - x_0}{h'} + x_0$$

If we now consider two pencils of rays E O and E′ O (Fig. 21), the influence of horizontal changes of density, depending on the distance of the particle from the axis of gyration, will obviously be greater for E O, which encounters particles at various distances from this axis, than for E′ O, which passes through absorbing elements all of which lie close to O C. But we have $\frac{\text{O G}}{\text{O C}'} = \frac{x^\circ}{h'}$ (Fig. 20), and this quotient, which is always less than unity, is smaller for E O than for E′ O. Hence we may conclude that for those rays that are mostly affected by the horizontal changes of density, $\frac{x_0}{h'}$ is a comparatively small fraction. Under these circumstances we may write

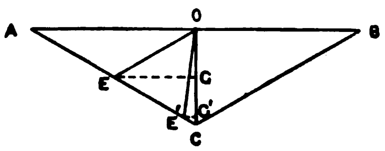


Fig. 21.

$$ln\,\frac{h' - x_0}{h'} = -\,\frac{x_0}{h'} - \frac{x_0^{\,2}}{2\,h'^2},$$

neglecting the higher terms of $\frac{x_o}{h'}$, whence we derive

$$\int_o^{x_o} \rho' \, dx = \rho_o \, Sec \, \theta \, \frac{h' x_o}{h' (h' - x_o)} [1 - \epsilon r_o^2] \qquad (7).$$

Theoretical relation between the radiation of a spot and that of the normal photosphere. The ratio U/C.

81. Now let us assume a pencil of rays with the radius dr, starting from a photospheric point O in the centre of the solar disc, and traversing the solar atmosphere P P′ Q′ Q in a vertical direction (Fig. 22).

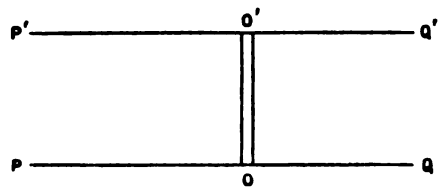


Fig. 22.

Denoting by A the intensity of the pencil when it starts from O, by C its intensity when it arrives at the upper limit of the solar atmosphere O′, and further putting S for the whole sum of absorbing particles encountered along O O′, we obtain, according to Bouguer-Lambert, the relation

$$C = A \cdot e^{-\alpha_o S}.$$

On the other hand, for a cylinder of rays with the same radius dr starting from a point E of the photosphere within a spot (Fig. 21), and traversing the vortex along E O, we have

$$U = A \cdot e^{-\alpha_o \int_o^{x_o} \rho' \, dx}.$$

From these two relations we derive

$$\frac{U}{C} = e^{-\alpha_o \int_o^{x_o} \rho' \, dx + \alpha_o S}.$$

But Prof. Frost's investigations (Astr. Nachr., Bd. 130) show that

$$e^{-\alpha_o S} = 0{\cdot}72,$$

so that we finally obtain

$$\frac{U}{C} = 1{\cdot}389 \, e^{-\alpha_o \int_o^{x_o} \rho' \, dx} \qquad (8).$$

The ratio U/C found from observations.

82. This fraction $\frac{U}{C}$, being the ratio between the thermal radiation of the spot's umbra and that of the photosphere at the centre of the solar disc, has been carefully determined by important series of observations recently made by Mr. Wilson* and by Professor Frost.†

* The Thermal Radiation of Sun Spots; Observations made at Daramona, Streete, Co. West Meath. Monthly Notices, Vol. 55, p. 457.

† Observations on the Thermal Absorption in the Solar Atmosphere made at Potsdam. A.N., Vol. 130, p. 129.

Both observers compared the heat emanating from the centre of the umbræ of a great number of spots in various positions on the sun's face with the photospheric radiation at the centre of the disc.

Mr. Wilson, whose series is by far the more extensive, has arrived at two remarkable results. The first of these brings out the fact already mentioned that the ratio $\frac{U}{C}$ is subject to scarcely any variation during the progress of a spot across the solar disc. The second refers to the changes of spot-radiation during a sunspot cycle. The umbral radiation is found to be smallest at the time of spot maxima, and to increase rapidly towards that of minima—thus indicating that the activity in the vortices, and consequently the dyamical force of the colliding currents, is much more powerful near the times of maxima.

For the present inquiry we are concerned only with the first of these conclusions; but, since the phenomenon brought out by the second might seriously affect the accuracy of our results, its influence has to be carefully examined. This may be accomplished by arranging the observations of $\frac{U}{C}$ in different groups, each covering an interval of about half a year. We can thus find the successive changes from year to year in the mean values of these groups; and since the ratio $\frac{U}{C}$ is practically independent of the position of the spot with regard to the centre of the solar disc, the somewhat irregular distribution of spots in the single groups relatively to their distance from the centre of the solar face will not appreciably affect the results. By these means it is possible to reduce to a fixed epoch a measurement made at any time during the period of observation, and thereby to eliminate the effects of Mr. Wilson's second result.

In the following Table we give for different epochs the factors by which his observed values must be multiplied to reduce them to 1893, October 1, the date adopted as the mean epoch for our further investigation.

1893—Aug.-Nov.,	1·00	1894—Sept. 1,	0·73
1894—June 1,	0·95	Oct. 1,	0·68
July 1,	0·87	Nov. 1,	0·64
August 1,	0·80	Dec. 1,	0·60

Having thus reduced all the observations to one and the same epoch, or, in other words, to a fixed standard of gyratory activity, we rearrange the corrected data into different groups according to the distances of the spots from the centre of the solar disc. The result thus obtained may be considered as a sufficiently accurate expression for the change in apparent radiant power which a normally developed spot of constant gyratory activity undergoes with change of distance from the centre of the sun's face. These changes are exhibited in the subjoined Table :—

Distance from centre in parts of ⊙'s diameter.	θ	U/C.
11·0	6°·3	0·344
33·5	19°·6	0·372
42·9	25°·4	0.359
56·3	34°·3	0·361
67·1	42°·1	0·359
81·4	54°·5	0·346
86·8	60°·2	0·335
92·3	67°·4	0 323
96·7	75°·2	0·334

$$Sin\ \theta = \frac{Distance.}{100}$$

Comparison between theory and observation. Determination of a vortex the conditions in which satisfy the observed values of U/C as nearly as possible.

83. Considering the uncertainty of the observations, the changes in $\frac{U}{C}$ are extremely slight, and afford a striking contrast to the remarkable falling off in the case of the light and heat radiation of the ordinary solar atmosphere. At first sight such a peculiar result might perhaps seem to be a strong argument in favour of Mr. Maunder's objection against the absorption theory. It can be shown, however, that this peculiarity of spot-radiation is perfectly reconcilable with the vortex theory.

In fact, taking the distribution of absorbing particles in a normal gyration as it has been assumed in the preceding investigation, there is no difficulty in finding a cone which perfectly satisfies the conditions of density necessary for the representation of the empiric values of $\frac{U}{C}$. By writing

$$\mu = -\ ln \left[0{\cdot}72 \times \frac{U}{C} \right]$$

we derive from (8) and (7)

$$Sec\ \theta \frac{h'\, x_\circ}{h' - x_\circ}(1 - \epsilon\, r_\circ^{\,2}) = \mu\, a \rho_\circ \qquad (9).$$

But we may express $x_\circ$ in terms of the height of the cone, h, and the angle at the base, γ. Evidently we have

$$x_\circ = \frac{h\ Cos\ \theta}{Cos\ \theta + tan\ \gamma\ Sin\ \theta} = h\ Cos\ \gamma \frac{Sec\ (\theta - \gamma)}{Sec\ \theta.}$$

By substituting this value in (9) we obtain the following relation :—

$$\left(\frac{1}{h} - \frac{1}{h'}\right) Cos\ \theta + \frac{1}{h} tan\ \gamma\ Sin\ \theta + \epsilon \frac{a\ \rho_\circ}{\mu} r_\circ^{\,2} = \frac{a\ \rho_\circ}{\mu} \qquad (10).$$

Since $\frac{a\rho_\circ}{\mu}$ is nearly a constant for all values of θ, we may put

$$\frac{\epsilon\, a\, \rho_\circ}{\mu} = \epsilon_\circ$$

where ρ_0 signifies, as before, the quantity of absorbing elements in a unit of volume of the uppermost layer A B.

Now, if we wish to express the heights h and h' in units of a distinct standard, we must fix upon a certain depth for the assumed unit of volume. Obviously the ordinary height of the dust-like matter in the sun's undisturbed atmosphere is the most convenient unit of depth for this purpose, since in this case we have

$$\rho_\circ = S,$$

and therefore

$$\alpha\, \rho_\circ = ln \left(e^{\, - \, \alpha\, S} \right) = - \frac{log\ 0{\cdot}72}{log\ e}, \qquad (11).$$

so that we obtain

$$\frac{\alpha\, \rho_\circ}{\mu} = \frac{log\ 0{\cdot}72}{log \left[0{\cdot}72 \times \frac{U}{C} \right]}$$

If now this expression be introduced into (10) there results the following equation :—

$$\left(\frac{1}{h} - \frac{1}{h'} \right) Cos\ \theta + \frac{1}{h}\ tan\ \gamma\ Sin\ \theta + \epsilon_\circ\, r_\circ^{\,2} = \frac{log\ 0{\cdot}72}{log \left[0{\cdot}72 \times \frac{U}{C} \right]} \qquad (12.)$$

$$\text{where } r_\circ = \frac{h\ Sin\ \theta}{Cos\ \theta + tan\ \gamma\ Sin\ \theta.}$$

It is clear that the unknowns h and γ cannot be separated. We therefore proceed in this way that we assume different values for the angle γ, and solve the system of equations for each of these values, thus obtaining in each case a set of numerical values for h, h', and $\epsilon_\circ$, as follows :—

γ	h	h'	$\epsilon_\circ'$
5°	0·80	0·98	0·01503
10°	1·69	2·70	0·00526
15°	2·72	6·93	0·00306
20°	3·95	35·84	0·00212

(13).

84. Each of these four systems of values satisfies the observed ratios $\frac{U}{C}$ almost equally well, the sum of the squares of the residuals being much the same in each case. No distinct preference can therefore be given to any one set, but to show by a single

Agreement between computed and observed values of U/C.

example the agreement between theory and observation we select the third system of values—viz., $\gamma = 15°$; $h = 2{\cdot}72$; $h' = 6{\cdot}93$; $\epsilon_o = 0{\cdot}00306$.

From (12) we then obtain

$$\log \frac{U}{C} = \frac{\log 0{\cdot}72}{\left(\frac{1}{h} - \frac{1}{h'}\right) \operatorname{Cos}\theta + \frac{1}{h} \tan\gamma \operatorname{Sin}\theta + \epsilon_o r_o^2} - \log 0{\cdot}72, \qquad (14).$$

a relation which gives the ratio $\frac{U}{C}$ for any value of θ.

The following Table contains the quotient $\frac{U}{C}$ thus computed, as compared with the results of the observations given on page 137.

θ.	$\frac{U}{C}$ *Computed.*	$\frac{U}{C}$ *Observed.*	*Obs.—Comp.*
6°·3	0·340	0·344	+ 0·004
19°·6	0·364	0·372	+ 0·008
25°·4	0·368	0·359	− 0·009
34°·3	0·368	0·361	− 0·007
42°·1	0·360	0·359	− 0·001
54°·5	0·341	0·346	+ 0·005
60°·2	0·329	0·335	+ 0·006
67°·4	0·322	0·323	+ 0·001
75°·2	0·337	0·334	− 0·003

To illustrate graphically the agreement between the computed and the observed results Fig. 23 has been prepared, in which the full drawn curve indicates the results of observation, and the dotted line represents those deduced from equation (14).

The chief result brought out by the vortex theory is the small change in umbral radiation with increased distance from the centre of the sun's disc, and even the slight changes shown in the observed values are closely followed by the curve derived from (14).

This agreement holds good at least as far as $\theta = 75°$, which corresponds to a central distance of 0·966 of the sun's radius, or to the extreme practical limit of observation.

Maximum value for angle at base of vortex.

85. Unfortunately the present state of the problem does not permit of separating h, the depth of the spot-cone, from γ, the angle of the slope, and it is therefore impossible as yet to derive the true proportions of the normal vortex.

But our formulæ indicate a special limiting value for the inner slope of the spot-cone. A reference to Table (13), page 139, will show the rapid increase of h' for increasing values of γ. As a matter of fact, when $\gamma = 21°$, h' becomes infinite; thus indicating, as we have seen, that the whole increase of density downwards would, in its thermal effects, be exactly counterbalanced by the increase of heat due to compression; so that all layers of equal thickness would have the same power of absorption, whatever be their distance from the apex.

If γ is greater than 21°, h' becomes negative. In such a case the absorptive power of a particle within the cone would decrease with increasing density. If this were so,

however, the centre of the umbra ought to be hotter and brighter than the margin of the spot—a conclusion quite at variance with the observed facts.

Hence the angle γ cannot exceed 21°, and thus the slope of an average spot-cone is relatively small.

86. One interesting fact resulting from these considerations may be mentioned. If R be the radius of the uppermost layer $A\ B$, *i.e.* of the visible surface of the spot, we have the relation

Height of absorbing envelope round the photosphere.

$$R = h \operatorname{Cot} \gamma.$$

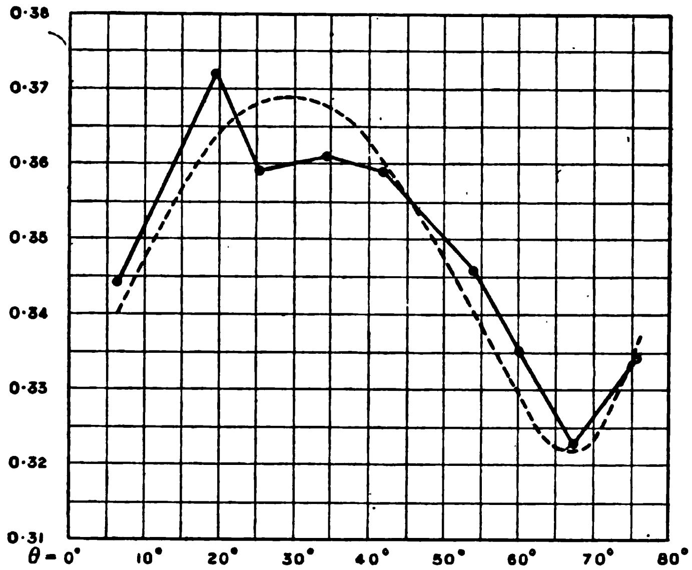


Fig. 23.

By substituting for h the values given in (13)—remembering that the unit adopted is the height of the ordinary absorbing atmosphere—we obtain the following values for R:—

γ.	R.
5°	9·2 × height of absorbing atmosphere.
10°	9·6 × " " "
15°	10·2 × " " "
20°	10·9 × " " "

Now, the average diameter of the spots investigated in Mr. Wilson's observations here discussed may safely be assumed to have been not more than 20,000 miles, whence the height of the absorbing solar envelope cannot be supposed to exceed 1000 miles—corresponding to about two seconds of arc. Thus the upper limit of the envelope which effects the principal absorption of light and heat would lie far below the surface of the chromosphere, and would, on the solar scale, represent an extremely thin layer immediately overlying the radiating photosphere.

Shallowness of absorbing layer confirmed by Langley's observations.

87. This conclusion, arrived at by essentially theoretical deductions, appears to be fully confirmed by observation. From a paper by Prof. Langley in "American Journal of Science," 1875, page 488, we quote the following passage to this effect:—"The portion of this atmosphere chiefly concerned in absorption I have been led to believe from several considerations to be extremely thin, and I am inclined to think it is nearly identical with the 'reversing layer' at the base of the chromosphere observed by Secchi and Young."

Recapitulation of results.

88. We are then, in the light of these results, entitled to say that Mr. Maunder's objection to the absorption theory does not hold good under the assumption that spots are caused by gyratory motions of the photosphere.

On the contrary, the peculiar characteristics of spot-radiation into space, which apparently offer a strong and, at first sight, convincing argument against the absorption theory, receive a most striking explanation from our point of view.

It is true that the mathematical solution can make no pretence of giving anything like an exhaustive treatment of the question, and is meant to serve only as a first approximation. But no physicist familiar with the intricacies of the subject can be surprised at this. The essential point is whether or not the assumptions we have been compelled to make are in accordance with the fundamental principles of the problem, and, judging from the results, and their close agreement with those of observation, the answer would appear to be distinctly affirmative.

Definition of Penumbra.

89. There are thus good reasons for believing that a solar spot is caused by the collision of two photospheric currents, resulting in a gyrating motion. The relative darkness and coolness of the spot are due, then, to the absorption which the waves of light and heat undergo through the influence of the atmospheric matter attracted from every side towards the centre of the vortex, and which in its downward rush is subjected to a rapidly increasing compression as the apex is approached.

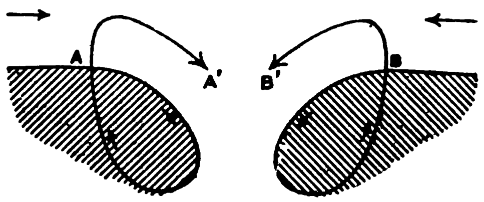


Fig. 24.

Now, when it is considered that near A and B in a vortex (Fig. 24) the ascending motion must be particularly energetic, the granulations at such points may be inferred to rise above the normal level of the photosphere, and, by falling a prey to the centripetal currents of the atmosphere, to be gradually deflected towards the axis of the vortex, thus forming the penumbra of the spot.*

Reason why spots as a rule exhibit no *visible* vorticose motions.

90. Here, however, it might be objected, that if a spot is really a gyration, its visible portions, and especially the penumbra, ought to exhibit some evidence of this rotatory motion. Now, observations, it is true, show very little evidence of such a motion. This, however, may be readily accounted for. The points A, B, at which the ascending currents rise above the level of the photosphere, being at the outer limit of

* This definition of the penumbra, so far as we know, was first propounded by Professor Hastings, of Baltimore. (*See* Young, page 337.)

the gyration, cannot show any appreciable signs of vorticose motion. Owing, moreover, to the slow rotation of the sun, the attracted atmosphere will move in an almost absolutely centripetal direction, thus offering little chance for the deflected granulation currents to exhibit appreciable deviations from a radial course, at least so long as they have not come into the sphere of action of the gyratory forces in the interior parts of the vortex. By the time this does happen, the granulations will have been forced down so deeply into the cone that their light will be obscured by the overlying absorbing matter. Thus the obvious fact that the visible parts of the penumbra can belong only to the surface layers of the vortex, where gyratory motion is insignificant, sufficiently explains why the vorticose rotation has been so rarely observed.*

Explanation of Mr. Maunder's observation that the umbra is less foreshortened than the entire spot.

91. It may not perhaps be out of place here to notice a somewhat enigmatical phenomenon mentioned by Mr. Maunder, the explanation of which now offers no great difficulty.

By an extensive series of observations Mr. Maunder endeavoured to discover whether or not the so-called "Wilsonian effect" could be proved to exist. He arrived at the conclusion that in a small majority of cases indications of this effect could indeed be traced, though on the whole these were extremely feeble. He then continues:—

"But a far more important result is, I think, clearly established. The umbra in a circular spot becomes less elliptical by foreshortening than does the entire spot. In other words, its minor axis is longer in proportion to that of the whole spot than its major. This is clearly marked. In extreme cases the umbra completely stretches across the spot when the latter is much foreshortened, and remains sensibly circular when the whole spot is perhaps twice as long as it is wide."

It has already been remarked that the currents of the penumbra can be visible only so long as the overlying absorbing matter allows the light-radiation to pass through. At a certain depth, however, the density of the gases and solid absorbing particles within the spot must become so great as to absorb all the penumbral luminosity. This, naturally, is the place where the visible umbra begins.†

* From this point of view we find an explanation of the fact that the few cases of gyratory motion observed by Secchi have always been found in spots in the first phase of their development. (*Cf.* Secchi-Schellen, page 84:—"Solche Drehströme beobachteten wir übrigens nur beim Entstehen und in den ersten Entwickelungsstadien des Flecks.")

Since some time must elapse before the cavity of a vortex is filled up with atmospheric matter, we find, immediately after the spot's formation, a favourable opportunity for looking into the interior of the vortex, where we then perceive the rapidly gyrating photosphere near the apex, which afterwards will be covered by dense layers of atmospheric matter. The vorticose nature of spots also explains the following fact observed by Secchi:—"Es gibt eine thatsächliche Erscheinung, welche beweist, dass die Flecke eine gewisse anziehende Kraft besitzen, die Erscheinung nämlich, dass die kleinen Flecke von den grossen absorbiert werden. Man sieht nicht selten, wie die ersteren sich immer mehr der Haupthöhlung nähern, und bald in derselben vollständig verschwinden." (Page 82.)

† That, as a general rule, the limit between umbra and penumbra is pretty sharply defined may perhaps explain itself on the assumption that the downward curvature of the penumbral filaments increases rapidly as they approach the axis.

In Fig. 25 let us assume this locality to be at B' at the time when the spot is in the centre of the solar disc. This means that a sufficient number of absorbing elements exist between F' and B' to prevent any luminous radiation from B' passing into space. A point B'', however, will still be seen in this position of the spot. When the spot approaches the solar limb, we see the same point B'' along $B''\,G''$. A ray in this direction traverses the same absorbing layers as does $B''\,E''$, save that its path through them is now considerably longer. For this reason, then, the point B'' may no longer be visible. In other words, the farther the spot travels away from the centre of the solar disc towards the limb, the farther will the boundary of the umbra stretch across the side of the spot turned towards the limb. The umbra will therefore appear less foreshortened than the outward limit of the spot, the latter thereby assuming the appearance sketched in Fig. 26. This phenomenon obviously counteracts, to a certain extent, the "Wilsonian" effect, and thus accounts for the very small majority of cases in which this latter phenomenon is exhibited.

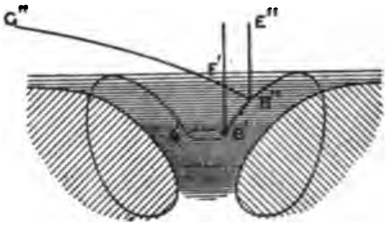


Fig. 25.

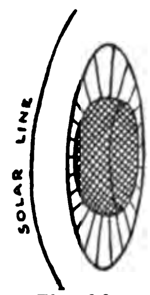


Fig. 26.

Heat-absorption in a spot less than light-absorption.

92. Still another important point receives additional light from the preceding considerations. We know from observation that the thermal spot-radiation at the time of a minimum greatly exceeds that near a maximum.

But in many cases the penumbra appears to be brighter at the inner edge, thus betraying a greater power of radiation notwithstanding the increased density of absorbing matter. Of this fact an ingenious explanation has been advanced by Professor Hastings, who says (Young, "The Sun," 1895, page 337) :—

"The upward convection currents in the region of the spots are bent horizontally by the centripetal winds. Yielding their heat now by the relatively slow process of radiation, the *loci* of precipitation are much elongated, thus giving the region immediately surrounding a spot the characteristic radial structure of the penumbra. This conception of the nature of the penumbra implies a ready interpretation of a remarkable phenomenon, amply tested by the most skilful observers, and, as far as my knowledge goes, wholly unexplained—namely, the brightening of the inner edge of the penumbra in every well developed spot.

"This interpretation is perhaps most readily imparted by a comparison of the hot convection currents in the two cases. When the convection current is rising vertically, the medium is cooled by expansion until the precipitation temperature is reached, when all the condensible material appears *suddenly*, save as it is somewhat retarded by the heat liberated in the act. Immediately afterwards the particles become relatively dark by radiation. In the horizontal currents a very different condition of things obtains. Here the medium does not cool dynamically, by expansion, but only by radiation ; hence, since the radiation of the solid particles is enormously greater than that of the supporting gas, practically by that of the particles themselves. Thus, after the first particle appears, it must remain at its brightest incandescence until all the material of which it is composed is precipitated. From this we see that such a horizontal current must increase gradually in brilliancy to its maximum, and then suddenly diminish—an exact accordance with the facts observed."

This theory, which certainly has much in its favour, is submitted to the following critical remarks by Professor Young :—

Wilson in his measurements, made in 1894 (very close to a maximum), found $\frac{U}{C} = 0{\cdot}35$; while Frost's observations, made in 1891-92, at a comparative minimum of solar energy, result in exactly double that value, viz. 0·70.

Now, if we take into consideration the fact that the radiation of a spot remains almost constant from the centre to the limb, while that of the photosphere decreases very considerably, the ratio of the radiation of a spot to that of the *neighbouring* photosphere must increase continuously as the spot approaches the limb. From observation, the radiation of the photosphere at the limb is known to be about 0·4 of that at the centre. It is thus evident that, at a minimum, the radiation of a spot near the limb may even exceed that of the photosphere in its vicinity. This remarkable fact is distinctly exhibited in Prof. Frost's observations (A. N. 3106, p. 144).

Here, one might well ask how it is possible that a spot can ever be warmer than the surrounding photosphere, and still remain, at the same time, so much darker in appearance. The mysterious character of this phenomenon has indeed lately evoked a considerable amount of interest.

93. Various reasons may be offered in explanation of it.

Influence of the heat generated by compression on the spot's radiation.

First, it is quite conceivable that although the heat generated by compression in the vortex must considerably raise the temperature of the absorbing particles, it may affect their state of luminosity in but a slight degree. For, while growing hotter, they may yet remain dark, as a block of metal may be much heated before it begins to glow and thus to alter its optical appearance.

"The received theory regards the general brightening at the inner edge of the penumbra as produced by the conveyance of the luminous filaments rendered horizontal by the indraught. The quasi-bulbous termination of the filaments occurs only occasionally, and may perhaps be accounted for in the way proposed by Mr. Hastings more satisfactorily than in any other. Still, many circumstances seem to show that the brightening at the end is due, like that of the faculæ, to mere protrusion through the smoke veil."

We cannot suppress a doubt respecting the accuracy of Professor Young's conjecture in the last sentence. If the brightness at the inner edge of the penumbra be really due to the same cause as the increased luminosity of the faculæ, we should expect to find it exhibited on the photographs of a solar spot with the same distinctness as the bright patches presented by the faculæ around the spot.

But on all photographs taken by Mr. Janssen, not only is there a much more pronounced darkness of the penumbra, but also a much less striking contrast between penumbra and umbra, than is usually seen in visual observations.

The brightening of the inner edge of the penumbra, generally so conspicuous in the telescope, is indeed almost completely absent on the photographs. We know, however, that the filaments of the penumbra, when coming nearer the centre, must gradually sink down to lower levels, and thus place a greater quantity of absorbent matter above them as they approach the inner edge. This must cause an increase of absorption. But, on the other hand, according to Professor Hastings, the radiating power of the filaments will at the same time increase. Thus we have two effects which tend to neutralise each other. Now, we are familiar with the fact that the absorbing matter has more effect on the *actinic* than on the *luminous* part of the spectrum. We therefore conclude that, in the first place, the penumbra must, on the whole, appear darker—*i.e.*, in stronger contrast to the surrounding photosphere—in photographs than it does in visual observations; and, in the second place, that the brightening of the inner edge must be less conspicuous in the photographs than when directly observed—both of which conclusions are in accordance with the known facts.

As a matter of fact, we are quite familiar with the phenomenon of "dark heat," *i.e.* those radiations of great wave-length which lie in the infra-red part of the spectrum. It is extremely probable that the heat generated by compression is at least partly of such a character, producing a great abundance of infra-red radiations in the spot, which have no effect on the eye, and yet will inevitably betray themselves when tested by the bolometer. Mr. Evershed, indeed, has lately expressed the opinion that this may be actually the case.

Rays of short wave-length more affected. Prof. Vogel's observations.

94. But the heat so generated is by no means the only agent causing the thermal radiation in the spot to exceed that of light.

We know that rays of short wave-length are more freely absorbed by the atmospheric matter than those of greater wave-length—a fact regarding which the observations of Professor Vogel (*see* Young, page 281) leave no doubt whatever.

Let us, for example, consider the two extreme measurements mentioned in Prof. Young's book at $\lambda = 408$ and $\lambda = 662$. The ratio between A, the radiation leaving the photosphere, and I, that entering into space, is represented by the exponential function

$$\frac{I}{A} = e^{-f \, Sec \, \theta},$$

where f indicates a quantity varying with the wave-length, and θ is determined by the relation

$$Sin \, \theta = \rho / R,$$

R being the solar radius, and ρ the distance of the observed place from the centre of the solar disc.

For the centre itself the formula becomes

$$\frac{I_\circ}{A} = e^{-f}.$$

From the observations referred to it is found that for the wave-length 408 in the violet, $e^{-f(v)} = 0{\cdot}54$ (approx.), while for 662 in the red, $e^{-f(r)} = 0{\cdot}78$.

If, instead of the normal atmosphere, we had a material with double the density, the quotient $\frac{I_\circ}{A}$ would be reduced to e^{-2f}.

The intensity of the violet ray would then become $(0{\cdot}54)^2 A_v$, and that of the red ray $(0{\cdot}78)^2 A_r$, etc. In this manner we obtain the following table:—

Density of Absorbing Material expressed in Units of the Solar Atmosphere.	Intensity of Rays transmitted into space after traversing the Absorbing Medium in a Vertical Direction.	
	$\lambda = 408.$	$\lambda = 662.$
1	$0{\cdot}54{\cdot}A_v$	$0{\cdot}78{\cdot}A_r$
2	0·29	0·61
3	0·16	0·47
4	0·08	0·37
5	0·05	0·29

Now, according to our theory, the interior of a sunspot is to be considered as filled with compressed atmosphere. If, then, the density of the absorbing matter in a spot were assumed to be five times that of the ordinary atmosphere, the ratio between the radiation of a spot and that of the surrounding photosphere would be $\frac{0\cdot05}{0\cdot54}$, or practically $\frac{1}{11}$ for the violet rays, and $\frac{0\cdot29}{0\cdot78}$, or more than $\frac{1}{3}$, for the red rays.

We may therefore at once conclude that the contrast between the light-radiation of the spot and of the photosphere must be much greater than that between their heat-radiation—and especially so in the not unlikely probability of a preponderance of *infra-red* radiation in the spot.

Proof from Mr. Wilson's observations that the absorbing atmosphere near a spot is denser than that over spotless parts of the photosphere.

95. Mr. Wilson's observations bring out another very interesting point. If, as is required by our theory, the solar atmosphere is attracted towards the centre of a spot, the absorbing matter, not only in the vortex itself, but even at some distance from it, must be denser than over a spotless photospheric region.

By comparing the thermal radiation of a spot with that of its surrounding photosphere, we should then find that the ratio $\frac{\text{Spot}}{\text{Photosphere}}$ must be greater than if the spot were compared with a region of the photosphere not affected by a spot-disturbance at the same distance from the centre of the solar disc.

For, suppose that the thermal radiation of a spot, at a certain distance ρ from the centre of the disc, is compared first with the radiation of the photosphere at the centre, and then with that of the photosphere in the immediate neighbourhood of the spot-vortex.

Denoting the ratio between umbra and photosphere in the first case by U/C and in the second by U/N, we obtain by division the value of N/C, *i.e.* the ratio of the thermal radiation of the photosphere at the distance ρ to that of the photosphere at the centre of the disc.

Now, if the atmosphere in the vicinity of the spot were in a normal condition, the values of N/C should exhibit the same rate of decrease towards the sun's limb as is shown in observations of the radiation extended over a spotless photosphere.

This, however, is not at all the case, as will be readily seen from the subjoined Table. In the first column are given, for various distances from the centre of the disc, the corresponding values of the ratio N/C, obtained from Mr. Wilson's observations. The second column contains Mr. Wilson's measurements of the radiation in spotless parts of the photosphere, for different values of ρ. The latter values are denoted by (N/C).

ρ.	N/C.	ρ.	$(N/C.)$
11·0	98·8	0	100·0
30·6	97·0	10	99·8
42·4	95·3	20	99·5
55·2	93·4	30	98·9
64·0	88·8	40	97·2
73·6	82·3	50	95·3
85·5	75·2	60	92·2
94·2	53·5	70	87·8
		80	82·5
		90	72·0
		100	42·9

If the atmosphere surrounding a spot-district were not influenced by the conflux towards the vortex, the values of N/C and (N/C) for the same distance ρ should be identical.

But it appears, as is well shown in Fig. 27, that N/C is always appreciably less than (N/C), thus clearly proving that the absorbing atmosphere near a spot is denser than in a spotless region. The fact that the distance between the two curves in Fig. 27 becomes greater as the limit is approached is in perfect accordance with the law of absorption.

Explanation by the vortex theory of phenomena associated with sub-division of spots.

96. The theory that spots are vortex motions in the photosphere may also help to explain some peculiar phenomena usually observed in connection with the sub-division of large spots.

In such a case, as is well known, the separating parts, usually two in number, tend for some time to increase their distance from each other. The angular rotation of the

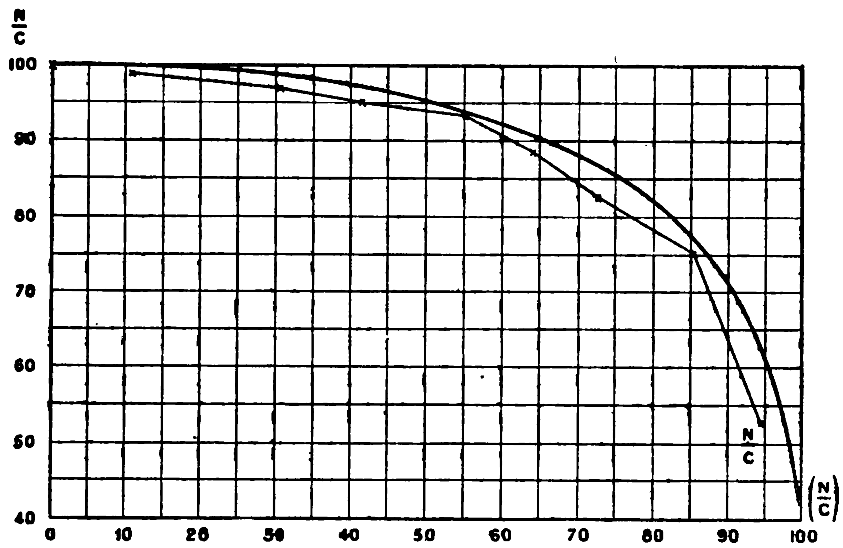


Fig. 27.

preceding spot is generally observed to increase, that of the following spot to decrease. Spörer's measurements, furthermore, reveal the curious fact that the forward drift of the preceding twin is considerably more rapid than the backward drift of the following one, and it is likewise established that the preceding spot, as a rule, has a longer life. Now a little reflection will show that the average direction of gyration, as seen from the earth, will, for a southern spot, be from left to right, *i.e.* clockwise; and for a northern spot in the opposite direction, or contra-clockwise. For a collision between the currents will occur most frequently when the pole-equator current is on the eastern and the equator-pole on the western side. (*See* Fig. 28.) No doubt there will be numerous exceptions to this rule, especially near the time of maximum, when the system of currents and counter-currents becomes extremely complicated. But it is nevertheless to be expected that the majority of cases must be in accordance with the above law, and, as stated by Flammarion,* observation goes far to corroborate this rule.

* L'Astronomie, 1893.

Now, to illustrate the phenomenon observed in spot-division, let us consider a normal vortex on the *northern* solar hemisphere, and gyrating, therefore, from right to left. (Fig. 29*a*.)

Let *B a*, *B′ a′* denote the boundaries of a photospheric current rushing into the vortex at *a a′*.

This current will deform the shape of the vortex in the manner shown in Fig. 29*b*.

Now, the gyration of the vortex at *a* is in a direction almost opposite to that of the intruding current B *a*; while at *a′*, on the contrary, the rotatory motion of the vortex is in the same direction as that of the current B′ *a′*. Thus the two motions at *a* act against one another, whereas at *a′* they act in conjunction. Hence the inrush at *a* will be impeded and its effect diminished, but that at *a′* will be aided and increased.

Consequently, since the current will flow in the direction of least resistance, it will spread chiefly towards the western side, thereby imparting to that part of the

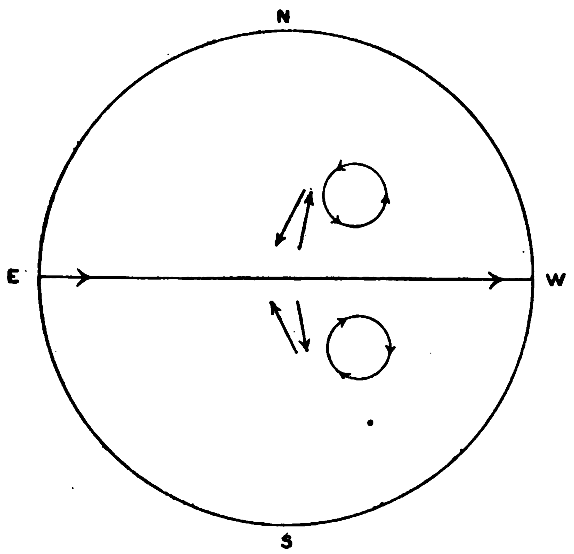


Fig. 28.

spots a greater impulse in the forward direction than it will to the eastern half in the backward direction.

In other words, the increase of angular rotation in the western half will be greater than the decrease of that of the eastern.

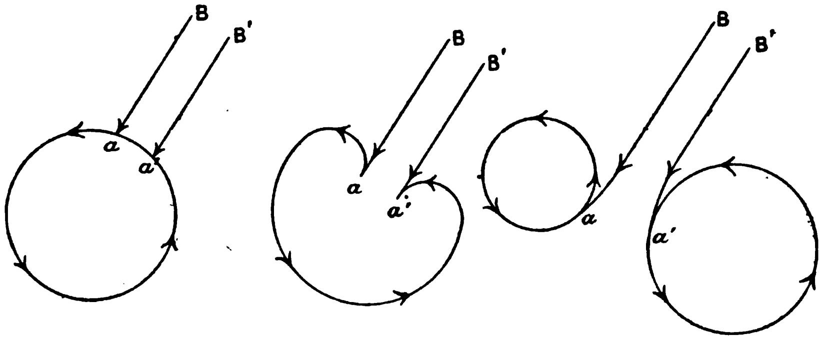
Seeing, moreover, that the inrushing current is continually acting against the rotatory motion of the eastern spot, while favouring that of the western, the activity of the latter will be sustained for a longer time. (Fig. 29*c*.)

The same considerations apply to the fission of a spot in the southern hemisphere

Secchi, in his admirable volume (German edition, Tafel ii., pp. 18-19), gives an excellent typical example of such a sub-division of a spot. The seven photographs there exhibited, taken by Mr. Rutherfurd during the 19th to the 26th September 1870, relate to an extensive group of spots in about 15° N. latitude. On the first two

plates, where the group is very near the limb, the preceding spot shows no extraordinary features. On plate 3, however, we begin to recognise the first attempts made by the western photosphere to break into the spot's interior. With each successive day the irruption grows more conspicuous, the sub-division becoming completed on plate 6, where the photograph shows in the place of the single original spot a widely separated pair of spots, the distance between which becomes still more pronounced on the last plate.

These photographs show still another phenomenon corroborating in a striking manner the conclusion regarding the direction of the gyratory motion. The spot under consideration, having appeared in the northern hemisphere, was gyrating in a counter-clockwise direction. Since the force of the vortex tends to carry along with it the inrushing photospheric current, the latter must gradually assume a rotatory



a. *b* *c.*

Fig. 29.

movement from west through north towards east. The photographs show that though the initial direction of the current was directly from west to east, it became more and more deflected into a direction from north to south, thus indicating the same direction of rotation as the spots observed by M. Flammarion.

CONCLUSION.

We hope that we have now sufficiently discussed most of the important points more or less closely connected with the causes of the periodic exhibitions of dynamical energy on our luminary. If the domain of spectroscopic research has not been touched upon, this is for the simple reason that the conclusions arrived at in the preceding theory with regard to the physical nature of eruptions and quiescent prominences, of faculæ and spots, are in perfect harmony with those to be derived from the spectroscopic appearance of these phenomena. So much is this the case that we could indeed have done no more than simply reiterate such well-known facts as have already been clearly demonstrated by solar spectroscopists.

The validity of a theory in natural science has to be judged not only by its final results and their agreement with fact, but also by the soundness and simplicity of the premises on which it is based. In this respect, at least, we hope that our theory will not be found wanting; for it is based mainly upon one great fundamental principle —the deficiency of the sun's contractile force—the soundness of which must, we feel assured, be admitted by all.

Starting, then, with this simple assumption—that the sun does not contract sufficiently to prevent the formation of an absorptive envelope round the radiating photosphere—there result, one by one, as direct and natural consequences, the periodic changes in solar activity, the formation and heliographic distribution of eruptions and spots, the peculiar law of solar rotation and its variations during a spot-cycle, the latitudinal changes in spot-positions, the vortex nature of spots and the remarkable law of radiation resulting therefrom, and many minor points in intimate connection with these more important features.

In short, we may assert that no observational evidence has yet come under notice which fails to find a sufficient explanation in our theoretical deductions. It is chiefly this comprehensive nature of the proposed theory, together with the new application of its fundamental principle, which encourage us to submit it to the criticism of investigators who have cultivated this field of astronomical science. We shall be amply rewarded if this research, imperfect as it doubtless is, should contribute to the solution of some of the difficulties connected with the theory of our luminary, and should at the same time stimulate solar observers to persevere in that careful and patient study which has already done so much to reveal the mysteries of the mighty life pulsating in the sun.

ON A NEW DOUBLE IMAGE MICROMETER.

BY J. GERHARD LOHSE.

It is well known that Airy's Double Image Micrometer labours under the disadvantage that the screw-reading for coincidence of the images varies rapidly with the distance of the observed objects from the optical axis. This peculiarity was fully discussed by the late Professor Kaiser in the 3rd volume of the *Annalen der Sternwarte in Leiden*, pp. 118 and 119. The images of two stars, for instance, brought into coincidence in the centre of the field become separated when they approach the margin. Accurate results can therefore only be obtained when the settings are made with the images exactly in the centre of the field. In this respect the micrometer under consideration resembles the heliometer and the sextant; but in Airy's double image micrometer the effect of deviation from the central axis is much greater than in the heliometer. To aid in keeping to the middle of the field a central square is formed in the eye-piece by two pairs of parallel thick wires, exactly as in the other instruments just mentioned. At the middle of this square all the coincidences ought to be made. Against a dark sky, however, it is often difficult, or even impossible, to distinguish the wires without artificial illumination, the introduction of which limits still further the range of stellar magnitudes within reach of an instrument in which the optical images have already been much enfeebled. Moreover, owing to the necessity of keeping the images scrupulously near the middle of the square, the double image micrometer loses the advantage over a filar micrometer of giving accurate results without a first-rate driving clock.

I therefore considered whether it might not be possible to construct a double image micrometer which would give correct results whether the images are in the centre of the field or not. This condition is very nearly fulfilled by the heliometer, where the error is only $0''{\cdot}01$ for every $300''$ of the measured distance, when the images are brought into coincidence at a distance of $20'$ from the central axis. It may easily happen that the setting is effected at that distance from the axis, considering that the field of view of a small heliometer of 25 inches focal length will be about $103'$, if the diameter of the field-lens of the eye-piece is $\frac{3}{4}$ inch. Generally, however, the images will be nearer the centre of the field at the time of setting, and the measure of a distance not exceeding $300''$ made with the heliometer may be considered as free from this class of error. For larger distances it will only be necessary to take care that the images are not too far from the centre.

In a heliometer, the semi-lenses of which are always placed symmetrically to the axis, the effect of the distance from the axis of the instrument of the point at which the images of the celestial objects are brought into coincidence is expressed by the formula

$$2 \tan \frac{s}{2} = \frac{m' - m}{206264 \cdot 8} \left\{ 1 - \left(\frac{\mu - a}{206264 \cdot 8} \right)^2 \right\}$$

in which s is the distance between the objects, $m' - m$ the angular value of the distance, deduced from the readings of the micrometer screw, and $\mu - a$ the angular distance from the axis of the instrument of the point at which the coincidence was effected, measured in the line of motion of the semi-lenses; since a displacement of the images at right angles to the line of motion of the semi-lenses does not affect the measures of distance. The factor $1 - \left(\frac{\mu - a}{206264 \cdot 8} \right)^2$ remains unaltered when the heliometer is used as a double image micrometer. The error for a distance of 300″ amounts therefore also in this case to only 0″·01, when the images are brought into coincidence at a distance of 20′ from the central axis. Very reliable measures can therefore be made with a heliometer converted into a double image micrometer, and there is no difficulty in effecting such a conversion. It is only necessary to transform the cone of light from the object-glass of a large instrument into parallel rays again, by means of a positive or negative collimating lens, as is done in nearly all spectroscopes. The advantages of a double image micrometer in the measurement of double stars, the diameters of planets, etc., being obvious, it was determined to construct an experimental instrument. Among the numerous apparatus of the Dun Echt Observatory is a heliometer of 25 inches focal length belonging to another instrument; this and a Barlow lens of 7·18 inches negative focal length were just the very things wanted. The small heliometer was mounted on the 15-inch refractor by means of a wooden adapter, which carries also the Barlow lens. The arrangement, therefore, is so very simple as almost to suggest itself, but I am not aware that the heliometer has been used before as a double image micrometer on another telescope, or that a suggestion has been made to use it in such a way. To avoid a reference line in the viewer of a spectroscope, the late Professor Zöllner, it is true, replaced the usual viewing telescope by a heliometer, with a reversing prism in front of one of the half-lenses. In this way two spectra were produced, the red end of the one being to the right and that of the other to the left. The positions of the lines were measured by bringing the corresponding lines of the two spectra into coincidence by means of the heliometer micrometer screw. There is, no doubt, a great resemblance between the modes in which the heliometer has been employed by Zöllner and myself, but, after all, there is only a resemblance. The introduction of the reversing prism complicates and alters the character of the instrument, and an altogether different end is aimed at. It is true that by taking away from Zöllner's apparatus the slit and the refracting prism, and then turning the viewing telescope so

as to look straight into the collimator, the new form of double image micrometer would be arrived at, but there is no indication in Zöllner's papers that he ever contemplated such a transformation of his ingenious spectroscope. The presence of the reversing prism would have the effect of "reversing" one of the images, and it is conceivable that this feature might even be turned to advantage.

The heliometer may be used in connection with a refractor or a reflector. The eye-end of the large instrument must be provided with a position circle by which the whole heliometer can be turned in position. The choice of a positive or negative collimating lens depends mainly on the construction of the large telescope; generally, however, the negative lens will make the instrument more compact and convenient. In that case the divided object-glass may even be placed several inches inside the principal focus of the large instrument if the range of the focussing arrangement of the latter is only sufficiently large; for the distance between the collimating lens and the divided object-glass has no influence on the optical performance of the micrometer, the rays of light being here parallel; to avoid loss of light, however, this interval should be as small as possible. It is useful and almost necessary to have spider lines* in the focal plane of the heliometer; they render the focussing of the instrument more accurate, and serve to determine the position angle of the objects measured, besides offering an easy mode of finding the angular value of the heliometer screw by observing the transits of the two images of a pole star. In the Dun Echt instrument two spider lines were inserted at right angles to each other. One of the lines was set parallel to the line of motion of the semi-lenses by directing the heliometer to a collimator, and turning it in position until the two images of the horizontal wire of the collimator remained superposed when the halves of the object-glass were moved forward or backward by the micrometer screw. The wire in the heliometer was then set parallel to the wire of the collimator and fixed in that position. At the same time the wires were brought carefully into the focal plane. The heliometer being now ready for use, the Barlow lens was attached to the adapter again, and the instrument mounted on the 15-inch refractor. The main focus of the refractor was then altered until a sharp image of a star was obtained in the plane of the wires of the heliometer. This adjustment can, of course, be made by simply sliding the Barlow lens along the axis of the telescope, but in our case it was more convenient to move both the lens and the heliometer without altering the distance between them. The focussing is important, and should be done carefully, as it affects the angular value of the heliometer micrometer screw.

The relations between the different parts of the instrument are best seen from the subjoined formulæ, which require little explanation. The numerical values of the formulæ refer to the Dun Echt instrument, and have been rounded off in most cases.

* If the heliometer is so constructed that a filar micrometer can be mounted on it, comparisons between the two micrometers can be made under optical conditions as nearly alike as possible. Such an arrangement would also prove useful for other investigations.

Let

D = $15^{in.}06$ = Diameter of object-glass O of the large instrument.
F = $182^{in.}3$ = Focal length „ „ „
L = $0^{in.}08262$ = Diameter of cone of light at 1 in. from the focal plane.
N = $0^{in.}0008837$ = Linear value of 1″ in the focal plane of the large instrument.
P = $0^{in.}02$ = Pitch of the filar micrometer screw, or the linear value of one revolution.
W = 22″·6326 = Angular value of one revolution of the filar micrometer screw.
d = $2^{in.}0$ = Diameter of the object-glass of the small heliometer.
d_e = Effective diameter „ „ „
f = $25^{in.}0$ = Focal length „ „ „
p = $0^{in.}02$ = Pitch of the micrometer screw of the heliometer.
w = 165″·0 = Angular value of one revolution of the heliometer micrometer screw.
w^1 = 6″·5002 = Angular value of one revolution of the heliometer micrometer screw, the heliometer being used as a double image micrometer.
s = 3300″ = Greatest distance which can be measured by the heliometer.
s^1 = Greatest distance which can be measured by the heliometer, used as a double image micrometer.
M = Magnifying power of an eye-piece on the large instrument.
m = Magnifying power of the same eye-piece on the heliometer
m^1 = Magnifying power of the same eye-piece on the heliometer, used as a double image micrometer.
d_1 = $0^{in.}70$ = Effective diameter of the collimating lens (Barlow lens).
f_1 = $7^{in.}18$ = Focal length „ „ „
m_1 = 25·4 = Magnifying power „ „ „
ρ = 206264·8 = Length of radius expressed in seconds of arc.

Then we have

$$\frac{N}{F} = \frac{1''}{\rho} \quad \text{or} \quad N = \frac{182^{in.}3}{\rho} = 0^{in.}\,0008837$$

$$\frac{L}{D} = \frac{1^{in.}}{F} \quad \text{or} \quad L = \frac{15^{in.}06}{182\cdot3} = 0^{in.}\,08262$$

$$\frac{F}{P} = \frac{\rho}{W} \quad \text{or} \quad \frac{182^{in.}3}{0^{in.}02} = \frac{\rho}{22''\cdot6326}$$

$$\frac{f}{p} = \frac{\rho}{w} \quad \text{or} \quad \frac{25^{in.}}{0^{in.}02} = \frac{\rho}{165''\cdot015}$$

The linear value of the micrometer screw of the filar micrometer being practically the same as that of the heliometer micrometer screw, 0·02 inch, the magnifying power of any eye-piece will be inversely proportional to the screw value of the instrument on which it is used.

$$\frac{m}{M} = \frac{f}{F} = 0\cdot13715 = \frac{W\cdot p}{w\cdot P} \text{ (in our case } p = P)$$

$$\frac{m^1}{M} = \frac{W\cdot p}{w^1 P} \quad \text{or} \quad \frac{m^1}{M} = \frac{22\cdot6326\cdot p}{6\cdot5002\cdot P} = 3\cdot48185$$

The magnifying power of an eye-piece applied to the double image micrometer is therefore nearly three and a half times as great as when used on the filar micrometer.

$$m \cdot m_1 = m^1, \text{ or } m_1 = \frac{m^1}{m} = \frac{3\cdot48185\, M}{0\cdot13715\, M} = 25\cdot386 = \frac{F}{f_1}; \quad w^1 = \frac{w}{m_1}$$

$f_1 = \dfrac{F}{m_1} = \dfrac{182\cdot3}{25\cdot386} = 7^{in.}\!\cdot18$ = the focal length of the collimating lens, which is in this case negative. $f_1 L = 0^{in.}\!\cdot59$ is therefore the diameter of the cone of light at the collimating lens.

The range of the divided object-glass being only 20 revolutions, not more than 3300″ or 55′ can be measured with it when used as a heliometer.

We therefore have

$$s = 3300'' \text{ and } sN = 2^{in.}\!\cdot92$$

$$s^1 = \frac{s}{m_1} = 130'' \text{ and } s^1N = 0^{in.}\!\cdot115$$

s^1N is the linear distance in the focal plane of the 15-inch between the images of two stars 130″ apart.

This quantity s^1N affects the effective diameters of the Barlow lens and the divided lens, and the expression for the former will be

$$d_1 = f_1 L + s^1 N - \frac{s^1 N}{m_1} = \frac{D + sN - s^1 N}{m_1} = 0^{in.}\!\cdot70,$$

while the effective diameter of the divided lens, which may be assumed to be situated in the focal plane of the large instrument,* takes the form

$$d_e = d_1 + \frac{g}{f_1} s^1 N + 20\,p = 1^{in.}\!\cdot22$$

$$= f_1 L + s^1 N - \frac{s^1 N}{m_1} + \frac{g}{f_1} s^1 N + 20\,p = f_1 L + 2\,b,$$

where g is the distance between the collimating lens and the divided lens; in our case $g = f_1$; b is the distance of the centre of the cylinder of light from the centre of the corresponding semi-lens.

For a positive collimating lens the formulæ are different. In that case the expression for the effective diameter of the collimating lens will be

$$d_1 = f_1 L + s^1 N + \frac{s^1 N}{m_1} = \frac{D + sN + s^1 N}{m_1},$$

* It will be seen further on that it is desirable to have the heliometer lens as close as possible to the Barlow lens, but in most cases it will be difficult to find room for it within the focal plane of the chief instrument.

while the effective diameter of the divided lens becomes

$$d_e = f_1 L - s^1 N - \frac{s^1 N}{m_1} + \frac{g}{f_1} s^1 N + 20 p = f_1 L + 2 b.$$

In treating of this case the interval g between the lenses will be assumed to be two inches. In these formulæ the thickness of the lenses has been neglected.

It is easily seen that the optical conditions are far more favourable if a positive collimating lens is employed; the light then falls nearly centrally on the divided lens, $2 b$ being a small quantity, and practically no light is lost. With a negative collimating lens, as will be shown, the loss of light becomes great if the distance between the objects measured is considerable.

It will be found that the positive collimating lens renders the instrument rather long for most refractors. In the case of reflectors, however, it is easy to give a compact and convenient form to the double image micrometer by simply placing the ordinary flat or a smaller one near the principal focus, the collimating lens being fixed in a fresh hole in the side of the tube nearer the mouth. The heliometer would then be the only protruding part at the side of the tube.

The case where a negative collimating lens is used requires further consideration.

Generally the quantities d, f, and s are known, and it is desirable to select a collimating lens which will give the greatest value of s^1 without sacrificing much light, or, in other words, which will allow the instrument to measure the greatest distance practicable.

In the Dun Echt heliometer the diameter of the object-glass d is about two inches, and the instrument will measure the greatest distance without loss of light in combination with a negative collimating lens, for which

$$m_1 = \frac{D + s_1 N + s N g / f_1 - s^1 N}{d - 20 p} = \frac{20{\cdot}6671}{1{\cdot}6} = 12{\cdot}917.$$

The focal length of this lens will be $f_1 = 14^{in.}{\cdot}111$.

If f_1 is larger than $14^{in.}{\cdot}111$, some loss of light will occur for distances exceeding a certain distance σ, which is defined by the condition that the effective diameter of the divided lens corresponding to this distance equals the real diameter of the lens; but in this case also there is, of course, no loss of light for smaller distances. As long as the loss of light is not very considerable, the observations are not likely to suffer much from this cause, unless the objects are very faint. The advantage, however, of being able to measure considerable distances is so obvious that for general use the preference will always be given to a large lens. For some purposes, especially the measurement of double stars, a smaller collimating lens is desirable; the micrometer screw is then more sensitive, and high magnifying powers are obtained with comparatively flat lenses. By adapting the ordinary micrometer eye-pieces to the heliometer a greater number of them is at once available for the heliometer, and a

direct comparison of their magnifying powers on either instrument is possible, which has the further advantage of affording a check on the determination of other instrumental constants.

For the selection of a larger collimating lens it is necessary to know σ, the distance at which the loss of light begins, and λ, the amount of light lost in measuring s^1, the greatest distance measurable with that combination.

The value of σ is easily found from the following formulæ :—

$$\sigma = \frac{n}{20} . s^1$$

$$\frac{n}{20} = \frac{d - f_1 L}{2b}$$

To find λ we put

$$\frac{d}{2} = r_o \qquad \frac{f_1 L}{2} = r, \quad \text{while } 20p + s^1 N + \frac{g}{f_1} s^1 N - \frac{s^1 N}{m_1} = 2b.$$

If now a plane is passed through the semi-lenses at right angles to the axis of the instrument, the position of the point where the cylinder of light falling on the lens intersects the rim of the semi-lens may be expressed by rectangular co-ordinates passing through the centre of the semi-lens. Let the axis of x be in the plane of separation of the semi-lenses; we shall then have

$$x = \frac{b}{2} + \frac{r_o^2 - r}{2b}$$

$$Cos\ \alpha = \frac{x}{r_o}$$

$$y = r_o . \sin \alpha$$

$$\frac{y}{r} = \sin \alpha^1 \text{ and}$$

$$\lambda = \frac{r^2 . \operatorname{arc} \alpha^1 - r_o^2 . \operatorname{arc} \alpha + b . y}{r^2 \pi} = \frac{\alpha^1}{180} + \frac{b . \sin \alpha^1}{r \pi} - \frac{\alpha}{180} . \frac{r_o^2}{r^2}$$

A sufficiently accurate value of λ may more simply be obtained graphically.

For the Dun Echt instrument several collimating lenses were computed, for which the values of the different quantities are given in the subjoined Table, showing the gradual increase of s^1 and λ and the decrease of m^1 and σ for an increasing f_1. The quantity d_s in this Table is greater than the real diameter, and indicates how large the diameter of the divided lens would have to be to prevent any loss of light taking place in measuring the distance between two objects s^1 seconds apart.

f_1	$14^{in}.111$	$15^{in}.0$	$16^{in}.0$	$17^{in}.0$	$18^{in}.0$	$19^{in}.0$	$20^{in}.0$	$21^{in}.0$	$22^{in}.0$	$23^{in}.0$	$24^{in}.0$
m_1	12·917	12·152	11·392	10·722	10·127	9·594	9·114	8·680	8·285	7·925	7·595
m^1	1·772M	1·667M	1·562M	1·471M	1·389M	1·316M	1·250M	1·190M	1·136M	1·087M	1·042M
s^1	$255''.5$	$271''.6$	$289''.7$	$307''.8$	$325''.9$	$344''.0$	$362''.1$	$380''.2$	$398''.3$	$416''.4$	$434''.5$
s^1N	$0^{in}.2258$	$0^{in}.2400$	$0^{in}.2560$	$0^{in}.2720$	$0^{in}.2880$	$0^{in}.3040$	$0^{in}.3200$	$0^{in}.3360$	$0^{in}.3520$	$0^{in}.3680$	$0^{in}.3840$
f_1L	1·1659	1·2393	1·3219	1·4045	1·4872	1·5698	1·6524	1·7350	1·8177	1·9003	1·9829
d_1	1·3742	1·4596	1·5555	1·6512	1·7468	1·8421	1·9373	2·0324	2·1272	2·2219	2·3164
d_0	2·0000	2·0996	2·2115	2·3232	2·4348	2·5461	2·6573	2·7684	2·8792	2·9899	3·1004
$2b$	0·8341	0·8603	0·8896	0·9187	0·9476	0·9763	1·0049	1·0334	1·0615	1·0896	1·1175
σ	$255''.5$	$240''.1$	$220''.8$	$199''.5$	$176''.4$	$151''.6$	$125''.3$	$97''.5$	$68''.4$	$38''.1$	$6''.6$
λ	0·000	0·021	0·058	0·098	0·139	0·179	0·216	0·252	0·286	0·318	0·347
l	$0^m.00$	$0^m.02$	$0^m.06$	$0^m.11$	$0^m.16$	$0^m.21$	$0^m.26$	$0^m.32$	$0^m.37$	$0^m.42$	$0^m.46$

The quantity l denotes the loss of light expressed in stellar magnitudes, and has been introduced to give a better idea of the effect of λ on the apparent brightness of a star.

The Table shows that even with a collimating lens of 24 in. focal length the instrument cannot measure a greater distance than 434 seconds. At the same time, the increase in the magnifying power of an eye-piece is by this combination practically reduced to zero; the loss of light, however, although beginning at very small distances, is not serious. The employment of a larger collimating lens not being advisable, it would seem as if the greatest distance measurable with a heliometer of 2 ins. aperture, used as a double image micrometer on the 15-inch refractor, were only 434 seconds, and this is really the case for the instrument as it is at present constructed. There is, however, no necessity for limiting the range of the heliometer to 20 revolutions or 55 minutes. A heliometer intended to be used as a double image micrometer ought to be so constructed that it will measure two degrees. This condition would be nearly fulfilled by making the range of the Dun Echt heliometer 40 revolutions or 110 minutes. To see how this alteration affects the different quantities, the computation was repeated for several collimating lenses. The only alteration in the formulæ consists in replacing $20\,p$ by $40\,p$, remembering at the same time that the value of s is now 6600 seconds. In the subjoined Table a quantity λ_{15} has been added, indicating the loss of light taking place in measuring the distance s^1, given in the same column in combination with a collimating lens of 15 in. focal length. The quantities l and l_{15} are the equivalents of λ and λ_{15} expressed in stellar magnitudes.

For the sake of comparison two positive collimating lenses were also computed; the quantities found for them are given in the last two columns. It will be noticed there that, although the cylinder of light produced by the lens of 24·527 in. focal length

is already greater than the divided lens, the loss of light is only nominal. If it is intended, therefore, to measure distances exceeding six minutes (the limit will be different for other instruments), better results may be expected from a double image micrometer with a positive collimating lens. In fact, this form of the micrometer is always preferable where the considerable length of the instrument is no bar to its employment.

COLLIMATING LENSES AND QUANTITIES DEPENDING ON THEM.

NEGATIVE LENSES.

f_1	$8^{in}.266$	$9^{in}.0$	$10^{in}.0$	$11^{in}.0$	$12^{in}.0$	$13^{in}.0$	$14^{in}.0$	$15^{in}.0$
m_1	22·051	20·253	18·228	16·571	15·190	14·021	13·020	12·152
m^1	3·024M	2·778M	2·500M	2·273M	2·083M	1·923M	1·786M	1·667M
s^1	299″·3	325″·9	362″·1	398″·3	434″·5	470″·8	507·″0	543″·2
s^1N	$0^{in}.2645$	$0^{in}.2880$	$0^{in}.3200$	$0^{in}.3520$	$0^{in}.3840$	$0^{in}.4160$	$0^{in}.4480$	$0^{in}.4800$
f_1L	0·6830	0·7436	0·8262	0·9088	0·9914	1·0741	1·1567	1·2393
d_1	0·9355	1·0174	1·1287	1·2396	1·3502	1·4604	1·5702	1·6798
d_e	2·0000	2·1054	2·2487	2·3916	2·5342	2·6764	2·8182	2·9598
$2\,b$	1·3170	1·3618	1·4225	1·4828	1·5428	1·6023	1·6615	1·7205
σ	299″·3	300″·7	298″·8	293″·1	284″·1	272″·0	257″·3	240″·2
λ	0·000	0·038	0·115	0·192	0·262	0·324	0·379	0·426
λ_{15}	0·051	0·086	0·137	0·193	0·251	0·310	0·369	0·426
l	0^m00	0^m04	0^m13	0^m23	0^m33	0^m42	0^m52	0^m60
l_{15}	0·06	0·10	0·16	0·23	0·31	0·40	0·50	0·60

	NEGATIVE LENSES.					POSITIVE LENSES.	
f_1	$16^{in}.0$	$17^{in}.0$	$18^{in}.0$	$19^{in}.0$	$20^{in}.0$	$20^{in}.0$	$24^{in}.527$
m_1	11·392	10·722	10·127	9·594	9·114	9·114	7·432
m^1	1·562M	1·471M	1·389M	1·316M	1·250M	1·250M	1·019M
s^1	579″·4	615″·6	651″·8	688″·0	724″·2	724″·2	888″·2
s^1N	$0^{in}.5120$	$0^{in}.5440$	$0^{in}.5760$	$0^{in}.6080$	$0^{in}.6400$	$0^{in}.6400$	$0^{in}.7849$
f_1L	1·3219	1·4046	1·4872	1·5698	1·6524	1·6524	2·0265
d_1	1·7890	1·8978	2·0062	2·1144	2·2222	2·3627	2·9170
d_e	3·1010	3·2418	3·3822	3·5224	3·6622	1·8062	2·0000
$2\,b$	1·7791	1·8372	1·8950	1·9526	2·0098	0·1538	−0·0265
σ	220″·8	199″·5	176″·4	151″·6	125″·3	..	..
λ	0·469	0·505	0·538	0·567	0·593	0·000	0·026
l	0^m69	0^m76	0^m84	0^m91	0^m98	0^m00	0^m03

With the lens (15)—where the figure indicates the focal length of the lens—more than two-fifths of the light is lost in measuring a distance of 543 seconds, yet the effect on the images of the stars is only a reduction of the apparent brightness by six-tenths of a magnitude. There is, however, reason to believe that a further increase in the focal length of the collimating lens would affect the definition of the images of objects more than 540 seconds apart in such a way as to impair the accuracy of the measures. A collimating lens of 15 ins. focal length is therefore to be considered as the largest lens that may be used with advantage in connection with the Dun Echt refractor and heliometer, assuming the range of the latter to be increased to 40 revolutions. In using such a lens instead of the Barlow lens the sensitiveness of the micrometer screw and the magnifying power of the eye-pieces are much reduced, but after all they are still greater than they would be if the same micrometer screw and eye-pieces were used in connection with an ordinary filar micrometer. The readings of the micrometer screw, therefore, determine the relative position of the semi-lenses with abundant accuracy, and $\lambda_{15}-\lambda$, the difference in the loss of light, compared with that taking place for smaller lenses, is so insignificant that it would hardly be noticed. Such an apparatus would give a high degree of accuracy in measuring any distance not exceeding 9 minutes. If it is intended, however, to restrict the measures to moderate distances a smaller collimating lens would be more advantageous.

For negative collimating lenses it has been shown that s^1, the greatest distance measurable, cannot be increased beyond a certain limit by increasing f_1, the focal length of the collimating lens. But there are other quantities affecting s^1, and we shall now examine their separate effects. Starting from the equation

$$s^1 = \frac{s}{m_1} \text{ and making the substitutions}$$

$$s = 40\,w \qquad w = \frac{p\,\rho}{f} \qquad m_1 = \frac{F}{f_1}, \text{ we obtain}$$

$$s^1 = \frac{40\,p \,.\, f_1 \,.\, \rho}{f \,.\, F}$$

$40\,p$ is the distance through which the semi-lenses can be moved, and shall be called t to make the expression quite general. An increase of t would also increase s^1. Figure 2, however, shows that the general conditions would become very unfavourable if t were greater than $0{\cdot}44\,d$. As a limiting value of t we therefore adopt $t = 0{\cdot}44\,d$. We see further that the focal length f of the divided lens should be made as short as is compatible with the proper optical performance of the same; while F in the denominator indicates that the heliometer will measure a greater distance in combination with a small instrument than with a large one. A slight transformation of the formula makes this more obvious still. Putting $q\,d = f_1\,L$, where q is an abstract number and $f_1\,L$ the diameter of the cylinder of light falling on the divided lens we obtain

$$s^1 = \frac{0{\cdot}44\, d \,.\, q \,.\, d \,.\, \rho}{f \,.\, F \,.\, L}, \text{ or}$$

$$s^1 = \frac{0{\cdot}44\, d}{f} \cdot \frac{d}{D} \cdot q \cdot \rho.$$

The quantity $\frac{d}{f}$ is practically a constant; s^1 depends therefore mainly on the ratio between the diameters of the divided lens and the large lens, and on the factor q, which determines the collimating lens.

If we wish to know the diameter of the divided lens that will render the instrument capable of measuring a certain distance, say 12 minutes, it is only necessary to insert in the formula for s^1 suitable values of $\frac{d}{f}$ and q. Putting, for instance, $\frac{d}{f} = \frac{1}{12{\cdot}5}$ and $q = 0{\cdot}6$, s^1 being 720″, we get the approximate value

$$\frac{d}{D} = \frac{1}{6}.$$

If the instrument is intended to measure only 480 seconds, we shall have

$$\frac{d}{D} = \frac{1}{9}.$$

In the case of a positive collimating lens we put $q = 1$; the respective values of $\frac{d}{D}$ are then $\frac{1}{10}$ and $\frac{1}{15}$.

The course taken by the light in passing through the different lenses is shown in Figures 1, 2, and 3. The dimensions in these figures are greatly out of proportion. Figure 1 represents a section through the axis of the telescope and micrometer. The plane of section passes also through the two stars, the images of which are brought into coincidence at I. O is the object-glass of the large instrument, C the collimating lens, and H the two semi-lenses of the heliometer. The rays passing through the left semi-lens are represented by dotted lines, while the rays forming the two outstanding images are omitted to avoid complicating the figure. The distance of the stars has been selected equal σ; the outermost rays therefore fall on the edge of the semi-lenses, but none are lost. It may be remarked that in the Dun Echt instrument the two semi-lenses move at the same time in opposite directions. They are therefore always placed symmetrically to the axis. No lateral motion can be given to the eye-piece; it is fixed in the axis.

Figures 2 and 3 illustrate the difference in the path of the rays forming the image in the axis of the instrument at I, according as a negative or a positive collimating lens is used. The diagrams show also how the semi-lenses must be situated in either case, the line $m\,I$ connecting the centre of the semi-lens with the image I being parallel to $c\,\epsilon\,k$, the direction of the rays between the collimating lens and the helio-

Fig. 1.

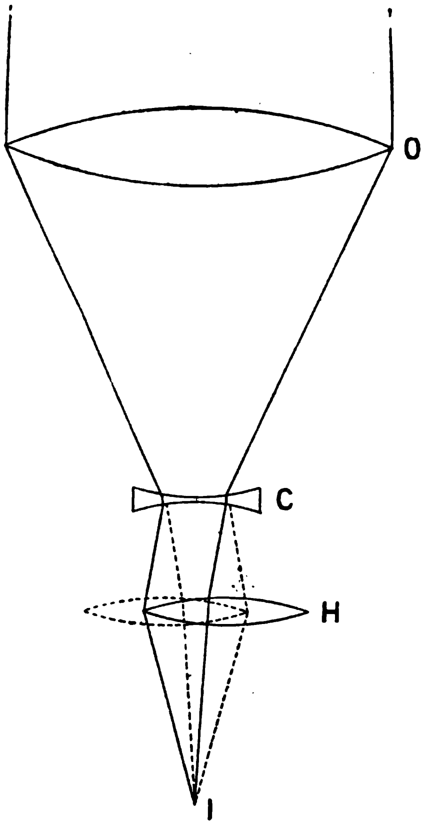
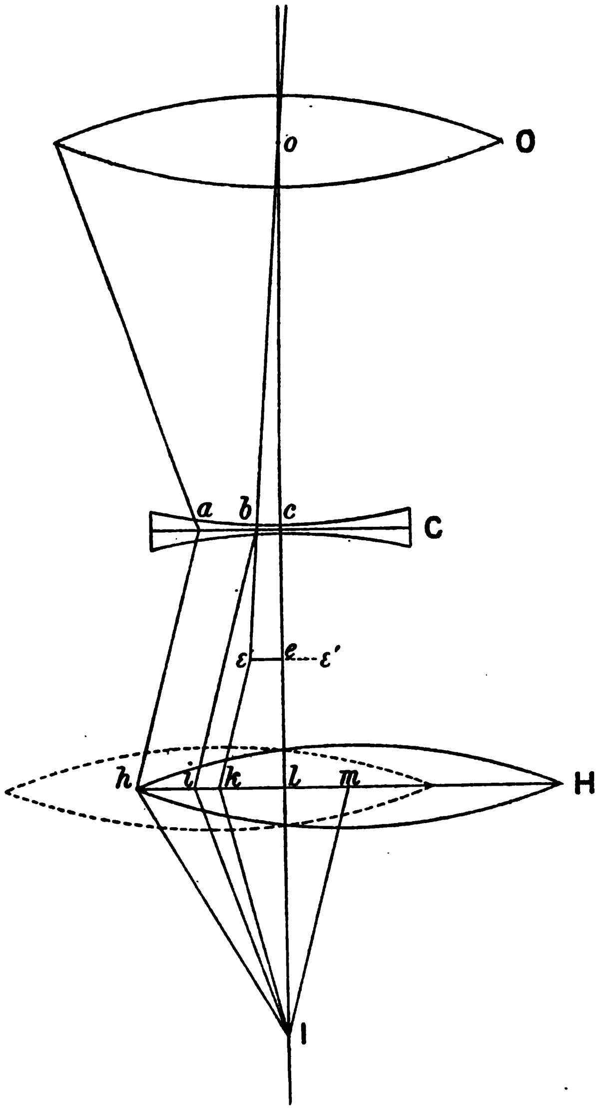


Fig. 2.



meter. The conditions are so chosen that the images of the two objects measured would be formed in the focal plane of the object-glass O at ϵ and ϵ', two points situated at equal distances from the central axis. The construction, however, is given only for one of the images.

The figures illustrate at the same time the geometrical significance of several of the expressions used in the formulæ. For instance :—

$$\begin{array}{lll} o\,e = F & \epsilon\,\epsilon' = s^1\,N & h\,m = h\,k + k\,l + l\,m = \tfrac{1}{2}\,d_e \\ c\,e = f_1 & a\cdot b = \tfrac{1}{2}\,f_1\,L & i\,m = b \\ l\,I = f & a\,c \text{ and } n\,c = \tfrac{1}{2}\,d_1 & l\,m = 20\,p \\ c\,l = g & & \end{array}$$

Figure 4 shows how the light falls on the semi-lenses in the special case when in combination with lens (15) the images of two stars, 543 seconds apart, are brought into coincidence. The portions of the semi-cylinders of light falling beyond the semi-lenses are represented by dotted lines, and evidently amount to more than one-third of the light available for each semi-lens; in fact, the value of λ may be obtained approximately from such a figure. To avoid confusion the semi-cylinders of light producing the two outstanding images are also omitted in this figure.

For obvious reasons there should be no friction between the two semi-lenses of the heliometer. A small interval must therefore always be left between them. Through this interval parallel rays, coming from the collimating lens, fall directly on the eye-piece, and produce the appearance of a bright streak passing through the centre of the field. Generally this streak is not troublesome; it indicates rather in a convenient way the position of the semi-lenses and the centre of the field; sometimes, however, the measures can be better made without it. In that case a diaphragm with two apertures, sufficiently large to admit all the rays from the two stars, is placed in front of the semi-lenses, the bar between the two apertures then covers the interval between the two lenses. If the objects to be measured are faint, the diaphragm has the further advantage of making the field darker by covering parts of the semi-lenses which do not receive direct rays from the two objects. Figure 5 represents such a diaphragm to be used in connection with the Barlow lens. To reduce the light of a very bright object one of the apertures in the diaphragm may be covered with gauze. The difference in brightness between the images of two objects of unequal brightness may also be lessened by allowing more light to fall on the semi-lens producing the fainter image. This can be easily effected by mounting the heliometer on a slide, by which it can be moved sidewise at right angles to the line of motion of the semi-lenses; the collimating lens, being mounted on the frame of the slide, remains in the axis of the large instrument while the heliometer is moved out of it. The cylinders of light falling on the semi-lenses are now unevenly divided; the one semi-lens receives more than one-half of the cylinder of light, and the other just as much less. This method of equalising the apparent images is more effective if the collimating lens is small. The heliometer being moved in such a manner that its axis remains parallel to the axis of the large instrument, no displacement of the images in the field of view is produced by the transverse movement.

FIG. 3.

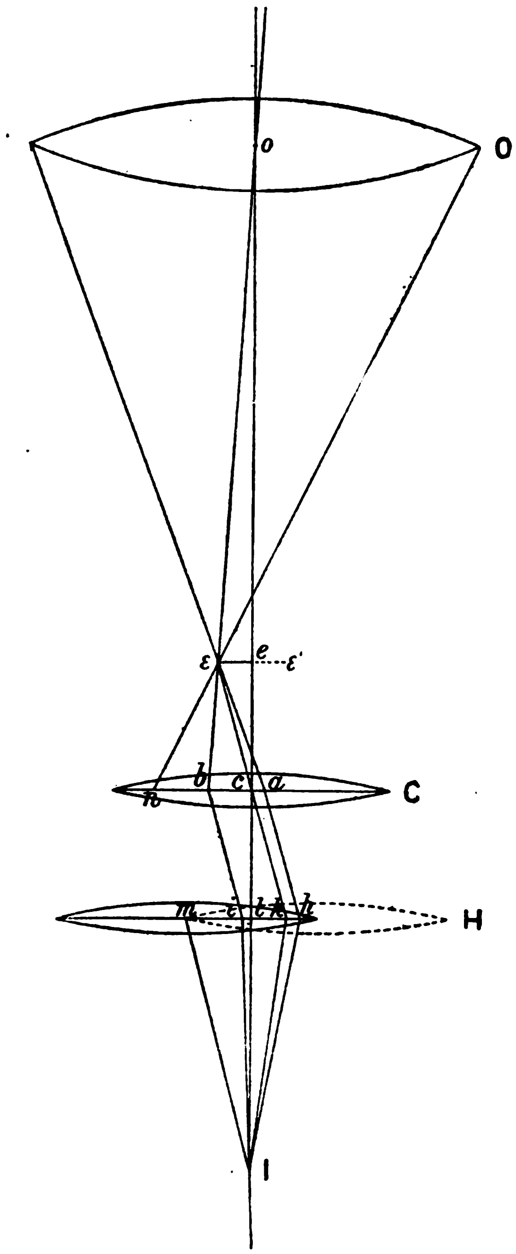


FIG. 4.

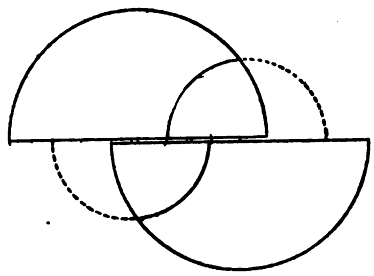
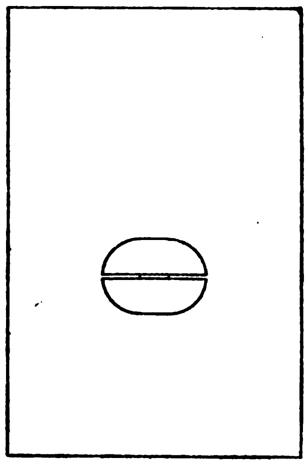


FIG. 5.



Compared with Airy's double image micrometer, the Astrometer, as the new instrument may be called, has several advantages besides those already referred to. By changing the eye-piece the magnifying power of the astrometer can be altered at any moment without interfering with the value of the micrometer screw or any of the adjustments. A change of the magnifying power in Airy's double image micrometer, however, is a serious matter; it simply alters everything; it alters the value of the micrometer screw, the focussing of the instrument, and generally also the zero correction of the position circle. The pencil of light falling on the divided lens of Airy's micrometer is very small, especially if the high powers are used; a large proportion of the light must therefore fall on the interval between the two semi-lenses if they are separated by a few hundredths of an inch.* A similar interval would cause only a very small loss of light in the astrometer, as is evident from Figure 4. Generally, therefore, the loss of light will be much greater in Airy's double image micrometer than in the astrometer, except perhaps for large distances.

The definition of the images depends mainly on the construction of the object-

* At Dun Echt is an Airy's double image micrometer made by Simms. It is supplied with five lenses, for each of which the magnifying power of the instrument when mounted on the 6-inch Simms refractor was determined. The diameters of the pencils of light at the divided lens were measured with Ramsden's dynamometer, and the interval between the halves of the divided lens was determined in the same way and found to be $0^{in}.00510$. From these quantities the loss of light produced by the interval between the halves of the divided lens is readily obtained. Let

$2\,r$ = diameter of pencil at the divided lens.

$2\,i$ = interval between the halves of the divided lens and $\sin\theta = \frac{i}{r}$, then we have

$$\text{Loss of light} = \frac{2}{\pi}\,.\,\cos\theta\,.\,\sin\theta + \frac{\theta^c}{90^c}.$$

The numerical values of these quantities appear in the subjoined Table. Mr. Simms has succeeded in making the interval between the semi-lenses very small. To show, however, what the loss of light would have been had the interval been greater, two columns have been added giving the loss of light for an interval of $0^{in}.01$ and $0^{in}.02$ respectively, the total amount of light at the divided lens being taken as unity.

Lens.	Magnifying Power.	Diameter of Pencil at Divided Lens.	Loss of Light.		
			$2\,i = 0^{in}.0051$	$2\,i = 0^{in}.01$	$2\,i = 0^{in}.02$
1	60·6	$0^{in}.13805$	·047	·092	·184
2	98·2	0·08518	·076	·149	·296
3	134·9	0·06201	·105	·204	·403
4	191·5	0·04370	·148	·292	·562
5	289·4	0·02892	·223	·431	·804

glass or mirror of the principal instrument, and a fair idea of what the images in an astrometer would be like may be obtained by covering one-half of the object-glass or mirror with cardboard; a diaphragm corresponding to the illuminated part of the semi-lens in Figure 4 may also be tried. The elongation of the images being at right angles to the line of motion of the lenses does not affect the measures.

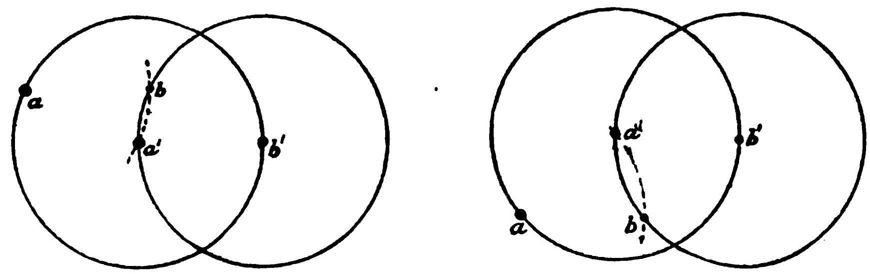
So far a small number of observations only have been made with the astrometer, owing to a combination of several unfavourable circumstances. The instrument was only ready for use three months before I left Dun Echt to take charge of the late Mr. Wigglesworth's new Observatory at Scarborough. The weather was very unfavourable during these three months, the sky being generally overcast. The short time at my disposal, I soon recognised, left me no choice but to take observations whether the definition was good or bad, if the night was only clear and the instrument free. Some observations were made on nights when the definition was so bad that nobody would have attempted to make similar observations with a filar micrometer, yet the results do not differ very much from those obtained under more favourable circumstances. Some nights, besides, were lost in consequence of some alterations of the refractor connected with the large spectroscope, which Messrs. T. Cooke & Sons at York were then constructing. No observations have been made with the astrometer since my departure from Dun Echt.

The observations were made in this way. The instrument was first directed to a bright star, and carefully focussed before setting it on the object to be measured. The semi-lenses were next brought into coincidence, so as to form a single lens, and the astrometer set in position by bringing the two objects to be measured on the position wire. The high magnifying power, combined with the flat field of view allow of doing this with a relatively high degree of precision. The usual bright field illumination is agreeably toned down by the intervention of the collimating and divided lenses. A complete measure of distance consisted generally of eight settings, four being made in one position of the lenses and four with the lenses crossed, thus eliminating the zero correction. The difference between the readings in the two positions of the lenses gives the fourfold, the double, or the single distance between the objects, according to the mode of observation adopted. If a and b are the two images produced by the semi-lens I., and a', b' those formed by the semi-lens II., the respective arrangement of the images in the three cases will be as follows :—

	Fourfold Distance.				Double Distance.			Single Distance.			
1st Observation, .	a ○	b •	a' ○	b' •	a ○	b ⊙ a'	b' •	a ○	a' ○	b •	b' •
2nd Observation, .	a' ○	b' •	a ○	b •	a' ○	b' ⊙ a	b •	a' ○	a ○	b' •	b •

The method of the fourfold distance is the best for close double stars. Each setting depends on two estimations; b must be half-way between the equally bright images a and a', and a' must be half-way between the equally bright images b and b'; a bias in comparing an interval to the right with one to the left is thus, in a great measure, eliminated, and the resulting distance is only affected by half the error made in setting. For large distances and for the determination of the diameter of a disk, the second method is best adapted. The coincidence of a' and b can be effected by placing b first to the right and then the same distance to the left of a', or b may be placed below a' by a slight motion in position; by another motion in position it is then moved across a' until it appears above it; in that way b describes a small arc of a circle of which b' is the centre, and by noticing the places of ingress and egress of the fainter star it is easy to see whether the transit is central or not.

The setting may also be controlled by noticing whether the line $a'\,b$ is at right angles to $a\,b'$ or not. This is a good test, especially if the distance between the stars a and b is small. b and b' being the two images of the same star, the distance between them is not altered while the instrument is turned round in position, b therefore



describes a circle round b', if the latter image is considered as fixed. At the same time a describes a circle round a' as a centre. If the setting in distance is correct, the circle in which b moves must pass through the centre of a', and the circle in which a' moves round a, if a is considered as stationary, must pass through the centre of b. b and a' are therefore at the same time situated on two circles, of which b' and a are the centres. The line $a'\,b$ is therefore always normal to $a\,b'$, if the setting in distance be but right, no matter how the instrument may be turned in position. In the special case where the instrument is turned 90° from the position at which a, a', b, and b' are in line, the parallelogram formed by the four images is converted into a perfect square. If the setting in distance is not correct, $a'\,b$ can never be normal to $a\;b'$, the deviation from the perpendicular being the greater the closer b is brought to a'.

The last method, in which the distance between the two stars a and b is bisected by a', is not a good one, although recommended by Chauvenet in his "Spherical and Practical Astronomy." The arrangement is not so symmetrical as in observing fourfold distances, and the result is vitiated by double the error of setting. The setting must therefore be four times as accurate to obtain results of the same degree of precision as those which the method of observing fourfold distances affords. But there

is no reason why the settings should be better in this case, the smaller distance between the images being counterbalanced by the unsymmetrical arrangement, for a' must be placed in the centre between the two unequal stars a and b.

With a double image micrometer or with a heliometer the position angle of a double star is generally determined by bringing the four images into line. Such a setting, however, is likely to be influenced by the images being elongated in a direction at right angles to the line of section.* To eliminate an error due to this cause, the lenses are crossed and the setting is repeated. The images being now interchanged, the error acts in the opposite direction; the mean of the two settings, therefore, gives a correct result.

If field or bright wire illumination is available, the position angle may also be determined by means of the wires as in a filar micrometer. To that end the optical centres of the semi-lenses are brought into coincidence; the superposed images being now quite round and well defined, no error can arise from the form of the two images.

Experience has shown that in both kinds of settings the eye of the observer tolerates errors up to a certain limit ϵ, which varies for different observers and instruments, and may be considered as inversely proportional to the apparent distance Δ of the stars. At any rate, for our present purpose it will be near enough to consider the product $\epsilon\,\Delta$ as a constant, or $\epsilon\,\Delta = \epsilon'\,\Delta' = \epsilon''\,\Delta'' = \ldots, \epsilon, \epsilon', \epsilon'', \ldots$ being the greatest apparent angular deviations from the correct position which the eye will pass without being offended, if the apparent distances between the components are respectively $\Delta, \Delta', \Delta'', \ldots$. In the expression—

$$\epsilon\,\Delta = \text{constant}$$

a reduction of ϵ means an increase of Δ, and in reality a lessening of ϵ has hitherto always been effected by increasing the apparent distance Δ. This may be done in several ways. The most effectual way of doing it is by using a large instrument, which separates the stars wider and better. Practically, however, it is more important to know how to accomplish it best with the special instrument at one's disposal. An eye-piece of high magnifying power increases the apparent distance Δ in the simplest manner. The very good results, however, obtained by the Rev. W. R. Dawes, who doubled the power of his eye-pieces by inserting a Barlow lens† in the cone of light, seem to indicate that it is preferable to secure high magnifying powers by employing a Barlow lens. The effect of such a lens is the same as if the focal length of the object-glass were increased. In a modified manner all the advantages of the Barlow lens are found in the new double image micrometer. The instrument has besides the advantage of bearing high magnifying powers well even when the images are dancing, but otherwise well defined. It occupies, therefore, a favourable position as far as the lessening of ϵ through the employment of a high magnifying power is concerned.

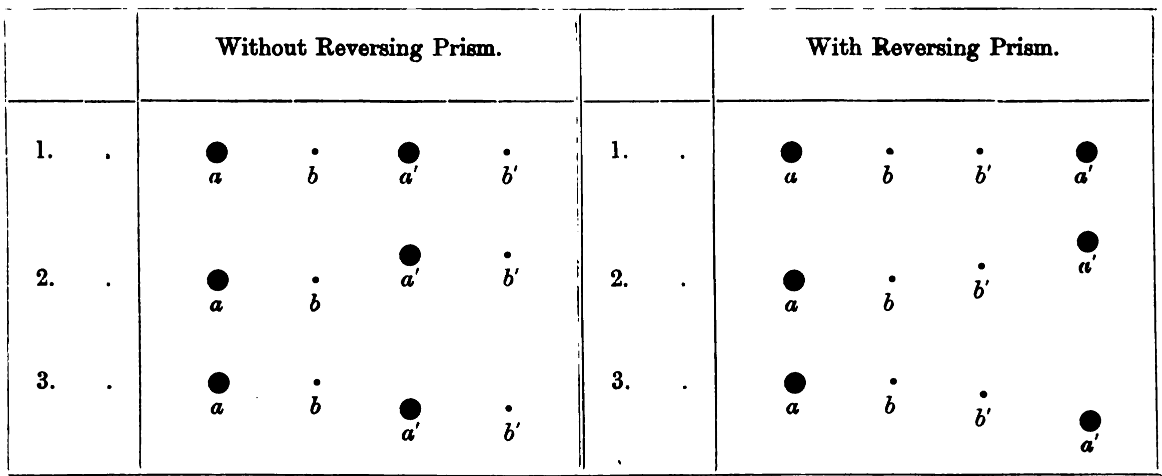
It is, however, conceivable that, apart from the greater or smaller skill and sensitiveness of perception of the observer, the size of ϵ may be reduced by some

* It may be mentioned that Dr. H. Schröder has constructed object-glasses where the images produced by one half of the lens are practically achromatic.

† Memoirs R. Ast. S., vols. xxxv. and xix.

mechanical contrivance which renders the error in the setting of position more conspicuous and consequently offensive to the eye. In that case $\epsilon\ \Delta$ ceases to be a constant; we must now write $p\ \epsilon^{\ \Delta}$ = constant, where the factor p indicates the degree of sensitiveness with which the instrument renders a deviation from the correct position angle conspicuous, $^{\Delta}$, whose influence on ϵ is known, being now regarded as constant. The problem of lessening ϵ without altering at the same time the apparent distance Δ has, as far as I am aware, never been discussed before, because this cannot be accomplished with the ordinary appliances hitherto in use, the heliometer excepted, and even for the heliometer the solution of the problem will always remain a theoretical one, the technical difficulties being too great. The smaller dimensions of the new double image micrometer, however, render a practical solution quite feasible. The object in view, namely, is accomplished by placing, as Zöllner did, a reversing prism in front of one of the semi-lenses. We will call this semi-lens the *prism-lens* and the other the *free lens*. Let the images of a double star, produced by the free lens, be a and b, and those produced by the prism-lens a' and b', and let us see what happens, if the instrument is now turned in position. We will first consider the images a' and b' of the prism-lens. In optics it is shown that the images seen through a reversing prism turn in the same direction through the angle $2\ \alpha$, while the reversing prism is turned through α. The images a' and b', therefore, turn in position twice as fast as the instrument; if the position circle indicates that the instrument has turned 1°, then the images a' and b' have altered their apparent position 2°. The apparent position of the images a and b of the free lens, on the other hand, is not affected at all by the turning in position of the instrument. Assuming now the setting in position to be perfectly right, so that a, b, a', and b' are in line. The result of turning the instrument then through the angle ϵ will be an alteration of $2\,\epsilon$ in the apparent position angle of $a'\ b'$ with regard to $a\ b$. This angle $2\ \epsilon$ will now be highly offensive to the eye, if ϵ is the greatest deviation from the correct position which the eye would tolerate, in case the reversing prism were removed. Without experiments it is impossible to say how much ϵ would have to be lessened to become insensible, the cases not being strictly analogous: probably a reduction to less than one-half would be required, or $p > 2$. The following corresponding illustrations of the positions of the images *without* and *with* a reversing prism will perhaps render the matter clearer.

POSITION OF IMAGES.



Let c be a point in $a\ b$ produced to the right and at an infinite distance from a, then the following angles will have the same value, provided the deviation from the zero position is in both cases the same, the distance between the images being immaterial:—

Without Reversing Prism.		With Reversing Prism.
Angle $a'\ a\ c$ = angle $b'\ b\ c$	=	angle $a'\ a\ c$ = angle $b'\ b\ c$.

So far the two cases agree; the difference between them is in the size of the angle $a'\ b'\ c$: without a reversing prism it is 180°, or $a'\ b'$ parallel to $a\ b$; while with a reversing prism the angle $a'\ b'\ c$ is $= 2 \times$ angle $a'\ a\ c$.

The construction of the images, as they would appear if a reversing prism were used, is very simple. a and b, which do not alter their position angle, may be considered as fixed. As zero-position of the instrument we take that where the four images are in line. The rotation of the micrometer from this position is then indicated by the angle $b'\ b\ c$. We, therefore, draw through b a line forming the requisite angle with $a\ b\ c$, b' is then a point on this line, the distance from b being arbitrary. We now assume a looking-glass to be placed half-way between b and b' and normal to that line, the image of a in the looking-glass determines then the place of a'. Or we may draw a line through a parallel to $b\ b'$, a' is then a point on this line at the same distance from b' as b is from a.

From the fact of angle $a'\ b'\ c$ being twice as great as angle $a'\ a\ c$, it is evident that the introduction of a reversing prism in front of one of the semi-lenses would render the instrument very efficient for obtaining accurate position angles, but it has the great disadvantage of making it at the same time absolutely unreliable for observing distances —the reason being simply this: the image produced by the prism-lens is essentially a looking-glass image; if the position of the instrument is now altered in a direction at right angles to the base of the reversing prism, for instance, by the slow-motion handle, then this looking-glass image must move in the field of view in a direction opposite to that pursued by the image produced by the free lens. The least disturbance of the telescope, therefore, produces a displacement of the two images with regard to each other of double the amount, while the position angle remains unaffected. Under such circumstances accurate measurement of distances is almost impossible. In Zöllner's spectroscope this defect is not so obvious, but it exists; the least flexure in the spectroscope affects the measures by double its amount.

Should anybody be inclined to try the reversing prism for measuring position angles, I would advise the simplification of the apparatus by doing away with the divided object-glass. A reversing prism placed in front of one half of an ordinary object-glass would answer perfectly, the arrangement being otherwise the same as that of the new double image micrometer. A filar micrometer added at the eye-end for measuring the distances of the double stars would complete the instrument. The necessary crossing of the images is then effected by the slow-motion handles or more simply by the slipping-piece, if the instrument is mounted on such a convenient contrivance.

The reversing prism may also be found useful for other purposes. In photographing celestial objects, for instance, the going of the driving clock must be controlled. This is done by keeping the image of a conspicuous star on the point of intersection of two spider lines (or in a tiny square) in a telescope mounted alongside of the photographic telescope. To that end the wires or the field must be illuminated. Artificial illumination, however, is, under such circumstances, always objectionable, and should be avoided as much as possible. A reversing prism of moderate size placed in front of the object-glass of the controlling telescope would produce a second fainter image of the guiding star. If the position of the prism is adjustable, the two images may be brought into coincidence in any part of the field. With the base of the prism normal to the daily motion any irregularity in the going of the driving clock will at once be indicated by a separation of the superposed images. The separation being twice as large as the displacement of the main image with regard to the cross wires would appear, it cannot fail to be noticed more readily in the dark field, and the position of the faint image with regard to the bright one leaves no doubt in which direction the adjustment of the instrument has to be made, but it affords no check upon a change in the declination, unless another prism is placed before the object-glass at right angles to the first one.

The reversing prism having been mentioned several times, a few remarks about the same are perhaps not out of place. The glass should of course be perfectly clear, transparent, and weather-proof, and although the prism is not intended to produce spectra, the refractive power of the glass is by no means immaterial, the conditions being more favourable if the index of refraction of the glass is as large as possible. Generally the prism is made in the form of a right-angled prism, the angles at the base being 45°. In that case the base-half of the prism only transmits light, while the other half, with the right angle at the apex, is absolutely useless, and might just as well be removed. A prism, however, in which the angles at the base are about 35° transmits light nearly up to the apex, if the index of refraction of the glass is high, while the additional loss of light on account of the angle of incidence being 10° greater is only small.

The most favourable prism generally will be the prism occupying the smallest space, provided the effective, light-transmitting height of the prisms is the same. This condition is fulfilled by the prism having the shortest base; the angle at the base, however, should not be smaller than, say, 32°, otherwise the loss of light by reflection becomes serious. Such a prism does not necessarily transmit light up to the apex. The size of the prism depends on the size of the object-glass. The effective light-transmitting height of the reversing prism, namely, must be the same as the diameter of the object-glass in front of which the prism is to be mounted. The length of the base is found from the formula—

$$\text{Base} = \frac{\cos r}{\sin\phi\cos(\phi + r)} \times \text{effective height of prism,}$$

where ϕ is the angle at the base of the prism, and $\sin r = \frac{\cos\phi}{n}$, n being the index of refraction of the glass. The maximum of the relation $\sin\phi\cos(\phi + r)/\cos r$ is $= 0{\cdot}2569$ for $n = 1{\cdot}5$ and $\phi = 32°$, and $= 0{\cdot}2659$ for $n = 1{\cdot}55$ and $\phi = 32°\ 40'$.

If a prism transmitting light up to the apex is wanted, the condition is expressed by the formula—

$$n = \frac{\cos \phi}{\cos 2 \phi}$$

The head of the heliometer micrometer screw is divided into 500 parts; the angular value of one revolution ($1^{\text{rev.}} = 6''\cdot5002$) was determined by a few transits of the two images of Polaris, but no investigation has been made of the irregularities of the screw. It is, however, not likely that a fuller investigation of the screw will much alter the preliminary results now given.

Observations with the Astrometer.

In making the observations we were confronted with the inconvenience that, owing to the length of the heliometer, the observer could not reach the tangent screw of the position circle while looking into the eye-piece; and no simple way was found of overcoming that difficulty without calling in the assistance of an instrument-maker, which would have caused a considerable delay. Under these circumstances it was considered best not to attempt a determination of position angles, but to restrict the observations to measuring distances, a proceeding justified by the consideration that the main interest in the astrometer attaches to the determination of distances. The results are not so accordant as had been anticipated, for which probably my own inexperience in observing with an instrument of this kind and the state of the weather are mostly to blame.

Perhaps a defect in the small Dun Echt heliometer, the exact seat and nature of which are still unknown, is answerable for some of the discordances. If the settings were faultless and the working parts of the heliometer perfect, the sum of the readings for the two positions of the semi-lenses should be a constant quantity, one-half of the sum being the reading for coincidence or the zero of the micrometer-screw. The observations, however, show that this sum varies a good deal. The heliometer was carefully examined; the working of the principal springs, however, can only be seen by taking the instrument to pieces. But the complicated construction of the heliometer and the fact that the instrument in its present form was only intended for temporary experimental use induced us to defer a more thorough examination until it was provided with a permanent adapter. Assuming that the discordances arose from the imperfect action of one of the springs, we trusted to neutralising that imperfection by effecting the final turn of the screw always in the same direction. If the change in the zero-reading, however, was occasioned by a shifting of one of the semi-lenses in its cell, no precaution could eliminate the errors thus introduced. The readings may also be affected by an irregularity of the screw, and indeed the observations strongly indicate the existence of such an irregularity, the sum of the readings being larger for great distances than for small ones.

The observations given below contain the following quantities: the readings of the micrometer screw for both positions of the semi-lenses, their means, and the uncorrected results. R.C. indicates that the measures were made by Dr. Copeland.

The corrections for refraction, phase, &c., are applied in the summary of the observations:—

1885, February 16.—Diameters of Jupiter Measured by the Method of Double Distances.

Equatorial Diameter. Observer: R. C.

	13^r	215^p	27^r	31^p
		180	26	485
		222	27	26
Mean	13	206	27	14

Difference - - - - - $13^r{\cdot}616 = 88''{\cdot}50$
Equatorial diameter - - - - - $= 44''{\cdot}25$

Polar Diameter. Observer: R. C.

	13^r	363^p	26^r	315^p
		355		318
		399		307
Mean	13	372	26	313

Difference - - - - - $12^r{\cdot}882 = 83''{\cdot}73$
Polar diameter - - - - - - $= 41''{\cdot}87$

Polar Diameter. Observer: J. G. L.

	13^r	363^p	26^r	299^p
		378		317
		376		314
Mean	13	372	26	310

Difference - - - - - $12^r{\cdot}876 = 83''{\cdot}69$
Polar diameter - - - - - - $= 41''{\cdot}85$

Equatorial Diameter. Observer: J. G. L.

	13^r	155^p	27^r	42^p
		150		2
		139		22
Mean	13	148	27	32

Difference - - - - - $13^r{\cdot}768 = 89''{\cdot}49$
Equatorial diameter - - - - - $= 44''{\cdot}75$

γ Leonis Measured by the Method of Fourfold Distances by R. C.

	19^r	5^p	21^r	145^p
		13		127
Mean	19	9	21	136

Difference - - - - - $2^r{\cdot}254 = 14''{\cdot}65$
Distance - - - - - - - $= 3''{\cdot}66$

The measures of Jupiter are the first observations made with the new instrument. Observing with a double image micrometer was then new to me, but when I had become quite familiar with it, I felt sure that the setting on this day had not been effected in the right manner—the limbs of the two disks had not been brought close enough together. When the definition is bad the setting is mainly a matter of good judgment, but with good definition a very distinct and sensitive phenomenon is noticed, which leaves no doubt on the observer's mind about the setting. When the limbs are in proper

contact, one or more bright points or beads are seen between them periodically, indicating a temporary overlapping of the limbs at that point, brought about by small atmospheric disturbances. If the limbs are brought closer together the beads become brighter and more persistent, and show a tendency to coalesce and to form a bright line. The setting has been considered right when the alternate absence and presence of the bright points is of equal duration. Under these conditions the limbs may be said to be in the true osculating position.

1885, March 10.—Measures of Castor by the Methods of Fourfold and of Single Distances by J. G. L.

	18^r	196^p	21^r	425^p	Difference - - - - $3^r \cdot 410 = 22'' \cdot 16$
		222		424	
		234		417	Distance - - - - - - = $5'' \cdot 54$
		210		414	
Mean	18	215	21	420	

	19^r	343^p	20^r	290^p	Difference - - - - $0^r \cdot 904 = 5'' \cdot 88$
		368		296	
		335		301	Distance - - - - - - = $5'' \cdot 88$
		334		300	
					Weight = $\frac{1}{4}$
Mean	19	345	20	297	

Measures o Polaris by the Method of Fourfold Distances by J. G. L.

	14^r	308^p	26^r	36^p	Difference - - - - $11^r \cdot 454 = 74'' \cdot 46$
		238	25	438	
		161		438	Distance - - - - - - = $18'' \cdot 61$
		227		433	
Mean	14	234	25	461	

	14^r	231^p	25^r	487^p	Difference - - - - $11^r \cdot 576 = 75'' \cdot 24$
		181		500	Distance - - - - - - = $18'' \cdot 81$
Mean	14	206	25	494	

The images were very unsteady.

1885, March 17.—Measures of Saturn by the Method of Double Distances. Observer: J. G. L.

	Major Axis of Ring.				
	13^r	426^p	26^r	236^p	Difference - - - - $12^r \cdot 652 = 82'' \cdot 24$
		408		233	
		433		257	Major axis of ring - - - - = $41'' \cdot 12$
		400		245	
Mean	13	417	26	243	

Equatorial Diameter of Globe.					
	17r	242p	22r	375p	Difference - - - - - 5r·344 = 34″·74
		225		412	
		229		417	Equatorial diameter - - - - - = 17″·37
		234		410	
Mean	17	232	22	404	

Definition very poor.

Measures of Jupiter by the Method of Double Distances. Observer: J. G. L.

Equatorial Diameter.					
	13r	280p	26r	353p	Difference - - - - - 13r·164 = 85″·57
		273		364	
		283		379	Equatorial diameter - - - - - = 42″·78
		301		366	
Mean	13	284	26	366	

Polar Diameter.					Hour angle, 1h. 50m. W.
	13r	475p	26r	180p	Difference - - - - - 12r·432 = 80″·81
		466		185	
		467		183	Polar diameter - - - - - - = 40″·40
		467		191	
Mean	13	469	26	185	

Polar Diameter.					Hour angle, 2h. 8m. W.
	13r	497p	26r	192p	Difference - - - - - 12r·372 = 80″·42
		499		171	
		485		174	Polar diameter - - - - - - = 40″·21
		500		186	
Mean	13	495	26	181	

Equatorial Diameter.					Hour angle at end, 2h. 27m. W.
	13r	305p	26r	349p	Difference - - - - - 13r·166 = 85″·58
		307		396	
		289		387	Equatorial diameter - - - - - = 42″·79
		303		405	
Mean	13	301	26	384	

1885, March 30.—Measures of Jupiter by the Method of Double Distances.
Observer: J. G. L.

Equatorial Diameter.					Hour angle, 2h. 17m. to 2h. 25m. W.
	13r	383p	26r	274p	Difference - - - - - 12r·790 = 83″·14
		392		246	
		400		293	Equatorial diameter - - - - - = 41″·57
		351		295	
Mean	13	382	26	277	

Polar Diameter.					Hour angle at end, 2h. 35m. W.
	14r	30p	26r	182p	Difference - - - - - 12r·172 = 79″·12
		33		117	
		29		123	Polar diameter - - - - - - = 39″·56
		12		98	
Mean	14	26	26	112	

Definition poor.

It had been noticed that the slide regulating the separation of the semi-lenses was a little loose; this defect was remedied in the meantime by applying a clamp screw to the slide.

1885, April 1.—Measures of Saturn by the Method of Double Distances.
Observer: R. C.

MAJOR AXIS OF RING.

	14^r	29^p	26^r	182^p
		31		212
		48		203
		49		192
Mean	14	39	26	197

Difference - - - - - $12^r{\cdot}316 = 80''{\cdot}05$

Major axis of ring - - - - - $= 40''{\cdot}03$

EQUATORIAL DIAMETER OF GLOBE.

	17^r	283^p	22^r	388^p
		287		387
		305		406
		313		407
Mean	17	297	22	397

Difference - - - - - $5^r{\cdot}200 = 33''{\cdot}80$

Equatorial diameter - - - - - $= 16''{\cdot}90$

MINOR AXIS OF RING.

	17^r	140^p	22^r	458^p
		134		473
		126		443
		144		451
Mean	17	136	22	456

Difference - - - - - $5^r{\cdot}640 = 36''{\cdot}66$

Minor axis of ring - - - - - $= 18''{\cdot}33$

Definition very poor. It began to blow hard from the south-west.

1885, April 2.—Measures of Saturn by the Method of Double Distances.
Observer: J. G. L.

MINOR AXIS OF RING.

	17^r	119^p	22^r	470^p
		85		470
		90		453
		128		469
		116		
		110		
Mean	17	108	22	466

Difference - - - - - $5^r{\cdot}716 = 37''{\cdot}15$

Minor axis of ring - - - - - $= 18''{\cdot}58$

The definition was so poor that no further measures were attempted.

1885, April 4.—Measures of Jupiter by the Method of Double Distances.

POLAR DIAMETER.

Observer: J. G. L.

	14^r	72^p	26^r	84^p
		81		70
		78		82
		56		81
Mean	14	72	26	79

Difference - - - - - $12^r{\cdot}014 = 78''{\cdot}09$

Polar diameter - - - - - $= 39''{\cdot}05$

EQUATORIAL DIAMETER.

	13^r	423^p	26^r	253^p
		411		251
		438		239
		424		247
Mean	13	424	26	248

Observer: J. G. L.

Difference - - - - - $12^r{\cdot}648 = 82''{\cdot}21$

Equatorial diameter - - - - - $= 41''{\cdot}11$

EQUATORIAL DIAMETER.

	13^r	422^p	26^r	212^p
		428		188
		433		215
		434		233
Mean	13	429	26	212

Observer: R. C.

Difference - - - - - $12^r{\cdot}566 = 81''{\cdot}68$

Equatorial diameter - - - - - $= 40''{\cdot}84$

POLAR DIAMETER.

	14^r	130^p	26^r	35^p
		131		58
		89		88
		88		40
				74
Mean	14	110	26	59

Observer: R. C. Hour angle, 2h. 20m. W.

Difference - - - - - $11^r{\cdot}898 = 77''{\cdot}34$

Polar diameter - - - - - - $= 38''{\cdot}67$

Definition good. There is more movement of the limbs for the polar diameter measures, the limbs being almost vertical above each other.

Measures of γ Virginis by the Fourfold Method of Double Distances.

Power, 450.

	Position.					
	154° 48′					
	153 28	18^r	247^p	21^r	418^p
	156 30		267		421
	153 40		288		402
	153 54		277		415
Mean	154 28	18	270	21	414

Observer: R. C.

Difference - - - - - $3^r{\cdot}288 = 21''{\cdot}37$

Distance - - - - - - - $= 5''{\cdot}34$

Position - - - - - - - $= 154^\circ{\cdot}5$

Power, 792.

	18^r	270^p	21^r	399^p
		271		395
		257		396
		284		392
Mean	18	270	21	396

Observer: J. G. L.

Difference - - - - - $3^r{\cdot}252 = 21''{\cdot}14$

Distan - - - - - - - $= 5''{\cdot}29$

1885, April 7.—Measures of Saturn, Jupiter's Satellites I. and II., and Uranus by the Method of Double Distances.

SATURN.

MAJOR AXIS OF RING.

	14^r	80^p	26^r	177^p
		67		168
		47		154
		61		141
Mean	14	64	26	160

Power, 450. Observer: J. G. L.

Hour angle, 3h. 52m. W.

Difference - - - - - $12^r{\cdot}192 = 79''{\cdot}25$

Major axis of ring - - - - - $= 39''{\cdot}62$

MIDDLE OF CASSINI'S DIVISION.

	14^r	460^p	25^r	283^p
		453		274
		441		296
		428		271
Mean	14	446	25	281

Hour angle, 4h. 3m. W.

Difference - - - - - $10^r{\cdot}670 = 69''{\cdot}36$

Cassini's division - - - - - - $= 34''{\cdot}68$

JUPITER'S SATELLITE I.

DIAMETER PARALLEL TO JUPITER'S BELTS. Power, 792. Observer: J. G. L.

	20^r	14^p	20^r	136^p
	19	486		143
		496		162
		498		141
Mean	19	499	20	146

Difference - - - - - $0^r{\cdot}294 = 1''{\cdot}91$

"Equatorial" diameter - - - - $= 0''{\cdot}96$

DIAMETER PARALLEL TO JUPITER'S AXIS.

	19^r	491^p	20^r	150^p
		480		136
		493		146
		499		147
Mean	19	491	20	145

Difference - - - - - $0^r{\cdot}308 = 2''{\cdot}00$

"Polar" diameter - - - - - $= 1''{\cdot}00$

JUPITER'S SATELLITE II.

DIAMETER PARALLEL TO JUPITER'S AXIS. Power, 792. Observer: J. G. L.

	20^r	1^p	20^r	162^p
	19	491		149
	20	4		131
	19	482		135
Mean	19	494	20	144

Difference - - - - - $0^r{\cdot}300 = 1''{\cdot}95$

"Polar" diameter - - - - - $= 0''{\cdot}98$

URANUS.

VERTICAL DIAMETER. Power, 450. Observer: R. C.

	19^r	266^p	20^r	345^p
		261		355
		261		339
		265		351
Mean	19	263	20	348

Difference - - - - - $1^r{\cdot}170 = 7''{\cdot}60$

Vertical diameter - - - - - $= 3''{\cdot}80$

VERTICAL DIAMETER. Power, 792. Observer: R. C.

(With the high power the image is apparently much worse.)

	19^r	290^p	20^r	336^p
		291		348
		267		345
		280		346
Mean	19	282	20	344

Difference - - - - - $1^r{\cdot}124 = 7''{\cdot}31$

Vertical diameter - - - - - $= 3''{\cdot}65$

HORIZONTAL DIAMETER. Power, 792. Observer: R. C.

	19^r	279^p	20^r	355^p
		297		363
		291		339
		281		345
Mean	19	287	20	350

Difference - - - - - $1^r{\cdot}126 = 7''{\cdot}32$

Horizontal diameter - - - - - $= 3''{\cdot}66$

Measure of η Coronæ by the Method of Fourfold Distances.

Magnifying power, 1530. Observer: R. C.

	19^r	476^p	20^r	160^p
		492		161
		485		170
		488		158
Mean	19	485	20	162

Difference - - - - - $0^r{\cdot}354 = 2''{\cdot}30$

Distance - - - - - - - $= 0''{\cdot}575$

1885, April 13.—Measures of Jupiter's Satellites III. and IV. and of Uranus by the Method of Double Distances.

JUPITER'S SATELLITE III.

Power, 792. Observer: R. C.

DIAMETER PARALLEL TO JUPITER'S BELTS. Hour angle, 2h. 27m. W.

	19^r	457^p	20^r	179^p
		483		173
		473		185
		475		187
Mean	19	472	20	181

Difference - - - - - $0^r\cdot418 = 2''\cdot72$

"Equatorial" diameter - - - - $= 1''\cdot36$

DIAMETER PARALLEL TO JUPITER'S BELTS. Hour angle, 2h. 53m. W.

(Astrometer turned through 180°.) Observer: R. C.

	19^r	451^p	20^r	179^p
		465		187
		485		183
		473		197
Mean	19	468	20	186

Difference - - - - - $0^r\cdot436 = 2''\cdot83$

"Equatorial" diameter - - - - $= 1''\cdot42$

JUPITER'S SATELLITE IV.

Power, 792. Observer: R. C.

DIAMETER PARALLEL TO JUPITER'S BELTS. Hour angle, 3h. 9m. W.

	19^r	478^p	20^r	190^p
		489		186
		479		189
		486		178
Mean	19	483	20	186

Difference - - - - - $0^r\cdot406 = 2''\cdot64$

"Equatorial" diameter - - - - $= 1''\cdot32$

(Astrometer turned through 180°.)

DIAMETER PARALLEL TO JUPITER'S BELTS. Hour angle, 3h. 29m. W.

	19^r	472^p	20^r	172^p
		478		172
		470		179
		485		172
Mean	19	476	20	174

Difference - - - - - $0^r\cdot396 = 2''\cdot57$

"Equatorial" diameter - - - - $= 1''\cdot29$

Definition fairly good.

URANUS.

Power, 792. Observer: J. G. L.

HORIZONTAL DIAMETER. Hour angle, 1h. 52m. W.

	19^r	303^p	20^r	359^p
		290		356
		288		351
		281		369
Mean	19	290	20	359

Difference - - - - - $1^r\cdot138 = 7''\cdot40$

Horizontal diameter - - - - - $= 3''\cdot70$

Definition rather poor.

VERTICAL DIAMETER. Hour angle, 2h. 15m. W.

	19^r	248^p	20^r	381^p
		275		351
		268		389
		257		381
Mean	19	262	20	376

Difference - - - - - $1^r\cdot228 = 7''\cdot98$

Vertical diameter - - - - - $= 3''\cdot99$

1885, April 14.—Measures of Jupiter's Satellites, Jupiter, and Uranus by the Method of Double Distances.

JUPITER'S SATELLITE III.

"Polar" Diameter.

	19^r	493^p	20^r	174^p
		457		190
		481		176
		496		172
Mean	19	482	20	178

Power, 792. Observer: R. C.

Difference - - - - $0^r{\cdot}392 = 2''{\cdot}55$

"Polar" diameter - - - - - = $1''{\cdot}27$

It was found that the separation slide wanted adjustment; this measure must, therefore, be rejected.

"Polar" Diameter.

	19^r	443^p	20^r	180^p
		490		171
		461		199
		463		181
Mean	19	464	20	183

Power, 792. Observer: R. C.

Difference - - - - $0^r{\cdot}438 = 2''{\cdot}85$

"Polar" diameter - - - - - = $1''{\cdot}42$

SATELLITE I.

"Polar" Diameter.

	19^r	495^p	20^r	136^p
	20	21		154
		20		145
		10		140
Mean	20	12	20	144

Power, 792. Observer: R. C.

Difference - - - - $0^r{\cdot}264 = 1''{\cdot}72$

"Polar" diameter - - - - - = $0''{\cdot}86$

SATELLITE II.

"Polar" Diameter.

	20^r	7^p	20^r	159^p
		26		120
		20		146
		15		139
Mean	20	17	20	141

Power, 792. Observer: J. G. L.

Difference - - - - $0^r{\cdot}248 = 1''{\cdot}61$

"Polar" diameter - - - - - = $0''{\cdot}81$

SATELLITE IV.

"Polar" Diameter.

	19^r	490^p	20^r	201^p
		484		177
		477		175
		480		181
Mean	19	483	20	184

Power, 792. Observer: J. G. L.

Difference - - - - $0^r{\cdot}402 = 2''{\cdot}61$

"Polar" diameter - - - - - = $1''{\cdot}31$

JUPITER.

Polar Diameter.

	14^r	131^p	25^r	492^p
		149	26	22
		119		15
		129		14
Mean	14	132	26	11

Power, 792. Hour angle, 1h. 10m. W. Obs.: J. G. L.

Difference - - - - $11^r{\cdot}758 = 76''{\cdot}43$

Polar diameter - - - - - - = $38''{\cdot}21$

Equatorial Diameter.

	13^r	471^p	26^r	157^p
		463		160
		489		164
		482		158
Mean	13	476	26	160

Hour angle, 1h. 19m. W. Obs.: J. G. L.

Difference - - - - $12^r\cdot368 = 80''\cdot39$

Equatorial diameter - - - - = $40''\cdot20$

Images good.

URANUS.

Horizontal Diameter.

	19^r	282^p	20^r	345^p
		281		335
		289		363
		303		338
Mean	19	289	20	345

Power, 792. Hour angle, 0h. 24m. W. Obs.: R. C.

Difference - - - - $1^r\cdot112 = 7''\cdot23$

Horizontal diameter - - - - = $3''\cdot61$

(Astrometer turned through 180°.)

Horizontal Diameter.

	19^r	300^p	20^r	319^p
		299		356
		304		335
		285		348
Mean	19	297	20	339

Hour angle, 0h. 32m. W. Observer: R. C.

Difference - - - - $1^r\cdot084 = 7''\cdot05$

Horizontal diameter - - - - = $3''\cdot52$

Vertical Diameter.

	19^r	282^p	20^r	373^p
		299		361
		292		349
		289		342
Mean	19	291	20	356

Hour angle, 0h. 45m. W. Observer: R. C.

Difference - - - - $1^r\cdot130 = 7''\cdot34$

Vertical diameter - - - - = $3''\cdot67$

(Astrometer turned through 180°.)

Vertical Diameter.

	19^r	274^p	20^r	343^p
		277		349
		291		346
		281		345
Mean	19	281	20	346

Hour angle, 0h. 58m. W. Observer: R. C.

Difference - - - - $1^r\cdot130 = 7''\cdot34$

Vertical diameter - - - - = $3''\cdot67$

Vertical Diameter.

	19^r	262^p	20^r	395^p
		245		385
		245		388
		277		393
Mean	19	257	20	390

Power, 792. Hour angle, 1h. 50m. W. Obs.: J.G.L

Difference - - - - $1^r\cdot266 = 8''\cdot23$

Vertical diameter - - - - = $4''\cdot11$

Images not very good.

SUMMARY OF OBSERVATIONS.

Deduction of Final Values.

The final results were deduced by correcting the observations of the planets for refraction and phase and reducing them to the mean distance.

The formulæ employed are those given by Bessel, but Mr. Marth's paper in the *Monthly Notices*, vol. xl., pp. 490-7, was also consulted.

Observed equatorial diameter corrected for refraction $= 2a\,(1 - \sin^2 \frac{d}{2} \sin^2 w)$.

Correction for phase $= + 2a \sin^2 \frac{d}{2} \sin^2 w$.

Observed equatorial diameter corrected for phase $= 2a$.

Equatorial diameter reduced to the mean distance $\Delta_0 = 2a \frac{\Delta}{\Delta_0}$, Δ being the distance at the time of observation.

Observed polar diameter corrected for refraction $= 2a \cos \epsilon \;(1 - \sin^2 \frac{d}{2} \cos^2 w)$.

Correction for phase $= + 2a \cos \epsilon \sin^2 \frac{d}{2} \cos^2 w$.

Observed polar diameter corrected for phase $= 2a \cos \epsilon$.

Polar diameter reduced to the mean distance Δ_0, and corrected for the latitude B of the earth above the planet's equator $= 2a \cos \epsilon \; \frac{\Delta}{\Delta_0} \cdot \frac{\cos \epsilon_0}{\cos \epsilon}$

$$= 2b \; \frac{\Delta}{\Delta_0},$$

Where $\sin \epsilon = \sin \epsilon_0 \cos B$, and $\cos \epsilon_0 = \frac{b}{a}$.

The correction for phase is insensible for Uranus and the polar diameter of Jupiter.

The logarithms of the mean distance were taken from the *Berliner Jahrbuch*, and are :—

Jupiter - - - - - -	$\log a = 0{\cdot}7162168$*	Le Verrier.
Saturn - - - - - -	$\log a = 0{\cdot}9802194$	Le Verrier.
Uranus - - - - - -	$\log a = 1{\cdot}2837100$	Newcomb.

The logarithms of the distance at the time of observation were interpolated from the Nautical Almanac. In the case of Jupiter's satellites, the tabular quantities for the planet were reduced to the true distances of the satellites from the earth. The effect on the final quantities is, however, almost imperceptible.

The following table contains the measured quantities, the corrections for refraction and phase, the reduction to mean distance, and the final results :—

* It will be seen further on that Engelmann used a mean distance of 5·2028 (log = 0·7162371); the effect of the difference on the diameters of Jupiter is barely 0″·002.

[TABLE.

Jupiter.

Equatorial Diameter.

Date.	Measured Diameter.	Refraction.	Phase.	Log. Reduction to Mean Dist.	Corrd. Diameter at Mean Dist.	Observer.
1885.	"		"		"	
Feb. 16	44·25	..	..	9·92703	37·41	R. C.
" "	[44·75	..	..	9·92703	37·83]	J. G. L.
March 17	42·78	..	+·08	9·93766	37·13	"
" "	42·79	..	+·08	9·93771	37·14	"
" 30	41·57	..	+·15	9·94942	37·13	"
April 4	41·11	..	+·18	9·95474	37·20	"
" "	40·84	..	+·18	9·95474	36·96	R. C.
" 14	40·20	..	+·24	9·96649	37·44	J. G. L.
				Mean,	37·20 ±·04	

Polar Diameter.

Date.	Measured Diameter.	Refraction.	Phase.	Log. $\frac{\Delta}{\Delta_0} \frac{\cos \epsilon_0}{\cos \epsilon}$	Corrd. Diameter at Mean Dist.	Observer.
1885.	"	"			"	
Feb. 16	41·87	+·02	..	9·92702	35·41	R. C.
" "	[41·85	+·02	..	9·92702	35·39]	J. G. L.
March 17	40·40	+·02	..	9·93768	35·02	"
" "	40·21	+·02	..	9·93769	34·85	"
" 30	39·56	+·02	..	9·94941	35·23	"
April 4	39·05	+·02	..	9·95473	35·20	"
" "	38·67	+·02	..	9·95473	34·86	R. C.
" 14	38·21	+·02	..	9·96648	35·39	J. G. L.
				Mean,	35·14 ±·06	

The next Table shows once more the two diameters of Jupiter, their difference, and the ellipticity resulting from each pair of measures.

[Table.

Date.	Major axis $2a$	Minor axis $2b$	$2(a-b)$	Ellipticity $\frac{a-b}{a}$	Observer.
1885.	$''$	$''$	$''$		
Feb. 16	37·41	35·41	2·00	$\frac{1}{18\cdot70}$	R. C.
„ „	[37·83	35·39	2·44	$\frac{1}{15\cdot50}$]	J. G. L.
March 17	37·13	35·02	2·11	$\frac{1}{17\cdot60}$	„
„ „	37·14	34·85	2·29	$\frac{1}{16\cdot22}$	„
„ 30	37·13	35·23	1·90	$\frac{1}{19\cdot54}$	„
April 4	37·20	35·20	2·00	$\frac{1}{18\cdot60}$	„
„ „	36·96	34·86	2·10	$\frac{1}{17\cdot60}$	R. C.
„ 14	37·44	35·39	2·05	$\frac{1}{18\cdot26}$	J. G. L.
Means,	37·20 $\pm$·04	35·14 $\pm$·06	2·06 $\pm$·03	$\frac{1}{18\cdot07\pm\cdot27}$	

The probable errors of the mean values of $2(a-b)$ and $\frac{a}{a-b}$, deduced from the probable errors of $2a$ and $2b$, would be about $\pm 0''\cdot07$ and $\pm 0''\cdot61$. The actual quantities, however, are less than one-half of these, and indicate that the determination of the absolute values of $2a$ and $2b$ has suffered from the bad definition on some nights and probably from other causes far more than the determination of their relative value. If the equatorial diameter was measured too large, the tendency has been to measure also the polar diameter too large, and *vice versâ*.

The ellipticity, it may be remarked, is not affected by any uncertainty which may attach to the screw-value of the astrometer.

[TABLE.

THE SATELLITES OF JUPITER.

Date.	Measured Diameter.	Refraction.	Phase.	Log. Reduction to Mean Dist.	Corr^d. Diameter at Mean Dist.	Observer.
			SATELLITE I. EQUATORIAL DIAMETER.			
1885. April 7	″ 0·96	..	″ +·01	9·95797	″ 0·88	J. G. L.
			SATELLITE I. POLAR DIAMETER.			
April 7	″ 1·00	..	..	9·95797	″ 0·91	J. G. L.
„ 14	0·86	..	..	9·96613	0·80	R. C.
			SATELLITE II. POLAR DIAMETER.			
April 7	″ 0·98	..	..	9·95804	″ 0·89	J. G. L.
„ 14	0·81	..	..	9·96617	0·75	„
			SATELLITE III. EQUATORIAL DIAMETER.			
April 13	″ 1·36	..	″ +·01	9·96569	″ 1·27	R. C.
„ „	1·42	..	+·01	9·96569	1·32	„
			SATELLITE III. POLAR DIAMETER.			
April 14	″ 1·42	..	..	9·96702	″ 1·32	R. C.
			SATELLITE IV. EQUATORIAL DIAMETER.			
April 13	″ 1·32	..	″ +·01	9·96563	″ 1·23	R. C.
„ „	1·29	..	+·01	9·96563	1·20	„
			SATELLITE IV. POLAR DIAMETER.			
April 14	″ 1·31	..	..	9·96630	″ 1·21	J. G. L.

The diameter parallel to the belts of Jupiter is called the "equatorial" diameter although the positions of the equators of the satellites are unknown.

SATURN.

Date.	Measured Diameter.	Refraction.	Phase.	Log. Reduction to Mean Dist.	Corrd. Diameter at Mean Dist.	Observer.
			MAJOR AXIS OF RING.			
1885.	"				"	
March 17	41·12	..	..	9·98060	39·32	J. G. L.
April 1	40·03	..	..	9·99182	39·28	R. C.
" 7	39·62	..	..	9·99594	39·25	J. G. L.
				Mean,	39·28 ±·01	
			MINOR AXIS OF RING.			
	"				"	
April 1	18·33	+·01	..	9·99182	18·00	R. C.
" 2	18·58	+·01	..	9·99252	18·27 *	J. G. L.
				Mean,	18·09	
			MAJOR AXIS OF MIDDLE OF CASSINI'S DIVISION.			
	"				"	
April 7	34·68	..	..	9·99594	34·36	J. G. L.
			EQUATORIAL DIAMETER OF GLOBE.			
	"				"	
March 17	17·37	..	+·05	9·98060	16·66	J. G. L.
April 1	16·90	..	+·04	9·99182	16·62	R. C.
				Mean,	16·64 ±·01	

* The observation on April 2 has received half weight on account of the poor definition.

On April 1–2, therefore, by the above observations the elevation of the earth above the plane of the ring was 27° 25′·3; whereas the elevation according to Bessel as given in the Nautical Almanac is 27° 2′·4.

[TABLE.

Uranus.

Date.	Measured Diameter.	Refraction.	Phase.	Log. Reduction to Mean Dist.	Corrd. Diameter at Mean Dist.	Observer.
			Horizontal Diameter.			
1885.	"				"	
April 7	3·66	..	..	9·95560	3·30	R. C.
„ 13	3·70	..	..	9·95650	3·31	J. G. L.
„ 14	3·61	..	..	9·95667	3·27	R. C.
„ „	3·52	..	..	9·95667	3·19	„
				Mean,	3·27 ±·02	
			Vertical Diameter.			
	"				"	
April 7	3·80	..	..	9·95560	3·43	R. C.
„ „	3·65	..	..	9·95560	3·30	„
„ 13	3·99	..	..	9·95650	3·61	J. G. L.
„ 14	3·67	..	..	9·95667	3·32	R. C.
„ „	3·67	..	..	9·95667	3·32	„
„ „	4·11	..	..	9·95667	3·72	J. G. L.
				Mean,	3·45 ±·05	

Measures of Double Stars.

Date.	Star.	Position.	Distance.	Magn. Power.	Observer.
1885.			"		
Feb. 16	γ Leonis	..	3·66	450	R. C.
March 10	Castor	..	5·54	450	J. G. L.
„ „	„	..	5·88; ¼ wt.	450	„
„ „	Polaris	..	18·68	450	„
April 4	γ Virginis	154°·5	5·32	450	R. C.
„ „	„	..	5·29	792	J. G. L.
„ 7	η Coronæ	..	0·575	1530	R. C.

The number of the Dun Echt observations being small, an elaborate comparison with the results of other observers would be out of place. Instead of that a summary of the more reliable results, grouped according to the kind of micrometer used, is given, which may prove useful. Most of the values are taken from Houzeau's *Vade-mecum de l'Astronomie;* the others are from Engelmann and other sources. Engelmann, in his paper "Ueber die Helligkeitsverhältnisse der Jupiterstrabanten," has computed the mean errors for a number of the results, weighted them, and deduced mean values. To Engelmann's table a column has been added containing the ellipticity of Jupiter.

Diameters of Jupiter.

Results collected by Engelmann.

Observer.	Days of Obs.	Equatorial Diameter.	Mean Error.	Weight.	Polar Diameter	Mean Error.	Weight.	Ellipticity.	Instrument and Power.
		″	″		″	″			
Bessel.	12	37·60	±0·056	10·7	35·21	±0·035	17·1	$\frac{1}{15\cdot73}$	6-in. Heliometer (300)
Mädler	10	38·26	±0·116	1.7	36·41	±0·121	1·7	$\frac{1}{20\cdot68}$	3-in. Heliometer 200?
Beer	5	38·18	±0·224	0·9	36·16	±0·286	0·7	$\frac{1}{18\cdot90}$	
Johnson	13	37·38	±0·164	1·8	35·13	±0·137	2·2	$\frac{1}{16\cdot64}$ *	7-in. Heliometer 300
Main	10	37·31	±0·083	3·6	35·03	±0·056	5·4	$\frac{1}{16\cdot36}$	7-in. Heliometer 300
Main	25	37·91	±0·092	2·7	35·66	±0·092	2·7	$\frac{1}{16\cdot84}$	Double Image Microm. (250)
Kaiser	10	37·61	±0·034	9·6	35·16	±0·033	9·9	$\frac{1}{15\cdot35}$	Airy's D.I. Microm. 326
Kaiser	10	37·48	±0·033	6·7	35·14	±0·028	7·8	$\frac{1}{16\cdot02}$	Airy's D.I. Microm. 220
Mean,	..	37·609	±0·082	..	35·236	±0·107	..	$\frac{1}{15\cdot82}$	
									Filar Micrometer on
W. Struve	6	38·33	±0·081	13·3	35·54	±0·081	13·3	$\frac{1}{13\cdot71}$	9·in. Refractor 540
Secchi.	9·8	38·35	±0·059	16·9	35·96	±0·085	11·8	$\frac{1}{16\cdot06}$	9-in. Refractor 1000
J. Schmidt	10	38·91	±0·084	1·2	36·42	±0·183	0·5	$\frac{1}{15\cdot63}$	4-in. Refractor 100
Mädler	12	37·87	±0·148	3·4	35·21	±0·164	3·0	$\frac{1}{14\cdot22}$	9-in. Refractor (500)
Mean,	..	38·312	±0·104	..	35·694	±0·158	..	$\frac{1}{14\cdot63}$	

* Radcliffe Observations, Vol. XI., p. 273, read Ellipticity $= \frac{1}{16\cdot6}$ instead of $\frac{1}{15\cdot6}$.

Other Results, chiefly from Houzeau.

	Observer.	Equatorial Diameter.	Polar Diameter.	Ellipticity.	
		Observations with Double Refracting Micrometer.			
		"			
1824.	Arago	36·743	. .	$\frac{1}{17·7}$	
1842.	Laugier	38·01	. .	$\frac{1}{11·1}$	
		Observations with the Heliometer.			
		"			
1874.	Main and Bellamy . .	37·16	. .	$\frac{1}{17·93}$	
1875.	Bellamy	37·00	. .	$\frac{1}{17·91}$	
			"		
1891.	Schur	37·256 ±·032	34·882 ±·030	$\frac{1}{15·69\pm·280}$ *	Images horizontal.
"	"	37·599 ±·024	35·158 ±·022	$\frac{1}{15·40\pm·198}$	" vertical.
"	"	37·428	35·020	$\frac{1}{15·54}$	Final result.
		Observations with Airy's Double Image Micrometer.			
1859.	Lassell			$\frac{1}{17·852}$	
		Observations with Filar Micrometer.			
		"	"		
1857.	De la Rue	37·141	. .	$\frac{1}{18·62}$	
1873.	H. E. Vogel . . .	38·367	. .	. .	
1873.	O. Lohse and Vogel .	37·899	35·500	$\frac{1}{15·80}$	
1874.	Périgaud and Folain .	39·44	. .	. .	
1880.	Hough	38·704	36·319	$\frac{1}{16·23}$	Power, 638.
1880.	Colbert	38·316	36·030	$\frac{1}{16·73}$	Power, 638.
1880.	Hough and Colbert . .	39·764	37·388	$\frac{1}{16·76}$	Power, 389.

* By an oversight the probable error is given too small in Ast. Nachr. 3073: instead of $\frac{1}{15·69\pm0·001}$ and $\frac{1}{15·40\pm0·001}$ read $\frac{1}{15·69\pm·280}$ and $\frac{1}{15·40\pm·198}$.

The observations by Professor Schur demonstrate clearly the influence of the position of the images on the measures, the diameters obtained in the vertical position of the images being 0″·3 greater than those where the images were horizontal. The employment of a reversing prism in front of the eye-piece is therefore very important, not only for the determination of position angles, but also of distances.

Professors Hough and Colbert ascribe the large differences in the diameters, as measured with powers 638 and 389, to irradiation. The difference of 1″·56 in the actual measures, however, seems too large to be explained entirely by irradiation. It seems more likely that the eye-pieces employed differed in quality. To settle this point it would be necessary to make a series of measures with eye-pieces of various constructions.

Diameters of the Satellites of Jupiter.

Results collected by Engelmann.

Observer.	Days of Obs.	Diameter.	Mean Error.	Weight.	Instrument and Power.
			SATELLITE I.		
		″	″		
Schroeter	3	1·120	±0·043	2·4	7 and 14 ft. Reflector, 200 (about).
W. Struve	8	1·015	±0·052	1·9	9-in. Refractor, 540.
Secchi	1	0·985	(±0·088)	1·1	" " 1000.
Mädler	7	1·200	±0·111	0·9	" " (?)
Engelmann	4	1·050	±0·060	1·7	8-in. " 460.
Vogel	1	1·002	(±0·028)	1·5	11-in. " 620.
			SATELLITE II.		
		″	″		
Schroeter	4	0·883	±0·037	2·7	
W. Struve	8	0·911	±0·023	4·3	
Secchi	3	1·054	±0·039	2·6	
Mädler	10	1·132	±0·166	0·6	
Engelmann	4	0·971	±0·053	1·9	
Vogel	2	0·868	..	2·0	
			SATELLITE III.		
		″	″		
W. Struve	9	1·488	±0·031	3·2	
Secchi	6	1·609	±0·044	3·3	
Mädler	11	1·519	±0·111	0·9	
Engelmann	4	1·386	±0·088	1·1	
Vogel	2	1·420	..	2·0	
			SATELLITE IV.		
		″	″		
W. Struve	9	1·273	±0·044	2·3	
Secchi	2	1·496	(±0·062)	1·6	
Mädler	11	1·300	±0·083	1·2	
Engelmann	4	1·177	±0·063	1·6	
Vogel	1	1·163	(±0·028)	1·5	

From these Engelmann finds the following mean values reduced to the mean distance 5·2028.

			Mean Error for Weight 1.
Satellite I.	=1·059″±0·0296″	Weight 9·5	±0·091
Satellite II.	=0·944±0·0338	„ 14·1	±0·127
Satellite III.	=1·494±0·0340	„ 9·5	±0·105
Satellite IV.	=1·282±0·0525	„ 8·2	±0·150

Engelmann deduces, besides, the diameters of the satellites from the duration of ingress and egress observed by Schroeter, Herschel, Bond, and others—the results being—

Satellite I.	=237·2ˢ	±3·7ˢ	=1·089″	±0·0170″
Satellite II.	=247·5	±4·9	=0·900	±0·0178
Satellite III.	=549·6	(±7·9)	=1·579	(±0·023)
Satellite IV.	=505·5		=1·087	

Hough obtains from micrometric measures on three nights the following values :—

Sat. I.=1·114″ Sat. II.=0·980″ Sat. III.=1·778″ Sat. IV.=1·457″

Dimensions of Saturn and Rings, mainly from Houzeau.

	Observer.	Equatorial Diameter	Outer Diam. of A.	Cassini's Division. Outer Edge.	Middle.	Inner Edge.	Inner Diam. of B.	Division in C.	Inner Diam. of C.
	Observations with Double Refracting Micrometer.								
1847.	Laugier	17·698″	..	..	..	..	..	..	..
	Observations with the Heliometer.								
1831.	Bessel	17·0055″	..	..	..	..	..	..	..
1835.	Bessel	17·053	39·311″	35·289″	..	34·185″	26·671″	..	..
1848.	Main	20·05	..	..	..	..	..	..	..
1862.	Main	18·07	..	..	..	..	..	..	..
	Observations with Airy's Double Image Micrometer.								
1855.	Main	..	39·73″	..	[36·15″]	..	27·65″	..	..
1856.	De la Rue	17·66	39·83	35·33	..	33·45	26·91	..	..
1872.	Kaiser	17·274	39·471	..	..	..	27·859	..	..

DIMENSIONS OF SATURN AND RINGS, MAINLY FROM HOUZEAU—*continued.*

Observer.		Equatorial Diameter	Outer Diam. of A.	Cassini's Division. Outer Edge.	Cassini's Division. Middle.	Cassini's Division. Inner Edge.	Inner Diam. of B.	Division in C.	Inner Diam. of C.
	OBSERVATIONS WITH FILAR MICROMETER.								
		″	″	″	″	″	″	″	″
1811.	Bessel	17·4	38·2696	..	..	..	..	..	..
1829.	W. Struve	17·991	40·095	35·289	..	34·475	26·668	..	..
1834.	Hussey	18·487	..	..	..	..	..	..	..
1838.	De Cuppis and De Vico	16·9998	37·440	33·344	..	32·576	25·9185	..	..
1838.	Encke	17·677	40·929	35·690	..	34·140	26·146	..	..
1838-9.	Galle	17·976	41·059	..	34·317	..	26·458	..	22·191
1851-2.	O. Struve	17·61	40·12	35·52	..	34·53	25·29	23·57	21·22
1853.	Lassell	17·453	40·881	..	..	..	..	..	..
1856.	Jacob	17·969	40·061	35·887	..	34·905	26·313	..	22·260
1856.	Secchi	17·661	40·893	..	34·659	..	25·714	..	21·419
1857.	Secchi	17·689	40·661	..	34·635	..	..	..	..
1857.	Bond	16·84	39·35	34·75	..	33·84	25·81	..	21·25
1863.	Mädler	17·182	..	..	..	..	..	..	..
1875.	Périgaud and Folain	18·66	..	..	..	..	..	..	..
1880.	W. Meyer	17·40	40·43	..	..	..	26·30	..	21·15
1881.	W. Meyer	17·77	40·35	..	34·48	..	26·05	..	21·13
1884.	J. G. Lohse	17·53	40·38	..	34·51	..	25·12	..	21·28

[TABLE.

Diameters of Uranus.

Chiefly from Houzeau.

Observer.		Diameter.	Ellipticity.	
Observations with Double Refracting Micrometer.				
1814.	Arago	″ 4·284	..	
Observations with the Heliometer.				
1863.	Main	″ 2·87	..	
Observations with Airy's Double Image Micrometer.				
1864-5.	Lassell	″ 3·624	..	Power, 872.
1864-5.	Marth	3·517	..	„ „
1872.	Kaiser	3·62	..	
Observations with Amici's Double Image Micrometer.				
1878.	Doberck . . .	″ 3·615	..	Ast. Nach., xcii., p. 159. (Mean value reduced.)
Observations with Filar Micrometer.				
1788.	W. Herschel . . .	″ 3·906	..	
1842.	Mädler	4·27	$\frac{1}{10·85}$	
1843.	„	4·3274	$\frac{1}{9·92}$	
1845.	„	3·98	$\frac{1}{9·45}$	
1869.	H. C. Vogel . . .	3·624	..	

These measures fully exemplify the great uncertainty inherent in the various methods of measuring the dimensions of a luminous disk. Apart from the purely atmospheric and instrumental sources of error that affect these measures in common with those of double stars, we here encounter two phenomena that greatly hinder all attempts to secure a high degree of accuracy. These are Diffraction and Irradiation. Of course the former, in accordance with well understood principles, surrounds the luminous image in the telescope with a fading margin of light, followed by a series of luminous rings of rapidly decreasing brilliancy. The breadth of these

fringes is largely dependent on the size of the object-glass, but the way in which they are presented to our vision is greatly modified by the quality of the instrument and the state of the air. We may consider irradiation as that spreading of a vivid perception of light which causes us to see a bright object under a greater angle than it really subtends. It is scarcely necessary to mention the familiar instance of the "old moon in the new moon's arms," in which the bright part of our satellite seems to swell out beyond the circle filled by the less illuminated portion. The difficulties caused by the diffraction of the image are common to all classes of micrometer. But in the wire micrometer we have also the diffraction at the margins of the wires to contend with, added to the uncertainty attendant on placing the centre or edge of the relatively thick wire tangential to the luminous disk. If the edge of the wire is used, there is, of course, the further difficulty of making due allowance for the thickness of the web. An idea of the extent to which some of these sources of error affect the most careful measures may be obtained by an examination of the dimensions of Saturn and his ring system that have been arrived at by various observers. First of all, however, attention may be directed to one dimension of the Saturnian system that seems to be practically free from constant error—viz., the diameter of Cassini's division. A little consideration will show that the diameter of the centre line of this feature is obviously almost altogether free from the effects of irradiation and diffraction. Its measurement also is practically unaffected by the diffraction at the margins of the wires of a filar micrometer. The fact that the inner margin of the division is brighter than the outer margin may indeed have some influence on the apparent centre of the black interval, but the uncertainty arising from this source must be very slight. The perseverance and skill of the observers seem also to have so completely overcome the inconvenience attendant on imperfect driving of the telescope that it seems quite permissible to give equal weight to all the results, although they have been obtained with a great variety of apparatus. Omitting, therefore, Main's value on the ground that it is but the modified exterior diameter of the outer ring, and taking the simple mean of the other 14 values, we obtain:—Diameter of the centre of Cassini's division = $34''\cdot55 \pm 0''\cdot10$, with $\pm 0''\cdot375$ as the probable error of one observer.

It is almost needless to add that the probable error might have been greatly lessened by weighting the observations in a perfectly warrantable manner. But the same end may be obtained by using only those results where the centre of the division has been directly measured. This limits the number of contributing results to six, viz.:—

1838–9.	Galle - - - -	Filar micrometer	34″·317.
1856.	Secchi - - - -	„	34″·659.
1857.	Secchi - - - -	„	34″·635.
1881.	W. Meyer - - -	„	34″·48.
1884.	J. G. Lohse - - -	„	34″·51.
1885.	J. G. Lohse - - -	Astrometer	34″·36.
		Mean	$34''\cdot494 \pm 0''\cdot039$.

The probable error for a single result is $\pm 0''\cdot 095$.

The smallness of this probable error shows that the mean diameter of Cassini's division can be found more certainly by direct measurement than by combining the direct diameters of its margins.

At first sight it seems rather strange to find that the inner diameter of the crape ring has also been measured by various observers with strikingly accordant results. Perhaps it is not going too far if we assume that this accordance is due to the absence of perceptible irradiation and diffraction at the edge of so faintly illuminated an object. In this case the determinations of eight observers, nearly all of whom had the advantage of instruments of considerable size, yield $21''\cdot 488 \pm 0''\cdot 109$, the probable error of a single result being as small as $\pm 0''\cdot 308$.

Still adopting the plan of giving equal weight to each observer, we have:—

		Probable Error of Single Results.
Outer diameter of A	$39''\cdot 923 \pm 0''\cdot 146$;	$\pm 0''\cdot 635$.
Inner diameter of A	$35''\cdot 137 \pm 0''\cdot 191$;	$\pm 0''\cdot 543$
Outer diameter of B	$34''\cdot 050 \pm 0''\cdot 180$;	$\pm 0''\cdot 509$.
Inner diameter of B	$26''\cdot 325 \pm 0''\cdot 134$;	$\pm 0''\cdot 519$.
Equat. diameter of globe	$17''\cdot 670 \pm 0''\cdot 093$;	$\pm 0''\cdot 465$.

The best values of the diameters of the outer planets and their satellites can only be obtained near opposition, when the illumination of the limbs is as nearly uniform as possible. Observations made at other times with a double image micrometer and also with a filar micrometer would show how the phase affects the measures. The observations would of course be corrected for phase according to Bessel's formulæ; but there are other difficulties connected with the defective limb, especially if the phase be considerable. The limb where defective is always fainter than where it is fully illuminated, and does not present the same well-defined outline, but fades away gradually. As there is then a considerable contrast between the two limbs which have to be brought into contact when using the double image micrometer, it is conceivable that the settings may be made involuntarily quite differently from what would be the case if the limbs were equally bright. The tendency probably would be to make the bright limb encroach on the faint one until the combined brightness became appreciable, the result being to make the measured diameter too small; this, however, would be counteracted, and perhaps more than outweighed, by the action of the diffraction and irradation, if the contrast between the two limbs is sufficiently great for the irradiation to become sensible. With the filar micrometer there is probably no such tendency to encroach on the faint limb, provided it is sufficiently illuminated to present a distinct edge and is not overpowered by the light of the sky or the bright illumination of the field.

Probably the best plan of studying the relation between the two instruments would be to mount a filar micrometer on an astrometer. The observations could then be made alternately and under practically the same optical conditions.

Measures with a double image micrometer and a filar micrometer (neglecting the diffraction produced by the wire) are equally affected by diffraction, which depends mainly on the aperture of the object-glass, but the influence of irradiation, which may be considered as a physiological phenomenon connected with the action of the retina, producing an apparent enlargement of a strongly illuminated object seen at the side of a dark one, is different in the two cases. In the filar micrometer the contrast between the dark wire and the bright limb favours the visibility of the irradiation fringe, while in the double image micrometer it disappears entirely at the point of contact if the limbs are of the same brightness. In all other cases, where there is a difference between the two limbs brought into contact, the irradiation fringe will at least be enfeebled, its effect being less prejudicial the more nearly the surfaces under comparison approach each other in point of brightness. The complete or partial elimination of the effect of irradiation is most probably one of the reasons why double image micrometers give smaller values for the diameters of planets than those obtained by filar micrometers.

Even if the limbs are of unequal brightness, it is possible to avoid the effect of irradiation by mounting a Zöllner reversing prism in front of one of the semi-lenses of the astrometer, which interchanges right and left in the image produced by that half of the lens. The limbs brought into contact are then the two images of the same limb, and are consequently equally bright. A complete measure would then be made by bringing into contact, first, the two bright limbs, and then the two faint limbs, the difference of the screw-readings giving the double diameter of the object, altogether free from irradiation. The results thus obtained ought to differ from the real quantities only by the amount of the diffraction fringe. Although theoretically correct, such an arrangement is unfortunately of no practical value. The measured double diameter of the object can only be the correct diameter if during the observations no change whatever takes place in the position of the instrument with regard to the object. The instrument must follow the object with absolute accuracy, for the least deviation from the original relative position between instrument and object affects the relative position of the two images in the field of view by double the amount of the deviation, as has been pointed out previously, the relation between the two images being the same as that of an object and its image in a looking-glass. An approach or recession of the object from the looking-glass, however, alters the apparent distance between the object and its image by double the amount. There being no easy means of controlling the position of the telescope with regard to the object, observations obtained with such an apparatus cannot inspire much confidence, and the mounting of a Zöllner reversing prism in front of one of the semi-lenses of a heliometer or astrometer cannot be considered a success as far as the measurement of distances is concerned, however promising it may look at first sight. The usefulness of the prism in measuring position angles has been discussed earlier on.

The foregoing considerations are based on the assumption that we are dealing with steady images, which can be brought into contact with great accuracy. (Compare notes to Observations pp. 174-182.) The case, however, is different when the atmospheric

conditions are unfavourable; the images then overlap at one moment, and immediately afterwards are separated by a dark interval. The settings then depend on rendering the central thickness of the double concave "lens" representing the dark interval equal to the central thickness of the double convex bright "lens" produced by the overlapping limbs. To what extent such measures are affected by irradiation is difficult to say, the setting depending too much on the judgment of the observer.

We have seen that it is possible, under favourable atmospheric conditions, to free the measures, if the limbs are equally bright, from the effect of irradiation by using a double image micrometer. The diffraction, however, cannot be eliminated by any arrangement of the apparatus, although its effect may be reduced by increasing the size of the object-glass, and also by placing a perforated screen in front of the object-glass, which diminishes the brilliancy of the object, while the advantages of the full aperture are preserved. We must, therefore, try to determine for each instrument the influence of the diffraction upon the measured diameters.

Measures of the diameters of Mars and Venus at various distances would probably afford the best means of obtaining a reliable value of the breadth of the diffraction fringe. The conditions are favourable. The apparent diameters vary considerably, while the effect of the diffraction may be treated as a constant, since in a given instrument the intrinsic brightness of each planet remains very nearly unchanged. Mars and Venus are besides practically spheres, and a diameter not affected by phase or ellipticity is always available for measurement.

$D, D', D'' \ldots =$ measured diameter of the planet.
$\Delta, \Delta', \Delta'' \ldots =$ corresponding distance of the planet from the Earth.
$D_\circ =$ diameter at unit of distance, free from diffraction and irradiation.
$d =$ double breadth of diffraction fringe,

then every observation would give an expression for $D_\circ$—

$$\begin{aligned} D_\circ &= (D - d)\,\Delta \\ &= (D' - d)\,\Delta' \\ &= (D'' - d)\,\Delta''. \end{aligned}$$

Putting $D_\circ = x$, and $d = y$, the equations assume the general form:—

$$\begin{aligned} x + \Delta y - D\,\Delta &= o \\ x + \Delta' y - D'\,\Delta' &= o \\ x + \Delta'' y - D''\Delta'' &= o. \end{aligned}$$

Provided only that the distance varies sufficiently, these equations yield the true diameter of the planet and the breadth of the fringe, together with their probable errors.

The corrections for diffraction thus arrived at would indeed strictly apply only to the planets observed and to objects of equal brightness measured with the same instrument, but their determination in a few special cases would not fail to throw some light on this most difficult subject.

If the observations are made with a filar micrometer, the combined effect of diffraction and irradiation would be obtained, instead of the diffraction alone. The quantity y would besides contain errors common to all the observations.